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Privatization neutrality with quality and subsidies

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Abstract

This study introduces a subsidy policy on product quality in a quality-then-price game to remedy the quality distortion under a mixed oligopoly (one public firm and one private firm) framework. We show that the multi-stage setting for firms is crucial for the validity of privatization neutrality. Since firms have different objectives, their asymmetric strategic consideration on price will spill over to the quality competition if there exists price differentiation in equilibrium under partial privatization. This spillover effect results in lower social welfare levels than the first-best outcome, and the neutrality of privatization in White (Economics Letters 53:189–195) no longer holds in our multi-stage model. Specifically, the optimal privatization policy is either fully public or completely private, where the social welfare attains the first-best outcome.

Keywords Mixed oligopoly · Privatization neutrality · Quality subsidy · Multi-stage · Quality competition

JEL Classification H21 · L11 · L22 · L23

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1 Introduction

It is well known that output distortion is inevitable in a mixed oligopoly,[1] and when this distortion is remedied by a proper subsidy, the social welfare level is independent of the degree of privatization, which is called the privatization neutrality theorem [see White (1996), Payago-Theotoky (2001), Myles (2002), Sepahvand (2002), and Kato and Tomaru (2007) for different scenarios]. The above statement is valid when firms have one common choice variable (e.g., quantities). If firms have two choice variables (e.g., quality and prices), which are determined in different stages and the government has only one policy tool (subsidy) to correct the distortion,[2] then we will show that the neutrality of privatization is broken. Specifically, the best subsidy policy can only reach the second-best outcome, except when the public firm is either fully owned by the government or fully privatized, such that an identical price in equilibrium is obtained. Therefore, the social welfare level will depend on the degree of privatization.

Privatization neutrality was first discussed by White (1996), who found that privatization is irrelevant to social welfare under the optimal subsidy policy. However, this neutrality may not be valid in some scenarios. For instance, Fjell and Heywood (2004) considered the order of a firm's moves and found that if the public firm becomes the follower after privatization, then social welfare will be reduced. Matsumura and Tomaru (2012) obtained non-neutrality of privatization by considering foreign competitors, because the subsidy to those foreign firms cannot be counted in the domestic surplus.[3] Cato and Matsumura (2013) also showed that the privatization neutrality theorem does not hold in a model with free entry (multi-stage but no strategic effect). Finally, Matsumura and Tomaru (2013) obtained non-neutrality when the excess burden of taxation is considered.

All the above studies on privatization neutrality are concerned with quantity competition, and thus, the game is only one stage for firms, except Cato and Matsumura (2013), while the current paper demonstrates that, given that the government has only one subsidy tool based on firms' quality, a multi-stage (quality then price) structure for firms results in the social welfare level depending on the degree of privatization. In other words, privatization neutrality is no longer valid in our framework. The intuition of our result is as follows. Since firms' objectives are different

[1] Mixed oligopoly was first defined in De Fraja and Delbono (1989) as meaning the simultaneous presence of private and public enterprises in an economic system; see also Cremer et al. (1989), Fjell and Pal (1996), Anderson et al. (1997), and Pal and White (1998), all of whom assumed either fully public or fully private firms. Later, Matsumura (1998) creatively introduced a partially public firm (instead of a fully public firm) to the traditional mixed oligopoly framework, and then, the behavior of partially public firms became a research focus in the studies of mixed oligopoly.

[2] If these two choice variables are determined in the same game stage, say as in Ishibashi and Kaneko (2008), then the first-best outcome can be achieved, and thus, the degree of privatization and the optimal subsidy are unrelated issues.

