考試科目	微積分	系所別	國貿系、金融系、統計系、資管系 二年級	考試時間	7月6日(四)第四節

1. Consider the function $f : \mathcal{R} \longrightarrow \mathcal{R}$ defined by $f(x) = \frac{x-1}{\sqrt{x^2+1}}$. Note f is well defined as $x^2 + 1 > 0$ for all $x \in \mathcal{R}$.

   (a) (10pts). Please find $\lim_{x \to -\infty} f(x)$. Show your work.

   (b) (10pts). Please find $f'(x)$, the derivative of $f(x)$. Show your work.

   (c) (10pts). Please find the global maximum or minimum value of $f(x)$ for $x \in \mathcal{R}$. Show your work.

2. Suppose $f^2(x)$, $g^2(x)$, $f(x)g(x)$ are all integrable functions in the interval $[0, 1]$.

   (a) (10pts). Please prove the inequality

   $$\left(\int_0^1 f^2(x)\mathrm{d}x\right)\left(\int_0^1 g^2(x)\mathrm{d}x\right) \geq \left(\int_0^1 f(x)g(x)\mathrm{d}x\right)^2.$$

   (Hint: $(tf(x) + g(x))^2 \geq 0$ for all $x \in [0, 1]$ and $t \in \mathcal{R}$.)

   (b) (10pts). Please show that

   $$\int_0^1 \frac{1}{1+x^n}\mathrm{d}x \geq \frac{n+1}{n+2} \quad \text{for all } n > 0.$$

   (Hint: use (a).)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	統計學系、國貿系、資訊管理學系、金融系 二年級	考試時間	7 月 6 日(四) 第 四 節

3. (20 pts) Suppose that

$$f(x,y)=\begin{cases} x+2y, & \text{if } 2x+y\geq 0;\\ 3x+y, & \text{if } 2x+y<0. \end{cases}$$

(a) (5 pts) Determine whether f is continuous at $(1,-1)$. Justify your answer.

(b) (5 pts) Find $f_x(0,1)$.

(c) (5 pts) Let

$$S=\{(x,y): 0\leq x\leq 1 \text{ and } 2\leq y\leq 3\}.$$

Find $\int_S f(x,y)d(x,y)$.

(d) (5 pts) Let

$$S=\{(x,y): -3\leq 2x+y\leq 3 \text{ and } -3\leq x-y\leq 0\}.$$

Find $\int_S f(x,y)d(x,y)$.

4. (20 pts) Suppose that $f(x,y)$ is a differentiable function of (x,y) for $(x,y)\in\mathcal{R}^2$, and f_{xx}, f_{xy}, f_{yx} and f_{yy} are continuous on $\{(x,y): -2<x<2 \text{ and } -1<y<1\}$. Suppose that $f_x(1,0)=0=f_y(1,0)$, $f_{xx}(1,0)=4$, $f_{xy}(1,0)=f_{yx}(1,0)=3$, and $f_{yy}(1,0)=2$.

(a) (10 pts) Let $g(t)=f(e^{3t},\sin(t))$ for $t\in\mathcal{R}$. Does g have a relative minimum at 0? Justify your answer.

(b) (10 pts) Does f have a relative minimum at $(1,0)$? Justify your answer.

5. (10 pts) Let

$$S=\{(x,y): 0\leq x\leq y\leq 1\}.$$

Find the maximum of $f(x,y)=x^2-\frac{7y^2}{4}+xy-\ln(1+x)$ on S.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	統計學系二年級	考試時間	7 月 6 日(四) 第二節

1. A sample of 36 scooter (摩托車) riders showed a sample mean annual cost of scooter insurance to be $545. The population standard deviation is known to be $30.

(1) Develop a 90% confidence interval of the population mean. (10%)

(2) What is the recommended sample size if we want a margin of error of $5? Use 95% confidence. (6%)

2. A survey asked "Do you drive the same car you bought 5 years ago?" was conducted and obtained the following results.

	Brand A	Brand B
Sample size	100	200
Yes	45	120

(1) Develop a 95% confidence interval for the difference between the driving-the-same-car proportions of the two brands. (10%)

(2) At α=0.1, test whether brand A has a smaller driving-the-same-car proportion? (12%)

3. A big corporation is interested in the distribution of its employees' job-stress. A sample of 160 employees was chosen for clinical evaluation. Then, they were classified into one of 4 job-stress categories. The frequency table is:

Low	Low to moderate	Moderate to high	High
50	50	40	20

Test if the job-stress category is evenly distributed at α=0.05. (12%)

4. E-readiness refers to a country's capacity and state of preparedness to participate in the electronic world. It seems reasonable to expect that a country with a larger GDP would be more E-readiness. 10 countries were ranked from highest to lowest on their GDPs. Also available are their scores of E-readiness with larger score reflecting more readiness.

