

考試科目

微積分

系所別

資訊科學系 三年級

考試時間

7月6日(四)第二節

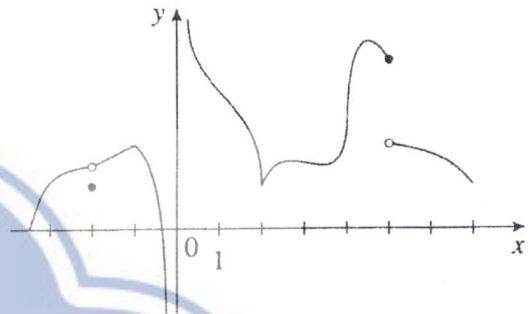
(10%) 1. Is there a number a such that the following limit exists? If so, find the value of a and the value of the limit.

$$\lim_{x \rightarrow -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

(20%) 2. The graph of $y = g(x)$ is given as follows.

(a) At what numbers is $g(x)$ discontinuous? Why?

(b) At what numbers is $g(x)$ not differentiable? Why?



(10%) 3. Find the points on the curve $y = (\cos x) / (2 + \sin x)$ at which the tangent is horizontal.

(10%) 4. Find a formula for $f^{(n)}(x)$ if $f(x) = \ln(x-1)$ where n is a nature number.

(10%) 5. Two people start from the same point. One walks east at 3 mi/hr and the other walks northeast at 2 mi/hr. How fast is the distance between the people changing after 15 minutes?

(10%) 6. For what values of the numbers a and b does the following function have the maximum value $f(2) = 1$?

$$f(x) = axe^{bx^2}$$

(10%) 7. For what values of a and b is the following equation true?

$$\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = 0$$

(20%) 8. The cost, in dollars, of producing x yards of a certain fabric is $C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$ and the company finds that if it sells x yards, it can charge $p(x) = 29 - 0.00021x$ dollars per yard for the fabric.

(a) Graph the cost and revenue functions and use the graphs to estimate the production level for maximum profit. (note: revenue function = $xp(x)$)

(b) Use calculus to find the production level for maximum profit.

備

註

- 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。

考試科目	計算機概論 (含C程式設計)	系所別	資訊科學系 二年級	考試時間	7月6日(四) 第四節
<ol style="list-style-type: none"> 1. (10%) Give two advantages of virtual memory. 2. (10%) Give a sorting algorithm whose worst-case running time on an n-element array is $O(n \log n)$. Describe the idea of the sorting algorithm. 3. (5%) Describe the difference between one's complement arithmetic and two's complement arithmetic. 4. (5%) Describe the difference between big-endian and little-endian. 5. (10%) Describe the fundamental difference between 32-bit CPUs and 64-bit CPUs. 6. Consider the 5 layer TCP/IP model. <ol style="list-style-type: none"> a. (5%) Give two protocols in Data Link Layer (Layer 2). b. (5%) Give two protocols in Transport Layer (Layer 4). c. (5%) Give two protocols in Application Layer (Layer 5). 7. (5%) Give an example of NP-hard problems. 8. Explain the following terms. <ol style="list-style-type: none"> a. (5%) Proxy b. (5%) Cloud computing 9. (10%) Write a C function <code>int NextPrime(int n)</code> that outputs the smallest prime number greater than n. Note that two is the smallest prime number. For example, <code>NextPrime(6)</code> should output 7. 10. (10%) Write a C function <code>double* CreateDoubleArray(int n)</code> that calls <code>malloc</code> to dynamically allocate an array of n doubles. Return <code>NULL</code> if <code>malloc</code> fails. 11. (10%) Describe the usage of <code>void*</code> in C programming language. 					
備註	<ol style="list-style-type: none"> 一、作答於試題上者，不予計分。 二、試題請隨卷繳交。 				

考 試 科 目	資料結構 (含C程式設計)	系所別	資訊科學系三年級	考試時間	七月六日(四)第二節
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Part 1. Single Selection. 45 points in total, 3 points each.

