國立政治大學 108 學年度 轉學生 招生考試試題

考試科目	微積分(二)	系所別	應用數學系二年級	考試時間	7月10日(三)第四節

1. (40 %) Evaluate the integrals.

   (a) (8 %) $\displaystyle\int_1^2 \frac{(\ln x)^2}{x^3}dx$

   (b) (8 %) $\displaystyle\int_0^2 \int_{y/2}^1 y\cos(x^3-1)dxdy$

   (c) (8 %) $\displaystyle\iiint xdV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$

   (d) (8 %) $\displaystyle\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xzdzdxdy$

   (e) (8 %) $\displaystyle\int_C (y+z)dx + (x+z)dy + (x+y)dz$, where C consists of line segments from $(0,0,0)$ to $(1,0,1)$ and from $(1,0,1)$ to $(0,1,2)$.

2. (16 %) Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   (a) (8 %) $\displaystyle\int_0^1 r\ln r dr$

   (b) (8 %) $\displaystyle\int_0^\infty e^{-\sqrt{y}}dy$

3. (14 %)

   (a) (6 %) Find the length of the curve

   $$y = \frac{x^4}{16} + \frac{1}{2x^2} \qquad 1 \le x \le 2$$

   (b) (8 %) Find the area of the surface obtained by rotating the curve in part (a) about the y-axis.

4. (10 %) Determine whether the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n \sin(\frac{1}{\sqrt{n}})\ln(1+\frac{1}{\sqrt{n}})$ is divergent, conditionally convergent, or absolutely convergent.

5. (10 %) Find the values of p for which the series $\displaystyle\sum_{n=1}^{\infty}\frac{n}{(1+n^3)^p}$ is convergent.

6. (10 %) Let E be the tetrahedron bounded by planes $-x+y+z=0$, $x-y+z=0$, $x+y-z=0$, and $-x+5y+7z=6$. Find the volume of E.

備註	一、作答於試題上者，不予計分 二、試題隨卷繳交

國立政治大學108學年度 轉學生 招生考試試題

考試科目	微積分(一)	系所別	應用數學系 二年級	考試時間	7月10日(三)第二節

1. (10 %) If the function f is defined by

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

prove that $\lim_{x\to 0} f(x)$ does not exist.

2. (24 %) Find the limit if it exists, or show that the limit does not exist.

   (a) (8 %) $\lim_{x\to 0^+} (\tan 2x)^x$

   (b) (8 %) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

   (c) (8 %) $\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

3. (10 %) Discuss the continuity of the function

$$f(x,y) = \begin{cases} \dfrac{\sin xy}{xy} & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$$

4. (18 %) Find the indicated derivatives.

   (a) (10 %) $P = \sqrt{u^2+v^2+w^2}$, $u = xe^y$, $v = ye^x$, $w = e^{xy}$. Evaluate $\frac{\partial P}{\partial x}$ and $\frac{\partial P}{\partial y}$ when $x = 0$, $y = 2$.

   (b) (8 %) $x^y = y^x + y$. Find $\frac{dy}{dx}$ at $(x,y) = (2,1)$.

5. (10 %) Find the extreme values of the function $f(x,y,z) = 3x - y - 3z$ subject to both constraints: $x + y - z = 0$, $x^2 + 2z^2 = 1$.

6. (12 %) Let $f(x)$ be a twice differentiable one-to-one function. Suppose that $f(2) = 1$, $f'(2) = 3$, $f''(2) = e$.

   (a) (6 %) Find $\frac{d}{dx} f^{-1}(1)$.

   (b) (6 %) Find $\frac{d^2}{dx^2} f^{-1}(1)$.

7. (16 %) From the equation $\sqrt{1+y} - \int_0^{x^2-1} \frac{dt}{1+t^2} + \tan(xy) = 1$, a differentiable $y = y(x)$ can be determined around $(x,y) = (1,0)$.

   (a) (8 %) Evaluate y' at $(x,y) = (1,0)$.

   (b) (8 %) Determine the concavity of $y = y(x)$ around $(x,y) = (1,0)$.

備註	一、作答於試題上者，不予計分 二、試題隨卷繳交

國立政治大學 108 學年度 轉學生招生考試試題

考試科目	微積分	系別	應用數學系三年級	考試時間	7 月 10 日 (三) 第二節

注意事項：

- 本試題共有 6 個問題，總計 100 分。
- 不得任意更改題目符號，否則依情節輕重扣分。

1. Determine whether each of the following functions is differentiable at $x = 0$:

   (a) (8 points) $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

   (b) (8 points) $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

2. Evaluate each of the following integrals:

   (a) (9 points) $\int \cos(x^{\frac{1}{3}})dx$

   (b) (9 points) $\int \frac{dx}{1 + \sqrt[4]{x-1}}$

   (c) (9 points) $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$

   (d) (9 points) $\int \frac{x+4}{x^3 + 6x^2 + 9x} dx$

3. (8 points) Show that a differentiable function $f(x)$ is continuous.

4. Consider the function

$$f(x) = \frac{1}{x^2 + x + 1}$$

   (a) (8 points) Find the Maclaurin seies for $f(x)$.

   (b) (8 points) Find the 36th order derivative $f^{(36)}(0)$.

5. Evaluate each of the following integrals:

   (a) (8 points) $\iint_R e^{x^3} dA$, where R is the region bounded by $y = x^2$, $x = 3$, and $y = 0$

   (b) (8 points) $\int_{\frac{\sqrt{2}}{2}}^1 \int_{\sqrt{1-x^2}}^x \frac{1}{\sqrt{x^2+y^2}} dy dx$

6. (8 points) Approximate $\int_0^1 \frac{\sin x}{x} dx$ correctly to six decimal places.

備註	一、作答於試題上者，不予計分 二、試題隨卷繳交

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考試科目	線性代數	系別	應用數學系三年級	考試時間	7 月 10 日 (三) 第四節

注意事項：

- 本試題共有 5 個問題, 總計 100 分。
- 不得任意更改題目符號, 否則依情節輕重扣分。

1. Let U and W be subspaces of a vector space V. Show that:
   (i) (10 points) U and W are contained in $U+W$.
   (ii) (10 points) $U+W$ is the smallest subspace of V containing U and W.

2. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by

$$T(x,y,z) = (x+2y-z, y+z, x+y-2z)$$

   Find a basis and the dimension of the
   (i) (10 points) image U of T
   (ii) (10 points) kernel W of T

3. Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T(x,y,z) = (2y+z, x-4y, 3x).$$

   (i) (12 points) Find the matrix of T in the basis $f = \{f_1 = (1,1,1), f_2 = (1,1,0), f_3 = (1,0,0)\}$.
   (ii) (8 points) Verify that $[T]_f[v]_f = [T(v)]_f$ for any vector $v \in \mathbb{R}^3$.

4. Consider the matrix

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

   (i) (15 points) Find all eigenvalues and a basis of each eigenspace for A.
   (ii) (5 points) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

5. Let $T: V \to V$ be a linear operator. Let $U = \ker T^i$ and $W = \ker T^{i+1}$. Show that:
   (i) (10 points) $U \subseteq W$
   (ii) (10 points) $T(W) \subseteq U$

備 註	一、作答於試題上者，不予計分 二、試題隨卷繳交