考試科目	統計學	系所別	統計學系二年級	考試時間	7 月 8 日(星期三) 第 2 節

(10%)Q1. For the following sample data,

56, 78, 34, 89, 52, 65, 72, 81, 96, 42

(a) Find the interquartile range

(b) Find the 80^{th} percentile.

(10%)Q2. The An Antibody test is used to determine whether someone has COVID-19. The false-positive rate is .035, the false-negative rate is .160. A physician has just received the Antibody test report that his patient tested negative. Before receiving the result, the physician assigned his patient to the low-risk group with only a .08% probability of having COVID-19.

(a) What is the probability that this patient tested negative?

(b) What is the probability that the patient actually doesn't have COVID-19?

(10%)Q3. A random variable X has the following density function

$f(x)=0.2-0.02x \quad 0<x<10 \quad =0 \quad$ otherwise

(a) Find $P(4<X<6)$

(b) Find the median of X

(10%)Q4. (a) In estimating population proportion p, what is the minimum sample size needed in order to guarantee that the sampling error is no more than 2% at 98% confidence level.

(b)In estimating population mean μ, what is the minimum sample size needed in order to guarantee that the estimation error is no more than B at 95% confidence level given that $\sigma=2$.

(15%)Q5. An experiment has been conducted for 4 treatments and 7 blocks. Given that SST(Sum of Square for Treatment)=800, SSB((Sum of Square for Block)=500 and SSTO(Sum of Square for TOtal)=1800.

(a) Test at .05 significance level whether there exists difference between treatments?

(b) Is the blocking effective? Use .05 significance level

(c) Ignore the blocking, test at .05 significance level whether there exists difference between treatments?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	統計學	系所別	統計學系二年級	考試時間	7 月 8 日(星期三) 第 2 節

(10%)Q6. Conduct a test at .05 significance level to determine whether reputation of company and quality of management are independent?

	Reputation of Company		
Quality of Management	Excellent	Good	Fair
Excellent	40	25	5
Good	35	35	10
Fair	25	10	15

(25%)Q7. In a simple linear regression analysis of quarter sales(y) and student population(x), a sample data for 10 restaurants are collected with the following summary statistics.

$\bar{x} = 14, \bar{y} = 130, s_y^2 = 153, s_x^2 = 56.8, s_{xy} = 284$

(a) Fine the least square line.
(b) Test the validity of this model at .05 significance level.
(c) Test the linear relationship between y and x at .05 significance level.(Use s=13.829 if you did not solve part (b)).
(d) Compute the coefficient of determination and interpret its meaning. (Use s=13.829 if you did not solve part (b)).
(e) Test whether the population coefficient of correlation ρ is 0 at .05 significance level.

(10%)Q8. What do we mean by p-value of a test? Describe its relationship with significance level α. Comment on the following statement :

"The p-value of the test is .04, therefore we reject the null hypothesis"

Statistical Table Values:

$F_{.05,3,18} = 3.16, F_{.05,4,18} = 2.93, F_{.05,6,18} = 2.66, F_{.05,7,18} = 2.58$
$F_{.05,3,24} = 3.01, F_{.05,4,24} = 2.78, F_{.05,1,8} = 5.32, F_{.05,1,9} = 5.12,$

$\chi^2_{.05,9} = 19.0, \chi^2_{.05,4} = 11.1, \chi^2_{.05,9} = 16.9, \chi^2_{.05,4} = 9.49$

$t_{.05,7} = 1.895, t_{.05,8} = 1.860, t_{.05,9} = 1.833,$

$z_{.01} = 2.33, z_{.02} = 2.06, z_{.05} = 1.645, z_{.025} = 1.96,$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	經濟系/國貿系/金融系/統計系/企管系/資管系 二年級	考試時間	7月 8 日(三) 第 四 節

Part I Problems (Multiple choice problems; 50 points; 5 points for each problem) 選擇題請在答案卡上作答，否則不予計分。

1. Which of the following statement is false?

   (a) $\frac{d}{dx}\ln(2x) = \frac{d}{dx}\ln(x)$ for $x > 0$.