- Which of the following is **not** a valid C++ variable name? (a) `i_am` (b) `i_am_2_digits` (c) `_I_underscores_` (d) `I_Dollar_Sign`
- What are the contents of string `s` after executing the following statements?

```
string s = "abc";
string t = "cde";
s += s + t[1] + s;
```

(a) `abcde` (b) `abccabc` (c) `abcdabc` (d) `abcabcdabc`
- Given the following two functions, `f` and `g`:

```
void f(int x)
{ std::cout << ++x; }
void g(int& x)
{ std::cout << ++x; }
```

Which statement is **incorrect**? (a) the argument to `f` is passed by value (b) the argument to `g` is called by reference (c) both values of the arguments are **not** changed (d) both produce the same output
- Consider the following attempt to allocate a 10-element array of pointers to doubles and initialize the associated double values to zero.

```
double* dp[10]
for (int i = 0; i < 10; i++) {
    dp[i] = new double;
    (next code)
}
```

Which of the following is the correct code? (a) `dp[i] = 0` (b) `dp[i] = 0.0` (c) `*dp[i] = 0` (d) `*dp[i] = 0.0`
- Suppose an initially empty stack `S` has performed a total of 25 *push* operations, 12 *top* operations, and 10 *pop* operations, 3 of which generated a *StackEmpty* exception that was caught and ignored. What is the current size of `S`? (a) 0 (b) 13 (c) 15 (d) 18
- Given the following sequence of queue operations: `enqueue(5)`, `enqueue(3)`, `dequeue()`, `enqueue(2)`, `enqueue(8)`, `dequeue()`, `dequeue()`, `enqueue(9)`, `enqueue(1)`, `dequeue()`, `enqueue(7)`, `enqueue(6)`, `dequeue()`, `dequeue()`, `enqueue(4)`, `dequeue()`, `dequeue()`. What is the last output? (a) 1 (b) 5 (c) 6 (d) 9
- Let `T` be an ordered tree with more than one node. Which of the following traversals of `T` visit the nodes in the reverse order of each other? (a) preorder vs. inorder (b) preorder vs. postorder (c) inorder vs. postorder (d) not possible
- At which node of a max-heap can an entry with the largest key (value) be stored? (a) the first leaf node (b) the last leaf node (c) any internal node (d) root node

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9. What is the worst-case running time for inserting n items into an initially empty hash table, where collisions are resolved by chaining? (a) $O(n \log n)$ (b) $O(n^2)$ (c) $O(n^3)$ (d) $O(2^n)$
10. How many different binary search trees can store the keys $\{1, 2, 3\}$? (a) 1 (b) 3 (c) 5 (d) 6
11. Suppose S is a list of n bits, that is, n 0s and 1s. How long will it take to sort S stably with the bucket-sort algorithm? (a) $O(n)$ (b) $O(n \log n)$ (c) $O(n^2)$ (d) $O(n^3)$
12. Trickle down by swapping is an action occurs when (a) inserting an item into a heap (b) removing an item from a heap (c) inserting an item into a 2-3 tree (d) removing an item from a 2-3 tree
13. The merge-sort tree associated with an execution of merge sort on a sequence of size n has height? (a) n (b) $n/2$ (c) $(n/2) + 1$ (d) $\log n$
14. A simple undirected graph is complete if it contains an edge between every pair of distinct vertices. What does a depth-first search (DFS) tree of a complete graph look like? (a) a path (b) a walk (c) a cycle (d) a star
15. A simple undirected graph is complete if it contains an edge between every pair of distinct vertices. What does a breadth-first search (BFS) tree of a complete graph look like? (a) a path (b) a walk (c) a cycle (d) a star

Part II. Short Questions. 55 points in total.