Country	A	B	C	D	E	F	G	H	I	J
GDP	1	2	3	4	5	6	7	8	9	10
E-readiness	8.24	8.41	8.49	8.4	8.38	8.21	8.36	8.22	8.07	8.23

(1) Compute the coefficient of rank correlation. (10%)

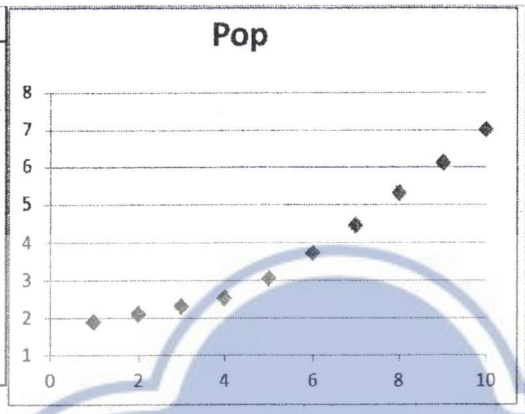
(2) At α=0.05, is there a positive relation between GDP and E-readiness? (10%)

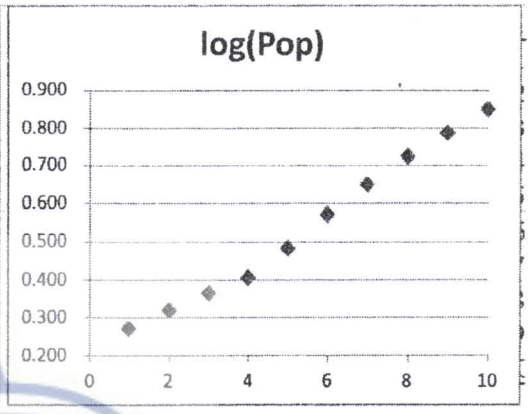
備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	統計學系 二年級	考試時間	7 月 6 日(四) 第二節

5. The table and plots show the world population size (in billion, i.e. 10^9) and log10 of population size from year 1920 to 2010.

Year	t	Pop	log(Pop)
1920	1	1.86	0.270
1930	2	2.07	0.316
1940	3	2.3	0.362
1950	4	2.52	0.401
1960	5	3.02	0.480
1970	6	3.69	0.567
1980	7	4.43	0.646
1990	8	5.26	0.721
2000	9	6.07	0.783
2010	10	7	0.845





Attached is the partial outputs using Pop and log(Pop) as the response variable, respectively.

Response variable: Pop
ANOVA

	自由度	SS
迴歸	1	27.6140
殘差	8	1.5680
總和	9	29.1820

	係數	標準誤
截距	0.6400	0.3024
t	0.5785	0.0487

Response variable: log(Pop)
ANOVA

	自由度	SS
迴歸	1	0.3713
殘差	8	0.0037
總和	9	0.3750

	係數	標準誤
截距	0.1702	0.0147
t	0.0671	0.0024

(a) Is it appropriate to fit a linear regression for world population size and t ? Why? (5%)

(b) Based on the plots and ANOVA tables, write down the appropriate estimated regression equation.
At α=0.05, test the significance of the slope. (10%)

(c) Estimate the world population size by year 2020. (5%)

(d) Establish 95% interval estimate for the population size by year 2020. (10%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	統計學系二年級	考試時間	7 月 6 日(四) 第二節

I. P(0< Z ≦ z) under Standard Normal distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

II. Upper critical values of Student's t distribution

df	Right-tail probability						
	0.2	0.1	0.05	0.025	0.01	0.005	0.001
1	1.3764	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088
2	1.0607	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271
3	0.9785	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145
4	0.9410	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732
5	0.9195	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934
6	0.9057	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076
7	0.8960	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853
8	0.8889	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008
9	0.8834	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968
10	0.8791	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437
11	0.8755	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247
12	0.8726	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296
13	0.8702	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520
14	0.8681	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874
15	0.8662	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328
16	0.8647	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862
17	0.8633	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458
18	0.8620	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105
19	0.8610	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794
20	0.8600	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518
21	0.8591	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272
22	0.8583	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050
23	0.8575	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850
24	0.8569	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668
25	0.8562	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502
26	0.8557	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350
27	0.8551	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210
28	0.8546	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082
29	0.8542	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962
30	0.8538	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852
31	0.8534	1.3095	1.6955	2.0395	2.4528	2.7440	3.3749
32	0.8530	1.3086	1.6939	2.0369	2.4487	2.7385	3.3653
33	0.8526	1.3077	1.6924	2.0345	2.4448	2.7333	3.3563
34	0.8523	1.3070	1.6909	2.0322	2.4411	2.7284	3.3479
35	0.8520	1.3062	1.6896	2.0301	2.4377	2.7238	3.3400
36	0.8517	1.3055	1.6883	2.0281	2.4345	2.7195	3.3326
37	0.8514	1.3049	1.6871	2.0262	2.4314	2.7154	3.3256
38	0.8512	1.3042	1.6860	2.0244	2.4286	2.7116	3.3190
39	0.8509	1.3036	1.6849	2.0227	2.4258	2.7079	3.3128
40	0.8507	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	統計學系 二年級	考試時間	7 月 6 日(四) 第二節