   (b) $\frac{d}{dx}\csc(x) = \cot(x)\csc(x)$ for $x \in (0, \pi)$.

   (c) $\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = 1 + \tan^2(x)$ for $x \in (-\pi/2, \pi/2)$.

   (d) $\frac{d}{dx}2^x = \ln(2) \cdot 2^x$ for $x \in (-\infty, \infty)$.

   (e) $\frac{d}{dx}(x^3 + 2x + 1) = 3x^2 + 2$ for $x \in (-\infty, \infty)$.

2. Let $f(x) = 1 + x^2$ for $x \in (-\infty, \infty)$ and let $g(x) = \tan(x)$ for $x \in (-\pi/2, \pi/2)$. Which of the following statement is true?

   (a) $\frac{d}{dx}\frac{g(x)}{f(x)} = \frac{2x\tan(x)}{(1+x^2)^2}$ for $x \in (-\pi/2, \pi/2)$.

   (b) $\frac{d}{dx}(f(x) + g(x)) = 2x + \sec(x)$ for $x \in (-\pi/2, \pi/2)$.

   (c) $\frac{d}{dx}(f(x)g(x)) = \sec^2(x)(1 + x^2) + 2x\tan(x)$ for $x \in (-\pi/2, \pi/2)$.

   (d) $\frac{d}{dx}f(g(x)) = 2g(x)$ for $x \in (-\pi/2, \pi/2)$.

   (e) None of the above statements is true.

3. Suppose that f is a differentiable function such that $f'(x) = -f(x)$ and $f(x) > 0$ for all $x \in (-\infty, \infty)$. Which of the followig statement is false?

   (a) $f(x) = f(0)e^{-x}$ for $x \in (-\infty, \infty)$.

   (b) $f''(x) = f(x)$ for $x \in (-\infty, \infty)$.

   (c) f is strictly increasing on $(-\infty, \infty)$.

   (d) Let $g(x) = \frac{d}{dx}\ln(f(x))$ for $x \in (-\infty, \infty)$. Then g is a constant function on $(-\infty, \infty)$.

   (e) $\lim_{x\to\infty} f(x) = 0$.

4. Suppose that f is a differentiable function on $(-\infty, \infty)$ such that $f(x) = ax^2 + bx + c$ for $x \in (-1, 0)$ for some constants a, b and c and $f(x) = -0.5$ for $x < -1$. Which of the following statements is false?

   (a) $c = a - 0.5$.

   (b) If $f(0) = 0$, then $f'(0) = 2$.

   (c) If $f'(0) = 2$, then $a = 1$.

   (d) If $f(0) = 0$, then $f(x) + f(-x) = 0$ for $x \in (-1, 1)$.

   (e) f is continuous at 0.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	經濟系/國貿系/金融系 統計系/企管系/資管系 二年級	考試時間	7月 8 日(三) 第 四 節

5. For $k > 0$, let $D_k = \{(x, y) : 0 \le x \le k\pi \text{ and } x\sin(x) \le y \le x\}$ and let A_k be the area of D_k. Which of the following statement is true?

   (a) $A_1 = 2\pi + \pi^2/2$.

   (b) $A_2 = 4\pi + 2\pi^2$.

   (c) $A_3 = 3\pi + 9\pi^2/2$.

   (d) $A_4 = 4\pi + 8\pi^2$.

   (e) None of the above statement is true.

6. Let $f(x, y) = x\sin(x^2 + y) + x + y$ for $x, y \in (-\infty, \infty)$. Which of the following statement is true?

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{f(x,y)}{x(x^2+y)} = 1$.

   (b) $\displaystyle\int_0^a f(x, y)dy = -x\cos(x^2 + a) + x\cos(x^2) + ax + \frac{a^2}{2}$ for $a > 0$.