1. (5 points) Using only addition (+) and /or subtraction (-), design a recursive algorithm to compute the product of two integers, m and n , e.g., $m \times n$.
2. (5 points) Given a stack S containing n elements and a queue Q that is initially empty. Design an algorithm that use Q to scan S to see if it contains a certain element x . Your algorithm must return the elements back to S in their original order. You may not use an array or linked list, only S and Q and a constant number of reference variables.
3. (5 points) A sequence is an abstract data type (ADT) that supports all the functions of the list ADT, but it also provides functions for accessing elements by their index. Describe, in pseudocode, how to perform a new sequence functions `makeFirst(p)` that moves an element of a sequence S at position p to be the first element in S while keeping the relative ordering of the remaining elements in S unchanged. S is implemented with a doubly linked list with the member functions, `getPrev()`, `getNext()`, `setPrev()`, `setNext()`. Your function should run in $O(1)$ time.

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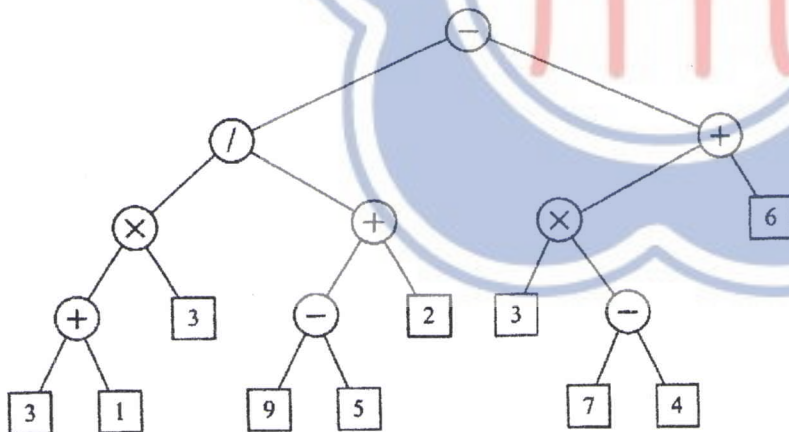
4. (5 points) Assume that the constructor Sequence creates an empty sequence of integer objects and insert(p, e) is a function that insert of copy of e just prior to the position p and shifts the subsequent elements one position to the right. Recall that division between integers performs truncation (for example, $7/2 = 3$). Consider the following fragment of C++ code:

```
Sequence<int> seq;
for (int i = 0; i < n; i++)
    seq.insert(i/2, i);
```

Show the sequence, seq, after each iteration of the loop for $n = 10$.

5. (5 points) Draw a binary tree T , such that:
- Each internal node of T stores a single character, and
 - A preorder traversal of T yields EXAMFUN, and
 - An inorder traversal of T yields MAFXUEN.

6. (10 points) Let T be the following binary tree.



(a) Give the output of the function `preorderPrint(T, T.root())` by following the code fragment below.

```
void preorderPrint(const Tree& T, const Position& p) {
    cout << *p; // print element
    PositionList ch = p.children(); // list of children
    for (Iterator q = ch.begin(); q != ch.end(); ++q) {
        cout << " ";
        preorderPrint(T, *q);
    }
}
```

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(b) Give the output of the function `printExpression(T, T.root())` by following the code fragment below.

```

Algorithm printExpression(T, p):
  if p.isExternal() then
    print the value stored at p
  else
    print "("
    printExpression(T, p.left())
    print the operator stored at p
    printExpression(T, p.right())
    print ")"
    
```

7. (10 points) Given two initially empty 2-3-4 trees.

(a) Draw the resulting 2-3-4 tree with the following insertion order: 4, 6, 12, 15, 3, 5.

(b) Draw the (another) resulting 2-3-4 tree with the following insertion order: 12, 3, 6, 4, 5, 15.

8. (10 points) Let G be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

Vertex	Adjacent Vertices
1	(2, 3, 4)
2	(1, 3, 4)
3	(1, 2, 4)
4	(1, 2, 3, 6)
5	(6, 7, 8)
6	(4, 5, 7)
7	(5, 6, 8)
8	(5, 7)

Assume that, in a traversal of G , the adjacent vertices of a given vertex are returned in the same order as they are listed in the table above.

(a) Draw G .

(b) Give the sequence of vertices of G visited using a DFS (Depth First Search) traversal starting at vertex 1.

(c) Give the sequence of vertices of G visited using a BFS (Breadth First Search) traversal starting at vertex 1.

