國立政治大學 109 學年度 轉學生 招生考試試題

考試科目	微積分(一)	系所別	應用數學系 二年級	考試時間	7月8日(三) 第二節

1. (20%) Use the definition of the limit to show that

   A. (10%) $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$.

   B. (10%) $\lim_{x \to \infty} \frac{3}{x^2-1} = 0$.

2. (15%) Find all local maximal, local minimal, and the saddle points of the function $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

3. (15%) Prove that if f is differentiable on $(-\infty, \infty)$ and $f'(x) < 1$ for all real numbers, then f has at most one fixed point. (A fixed point of a function f is the number c such that $f(c) = c$.)

4. (15%) Show that $\frac{d}{dx} e^x = e^x$.

5. (15%) Show that if the limit exists, then it is unique.

6. (10%) Show that if $f'(x) > 0$ on an interval, then f is increasing on that interval.

7. (10%) Show that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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考試科目	微積分(二)	系所別	應用數學系 二年級	考試時間	7月8日(三) 第四節

1. (10%) Let $f(x) = \int_{x^2}^{x^3+2x^2} e^{t^2} dt$. Find $\frac{d}{dx} f(x)$.
2. (15%) Compute the area of the hemisphere of radius $t > 0$ and centered at the origin given by $S = \{(x, y, z) \in R^3 : x^2 + y^2 + z^2 = t^2, z \geq 0\}$.
3. (10%) Evaluate the limit $\lim_{n\to\infty} \sum_{n=1}^{\infty} \frac{1}{n} \sqrt[3]{\frac{1}{n}}$.
4. (10%) Find the total length of the curve $r = 2(1 + \cos\theta)$.
5. (15%) Suppose that $f: [0,1] \to R$ satisfies $1 - f(x) = f(1 - x)$. Compute $\int_0^1 f(x)dx$.
6. (15%) Given the series $\sum_{n=0}^{\infty} nx^n$.
   - A. (5%) Find the radius of convergence.
   - B. (5%) Find the interval of convergence.
   - C. (5%) Find the sum.
7. (15%) Evaluate the integral $\iint_R \frac{y}{x^2+y^2} dA$, where R is trapezoid bounded by $y = x$, $y = 2x$, $x = 1$, and $x = 2$.
8. (10%) Evaluate the integral $\int_{-\infty}^{\infty} e^{\frac{1}{2}x^2} dx$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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考 試 科 目	微積分	系 所 別	應用數學系 三年級	考 試 時 間	7月8日(三) 第二節

1. (10%) Discuss the continuity of the function $f(x,y)=\begin{cases}\frac{\sin(x^2-y^2)}{x^2-y^2}, x^2\neq y^2\\ 1, x^2=y^2\end{cases}$.

2. (10%) Use $\varepsilon-\delta$ definition to prove that $\lim_{x\to 2}(x^2-2x)=0$.

3. (10%) Find the positive value of p for which the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ converges.

4. (20%) Let n be a non-negative integer and $a_n=\int_0^{\frac{\pi}{2}} sin^n x dx$.
   A. (10%) Compute $na_n a_{n-1}$.
   B. (10%) Prove that $\{a_n\}$ is a decreasing sequence.

5. (10%) Show that if $f''(x)>0$ on an interval, then f is concave upward on that interval.

6. (10%) Find the length of the curve $y=(\frac{x}{2})^{\frac{2}{3}}$ from $x=0$ to $x=2$.

7. (15%) Prove the Mean Value Theorem.

8. (15%) Prove that $\lim_{x\to 0} sin\frac{1}{x}$ does not exist at $x=0$.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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考試科目	線性代數	系別	應用數學系三年級	考試時間	7月8日(三)第四節

注意事項：
- 本試題共有 6 個問題, 總計 100 分。
- 不得任意更改題目符號, 否則依情節輕重扣分。

Please show all your work.

1. Given a 4×4 triangular Pascal matrix
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$
   (a) (10 points) Find the inverse A^{-1} of A and show that A is similar to A^{-1}.
   (b) (10 points) Find all the eigenvalue(s) of A, the corresponding eigenvector(s) and algebraic and geometric multiplicities.

2. (10 points) Find the Jordan canonical form for the matrix
$$A = \begin{bmatrix} 1 & -1 & -2 & 3 \\ 0 & 0 & -2 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

3. Let A be an $m \times n$ matrix and B an $n \times r$ matrix with $AB = 0$.
   (a) (10 points) Show that the null space of A contains the column space of B.
   (b) (10 points) Show that $rank(A) + rank(B) \leq n$.

4. Suppose A and B are square matrices such that $AB = I$, where I is the identity matrix.
   (a) (10 points) Show that $rank(AB) \leq rank(A)$.
   (b) (10 points) Show that A is invertible with inverse $A^{-1} = B$.

5. (10 points) Suppose $\{v_1, \cdots, v_n\}$ is a basis for $\mathbb{R}^n$ and the $n \times n$ matrix A is invertible. Show that $\{Av_1, \cdots, Av_n\}$ is also a basis for $\mathbb{R}^n$.

6. Let U be the subspace of $\mathbb{R}^5$ generated by
$$\Big\{(1,3,-2,2,3), (1,4,-3,4,2), (2,3,-1,-2,9)\Big\}$$
   and let W be the subspace generated by
$$\Big\{(1,3,0,2,1), (1,5,-6,6,3), (2,5,3,2,1)\Big\}$$
   Find a basis and the dimension for each of the following subspaces of $\mathbb{R}^5$.
   (a) (10 points) $U + W$.
   (b) (10 points) $U \cap W$.

備註	一、作答於試題上者，不予計分 二、試題隨卷繳交