   (c) $\dfrac{\partial}{\partial y} f(x, y) = 2x^2\cos(x^2 + y) + \sin(x^2 + y) + 1$.

   (d) The tagent plane to the surface $z = f(x, y)$ at the point $(0, 0, 0)$ is $z = 2x + y$.

   (e) None of the above statements is true.

7. Suppose that f is a differentiable function on $(0, 1)$ and f is continuous on $[0, 1]$. Which of the following statement is false?

   (a) If $f(0)f(1) < 0$, then there exists a number $c \in (0, 1)$ such that $f(c) = 0$.

   (b) If $f'(0.1)f'(0.9) < 0$, then there exists a number $c \in (0.1, 0.9)$ such that $f'(c) = 0$.

   (c) If $f(0) = 0$ and $f(1) = 1$, then there exists a number $c \in (0, 1)$ such that $f'(c) = 1$.

   (d) If $f'' > 0$ on $(0, 1)$ and $f'(c) = 0$ for some $c \in (0, 1)$, then $f(x) \ge f(c)$ for $x \in [0, 1]$.

   (e) If $f(x) = f(1 - x)$ for $x \in [0, 1]$, $f(0) > 0$ and $f(0.5) = 0$, then $f''(0.5) > 0$.

8. Define a sequence $\{a_n\}_{n=1}^{\infty}$ as follows. Let $a_1 = 1$ and $a_{n+1} = 0.5a_n + 1/n$ for $n \ge 2$. Which of the following statement is true?

   (a) $\lim_{n\to\infty} a_n = 1$.

   (b) $\lim_{n\to\infty} a_n = \infty$.

   (c) $\lim_{n\to\infty} a_n/n = 0$.

   (d) $\sum_{n=1}^{\infty} na_n < \infty$.

   (e) None of the above statements is true.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	經濟系/國貿系/金融系/統計系/企管系/資管系 二年級	考試時間	7月 8 日(三) 第 四 節

9. Let $S = \{r : r$ is a positive number such that $\sum_{k=0}^{\infty} x^k/k!$ converges absolutely for $|x| < r\}$. Which of the following statement is true?

   (a) S is empty.

   (b) S is nonempty and $r \leq 1$ for all $r \in S$.

   (c) S is nonempty and if for some $r \in S$, a function f is defined as

   $$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

   for $x \in (-r, r)$, then $f'(x) > f(x)$ for $x \in (0, r)$.

   (d) $S = (0, \infty)$.

   (e) None of the above statements is true.

10. Let $f(x, y) = 1/(1 + x^2 + y^2)$ for $x, y \in (-\infty, \infty)$ and let $D_a = \{(x, y) : x^2 + y^2 \leq a^2\}$ for $a > 0$. Which of the following statement is true?

    (a) $\int_{D_1} f(x, y)d(x, y) = \pi \ln(2)$.

    (b) $\int_{D_2} f(x, y)d(x, y) = \pi \ln(4)$.

    (c) $\int_{D_3} f(x, y)d(x, y) = \pi \ln(6)$.

    (d) $\int_{D_4} f(x, y)d(x, y) = \pi \ln(8)$.

    (e) None of the above statements is true.

Part II Problems (50 points)

Note: For Part II Problems, SHOW YOUR WORK TO GET THE POINTS

11. (20 points) Use Newton's method to find the zero of $f(x) = x^2 - 3$ with $x_0 = 2$. (Perform four iterations.)

12. (30 points) Let $f(x)$, $F(x)$ and $h(x)$ be defined as the following:

$$\begin{aligned} f(x) &= \frac{2}{\sqrt{\pi}\lambda} e^{-(x/\lambda)^2}, \\ F(x) &= \int_0^x f(t)dt, \\ h(x) &= \frac{f(x)}{1 - F(x)}. \end{aligned}$$

    (a) (15 points) Find $\lim_{x \to 0^+} h(x)$.

    (b) (15 points) Find $\lim_{x \to \infty} h(x)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。