備

註

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- 二、試題請隨卷繳交。

考 試 科 目	離散數學	系 所 別	資訊科學系 三年級	考 試 時 間	7 月 6 日(週四) 第 四 節
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- [15pts] Answer following questions:
 - If a tree has 60 vertices, then it has ____ edges.
 - If G is a planar graph with 3 connected components, 20 edges and 12 vertices, then a planar drawing of G will partition the plane into _____ regions.
 - Let $A = \{(x,y) \mid x \text{ and } y \text{ are integers and } 1 \leq x,y \leq 9\}$ and R an equivalent relation on A such that $(x_1,y_1) R (x_2,y_2)$ iff $x_1 y_2 = x_2 y_1$. Then the quotient set A/R has _____ equivalent classes.
- [10 pts] Let f be an increasing function satisfying the recurrence relation: $f(n) = 7 f(n/2) + 15 n^2/4$ whenever $n > 1$, and $f(1) > 0$. Mark each of the following statements with (O) if it is true and (X) if it is false. [2 pts per question]
 - $f(n) = O(n^3)$
 - $f(n) = \Omega(n^3)$
 - $f(n) = \Theta(n^3)$
 - $f(n) = O(n^3 / (\log n)^3)$
 - $f(n) = \Omega(n^2)$
- [20pts] Let $K_{s,t,u} = (V,E)$, where $s,t,u \geq 1$, be a complete undirected tripartite graph. Namely, $V = V_1 \cup V_2 \cup V_3$ where V_1, V_2 and V_3 are disjoint sets with s,t,u vertices, respectively, and $E = \{\{x,y\} \mid 1 \leq i,j \leq 3, x \in V_i, y \in V_j, i \neq j\}$. Answer the following questions about $K_{s,t,u}$.
 - The set $\{d \mid v \in V, v \text{ has degree } d.\}$ = _____.
 - There are _____ edges in $K_{s,t,u}$.
 - What is the sum of all degrees of all vertices in $K_{s,t,u}$? _____
 - What is the set $\{(s,t,u) \mid K_{s,t,u} \text{ has an Euler circuit}\}$? _____
 - There are _____ Hamilton circuits in $K_{2,2,2}$.
- [15pts] Let $S_n = \{a_1, a_2, \dots, a_k \mid 1 \leq k \leq n, a_1 = 1, a_k = n, \text{ and } a_j < a_{j+1} \text{ for } 1 \leq j < k\}$, where n is any positive integer, be the set of all strictly increasing sequences of positive integers that have 1 as their first term and n as the last term. Let $A_n = |S_n|$ be the number of sequences in S_n .
 - What is the value of A_1, A_2, A_3 ? [3pts]
 - Find a recurrence relation for A_n for all $n \geq 2$. [6pts]
 - Solve the recurrence relations you derived from (b) with A_1, A_2, A_3 you got in (a) as initial conditions to get a general solution to A_n . [6pts]

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5. [10 pts]

- (a) How many positive integer solutions are there to the equation $x*y*z = 5^9$?
- (b) How many positive integer solutions are there to the equation $x*y*z = 12^5$?

6. [15 pts] Let $A_n = \{1, 2, \dots, n\}$ (where n is a positive even number). A pairing P of A_n is a set of 2-element subsets of A_n such that every element of A_n belongs to exactly one member of P . For example, the set $\{\{1, 3\}, \{2, 4\}, \{5, 6\}\}$ is a pairing of A_6 .

- (a) Find the number of all possible pairings of A_{20} . [8pts]
- (b) Suppose all pairings of A_{20} have the same probability. Then what is the probability of getting a pairing of A_{20} in which every odd number is paired with an even number? [7pts]

7. [15pts] Consider the following two statements about non-negative integers:

- Goldbach's conjecture: Every integer > 1 is the average of two primes.
- SumOfThreePrimes: Every integer > 5 is the sum of three primes.

Show that both statements are logically equivalent. Namely,

- (a) Goldbach's conjecture implies SumOfThreePrimes [7pts].
- (b) SumOfThreePrimes implies Goldbach's conjecture [8pts].

備

註

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