III. Upper critical values of chi-square distribution with ν d.f.

ν	Right-tail probability 0.10	0.05	0.025	0.01	0.001
1	2.706	3.841	5.024	6.635	10.828
2	4.605	5.991	7.378	9.210	13.816
3	6.251	7.815	9.348	11.345	16.266
4	7.779	9.488	11.143	13.277	18.467
5	9.236	11.070	12.833	15.086	20.515
6	10.645	12.592	14.449	16.812	22.458
7	12.017	14.067	16.013	18.475	24.322
8	13.362	15.507	17.535	20.090	26.125
9	14.684	16.919	19.023	21.666	27.877
10	15.987	18.307	20.483	23.209	29.588
11	17.275	19.675	21.920	24.725	31.264
12	18.549	21.026	23.337	26.217	32.910
13	19.812	22.362	24.736	27.688	34.528
14	21.064	23.685	26.119	29.141	36.123
15	22.307	24.996	27.488	30.578	37.697
16	23.542	26.296	28.845	32.000	39.252

IV. Upper critical values of F distribution

$F_{0.05;\,v1,\,v2}$

$F_{0.05}$	$v_1 = 1$	2	3	4
$v_2 = 1$	161.448	199.500	215.707	224.583
2	18.513	19.000	19.164	19.247
3	10.128	9.552	9.277	9.117
4	7.709	6.944	6.591	6.388
5	6.608	5.786	5.410	5.192
6	5.987	5.143	4.757	4.534
7	5.591	4.737	4.347	4.120
8	5.318	4.459	4.066	3.838
9	5.117	4.257	3.863	3.633
10	4.965	4.103	3.708	3.478
11	4.844	3.982	3.587	3.357
12	4.747	3.885	3.490	3.259
13	4.667	3.806	3.411	3.179
14	4.600	3.739	3.344	3.112
15	4.543	3.682	3.287	3.056
16	4.494	3.634	3.239	3.007
17	4.451	3.592	3.197	2.965
18	4.414	3.555	3.160	2.928
19	4.381	3.522	3.127	2.895
20	4.351	3.493	3.098	2.866
21	4.325	3.467	3.073	2.840
22	4.301	3.443	3.049	2.817
23	4.279	3.422	3.028	2.796
24	4.260	3.403	3.009	2.776
25	4.242	3.385	2.991	2.759
26	4.225	3.369	2.975	2.743
27	4.210	3.354	2.960	2.728
28	4.196	3.340	2.947	2.714
29	4.183	3.328	2.934	2.701
30	4.171	3.316	2.922	2.690

$F_{0.025;\,v1,\,v2}$

$F_{0.025}$	$v_1 = 1$	2	3	4
$v_2 = 1$	647.789	799.500	864.163	899.583
2	38.506	39.000	39.166	39.248
3	17.443	16.044	15.439	15.101
4	12.218	10.649	9.979	9.605
5	10.007	8.434	7.764	7.388
6	8.813	7.260	6.599	6.227
7	8.073	6.542	5.890	5.523
8	7.571	6.060	5.416	5.053
9	7.209	5.715	5.078	4.718
10	6.937	5.456	4.826	4.468
11	6.724	5.256	4.630	4.275
12	6.554	5.096	4.474	4.121
13	6.414	4.965	4.347	3.996
14	6.298	4.857	4.242	3.892
15	6.200	4.765	4.153	3.804
16	6.115	4.687	4.077	3.729
17	6.042	4.619	4.011	3.665
18	5.978	4.560	3.954	3.608
19	5.922	4.508	3.903	3.559
20	5.872	4.461	3.859	3.515
21	5.827	4.420	3.819	3.475
22	5.786	4.383	3.783	3.440
23	5.750	4.349	3.751	3.408
24	5.717	4.319	3.721	3.379
25	5.686	4.291	3.694	3.353
26	5.659	4.266	3.670	3.329
27	5.633	4.242	3.647	3.307
28	5.610	4.221	3.626	3.286
29	5.588	4.201	3.607	3.267
30	5.568	4.182	3.589	3.250

備註	三、作答於試題上者，不予計分。 四、試題請隨卷繳交。