[3] They further discussed the leadership of firms and found that private leadership yields a larger (smaller) welfare than public leadership when the foreign investment in the private firms is non-zero and small (large). In other words, the privatization neutrality theorem does not hold, unless the share of foreign investors in the private firms is zero.

in a mixed oligopoly market, their quality and price strategies in both stages may be different.[4] The asymmetric strategic consideration on prices will spill over to the quality competition stage if there exists price differentiation in equilibrium under partial privatization. This spillover effect makes the policy of quality subsidy unable to achieve the first-best on social welfare. When the public firm is either fully owned by the government or fully privatized, the spillover effect will vanish, because the equilibrium prices are identical and the social welfare attains the first-best outcome [see the explanation after Eq. (11) for details]. The key point of our results is that the government has only one policy tool, while firms have two-stage competition strategies. In fact, if our mixed oligopoly market was reduced to a one-stage game for firms by a uniform price regulation, then the first-best outcome could be reached by a quality subsidy.[5]

Multi-stage frameworks are very common in the literature as well as in the real world. For example, Ishibashi and Kaneko (2008) and Laine and Ma (2017) are quality-then-price games.[6] Matsumura and Matsushima (2004) employed an R&D (cost-reducing)-then-price structure. Weiss (2003) is an innovation-then-quantity (or price) competition model. In the real world, location choices are often determined before price setting for most manufacturing firms. Quality positioning and pricing are strategic choice variables for many service industries, and the focus of the current paper.

For a real example, the education industry is highly mixed in its structure, with many public schools and private schools competing primarily on quality, instead of quantity.[7] The quality of a university can be evaluated by independent institutions and eventually shown in a ranking list. In Taiwan, the government tries to raise universities' global rankings by providing a huge subsidy to qualified universities. In fact, most countries provide various subsidies to both public and private schools, and these may depend on their quality performance. Another way to improve quality is privatization, such that the incentive for success is raised. For example, Japan's national universities have incorporated a form of partial privatization, to improve their quality in 2004.[8]

[4] If there are two identical private firms (and both have the same objectives), then the optimal subsidy can reach the first-best outcome.

[5] To demonstrate the importance of the multi-stage game, we provide an extra section (Sect. 3) to discuss a uniform price regulation (i.e., there is no price competition) in our story, and find that the neutrality of privatization is restored.

[6] Brekke et al. (2011) merely mentioned a quality-then-price game in their appendix. Fernández-Ruiz (2018) constructed a two-period Hotelling-type model and solved for the prices in the first and second periods.

[7] Quality competition is a traditional issue in industrial economics, such as Shaked and Sutton (1982), Spence (1975), Ma and Burgess (1993), Brekke et al. (2006), Ishibashi and Kaneko (2008), and Brekke et al. (2011).

[8] In Taiwan, many originally publicly owned firms were forced to privatize to enhance their product (or service) quality in the 1990s.

Our study adds an extra subsidy stage into Ishibashi and Kaneko (2008), where they showed that no privatization is necessary when there is no quality competition,[9] while partial privatization is optimal when quality competition is embedded. In contrast with Ishibashi and Kaneko (2008), our paper allows the government to use a uniform subsidy on product quality and finds that under the optimal subsidy, the level of social welfare depends on the degree of exogenous privatization. In other words, privatization neutrality does not hold in our study. Moreover, if the degree of privatization is a choice variable, then either full privatization or zero privatization is the best policy.

The rest of this paper is as follows. Section 2 is the main model and Sect. 3 discusses an alternative setting in prices and provides some discussion. Some concluding remarks are offered in Sect. 4.

2 The model

2.1 General cost functions on quality

Consider a mixed oligopoly framework in a unit-length market with two firms (1 and 2), respectively, located at the two ends of the market (i.e., $x_1 = 0$, and $x_2 = 1$). Following the setting of Matsumura (1998), assume that firm 1 is a public firm, whose objective is to maximize $\Omega = (1-\theta)W + \theta\pi_1$, where W is the social welfare (defined later in (6) and π_1 is its profit), in which θ represents the percentage of stock released to the private sector.[10] Firm 2 is a private firm whose objective is profit maximization. Consumers are uniformly distributed along the linear market. The utility for a consumer located at $x \in [0, 1]$ and purchasing from either firm 1 or firm 2, respectively, is:

$$U_1 = v + q_1 - t(x - x_1)^2 - p_1, \tag{1}$$

or

$$U_2 = v + q_2 - t(x - x_2)^2 - p_2, \tag{2}$$

where v is the reservation price, which is assumed to be large enough to ensure full market coverage, q_1 and q_2 are the quality of their products, t is the transport rate, and p_1 and p_2 are product prices. The disutility of distance is captured by $t(x - x_i)^2$.

Our game structure is three-staged. In the first stage, the government sets a per-unit subsidy (s) based on the product quality of these two firms, and chooses the

[9] When there is no difference on constant marginal cost between the public and the private firms, traditional wisdom says that privatization is not necessary.

[10] Actually, θ is endogenous in our model. However, in this subsection, θ is assumed to be exogenous for convenience. Later, we will discuss how a government can choose the best level of privatization. This arrangement is because the best θ is either 0 or 1 (corner solutions, see Proposition 2 later for details).

degree of privatization θ.[11] In the second stage, both firms choose their product quality simultaneously. In the third stage, both firms decide their product prices simultaneously. The equilibrium quality and prices will be solved by backward induction.

Given s, θ, q_1 and q_2 in the third stage, solving $U_1 = U_2$ yields an indifferent consumer:

$$\hat{x} = \frac{\Delta p - \Delta q + t}{2t}, \tag{3}$$

where $\Delta p \equiv p_2 - p_1$, $\Delta q = q_2 - q_1$. The demand for firm 1 is defined as $y_1 = \hat{x}$, and the demand for firm 2 is $y_2 = 1 - \hat{x}$. Then, the profit functions for firm 1 and firm 2 are:

$$\pi_1 = (p_1 - c) \cdot y_1 - K(q_1) + sq_1, \tag{4}$$

$$\pi_2 = (p_2 - c) \cdot y_2 - K(q_2) + sq_2, \tag{5}$$

where the total cost of producing quantity y_i at quality q_i is $C(y_i, q_i) = cy_i + K(q_i)$. The marginal cost c is constant and independent of quality, and $K(\cdot)$ are cost functions of quality for the firms. As per the settings in the literature, we assume $K' > 0$ and $K'' > 0$. The social welfare is defined as:

$$\begin{aligned} W &= \int_0^{y_1} U_1 \mathrm{d}x + \int_{y_1}^{1} U_2 \mathrm{d} + \pi_1 + \pi_2 - sq_1 - sq_2 \\ &= -c + v - \frac{(\Delta p)^2}{4t} + \frac{(\Delta q)^2}{4t} + \frac{q_1 + q_2}{2} - \frac{t}{12} - K(q_1) - K(q_2). \end{aligned} \tag{6}$$

The first-order conditions for the public firm and the private firm are:

$$\frac{\partial \Omega}{\partial p_1} = \frac{\Delta p}{2t} + \theta\left(\frac{1}{2} - \frac{p_1 - c}{2t} - \frac{q_2 - q_1}{2t}\right) = 0, \tag{7}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} - \frac{p_2 - c}{2t} - \frac{\Delta p}{2t} + \frac{q_2 - q_1}{2t} = 0. \tag{8}$$

Solving (7) and (8) simultaneously yields:

$$p_1(q_1, q_2; \theta) = c + t - \frac{(2\theta - 1)}{2\theta + 1}(q_2 - q_1), \tag{9}$$

[11] To compare with traditional models of privatization neutrality, we assume that the social cost of public funds is unity. In other words, we assume that there is no excess burden of taxation for public funding. For a case of considering subsidization with excess burden of taxation, please refer to Matsumura and Tomaru (2013) for details.

$$p_2(q_1, q_2; \theta) = c + t + \frac{(q_2 - q_1)}{2\theta + 1}. \tag{10}$$

The equilibrium prices in (9) and (10) describe the strategic effects of the quality levels q_1 and q_2, and of the parameter θ. Here, we have several points to note. First, the quality subsidy policy will not directly affect the decisions on equilibrium prices, because s is not included here. Second, given q_1, the higher q_2 is, the higher p_2 is, but given q_2, higher q_1 can result in either higher p_1 (when $\theta < 1/2$), or lower p_1 (when $\theta > 1/2$), meaning that the quality level of the public firm will be affected by the degree of privatization. Finally, Eqs. (9) and (10) imply:

$$\Delta p = \frac{2\theta \Delta q}{2\theta + 1}. \tag{11}$$

Equation (11) implies that the firm with a higher quality product has an advantage in that it can set a higher price. More important, $\Delta p = 0$ only when $\theta = 0$ (pure publicly owned firm)[12] or $\theta = 1$ (fully privatized firm, and thus $\Delta q = 0$ by symmetry). In other words, prices are differentiated ($\Delta p \neq 0$) when $0 < \theta < 1$. Note that our main result (see later in Proposition 2) is crucially derived from this property. When $\Delta p \neq 0$, it induces asymmetry in our model, and the asymmetric consideration on prices will spill over to the quality stage and make the quality subsidy unable to attain the first best.

Plugging (9) and (10) into the objective functions of these two firms yields:

$$\begin{aligned}\Omega(q_1, q_2, \theta) = {} & (1-\theta)\left(-c + v + \frac{(4\theta+1)(\Delta q)^2}{4(2\theta+1)^2 t} + \frac{q_1 + q_2}{2} - \frac{t}{12} - K(q_1) - K(q_2)\right) \\ & + \theta\left(\frac{t}{2} - \frac{\theta \Delta q}{(2\theta+1)} + \frac{(2\theta-1)(\Delta q)^2}{2(2\theta+1)^2 t} - K(q_1) + s q_1\right),\end{aligned} \tag{12}$$

$$\pi_2(q_1, q_2, \theta) = \frac{t}{2} + \frac{\Delta q}{(2\theta+1)} + \frac{(\Delta q)^2}{2(2\theta+1)^2 t} - K(q_2) + s q_2. \tag{13}$$

Thus, the first-order conditions are:

$$\Omega_1 \equiv \frac{\partial \Omega}{\partial q_1} = \frac{\theta+1}{(2\theta+1)}\left[\frac{1}{2} - \frac{\Delta q}{2t(2\theta+1)}\right] - K'(q_1) + s\theta = 0, \tag{14}$$

$$\pi_2^2 \equiv \frac{\partial \pi_2}{\partial q_2} = \frac{1}{(2\theta+1)} + \frac{\Delta q}{(2\theta+1)^2 t} - K'(q_2) + s = 0. \tag{15}$$

Solving (14) and (15) simultaneously yields $q_1(\theta, s) \equiv q_1^*$ and $q_2(\theta, s) \equiv q_2^*$. Then, we have the following comparative statics:

[12] When $\theta = 0$, prices are equal, and the intuition of this result is that in the price stage, for any given q_1 and q_2, the cost of quality can be seen as a sunk cost and can be ignored. Once prices are different, based on the social viewpoint, it will induce some welfare losses in misallocation of resources, which can be captured by the term $-\frac{(\Delta p)^2}{4t}$ in (6).

$$\frac{\partial q_1^*}{\partial s} = \frac{-\theta\pi_{22}^2 + \Omega_{12}}{D}, \tag{16}$$

$$\frac{\partial q_2^*}{\partial s} = \frac{-\Omega_{11} + \theta\pi_{21}^2}{D}, \tag{17}$$

where $D \equiv \begin{vmatrix} \Omega_{11} & \Omega_{12} \\ \pi_{21}^2 & \pi_{22}^2 \end{vmatrix} > 0$, is assumed by the stability condition, and $\Omega_{11} \equiv \frac{\partial^2\Omega}{\partial q_1^2} = \frac{\theta+1}{2(2\theta+1)^2 t} - K''(q_1^*) < 0$, $\pi_{22}^2 = \frac{1}{2t(2\theta+1)^2} - K''(q_2^*) < 0$, are based on the objective maximization. Note that $\Omega_{12} \equiv \frac{\partial^2\Omega}{\partial q_1 \partial q_2} = \frac{-(\theta+1)}{2t(2\theta+1)^2} < 0$ for $\theta \in [0, 1]$ and $\pi_{21}^2 \equiv \frac{\partial^2\pi_2}{\partial q_2 \partial q_1} = \frac{-1}{2t(2\theta+1)^2} < 0$ in our model, which imply that they are strategically substitutive in the second stage. In general, the signs of (16) and (17) are ambiguous. However, when θ is small enough, $\partial q_1^*/\partial s < 0$ and $\partial q_2^*/\partial s > 0$. Moreover, combining (16) and (17) leads to:

$$\frac{\partial \Delta q}{\partial s} = \frac{K''(q_1^*) - \theta K''(q_2^*)}{D}, \tag{18}$$

which is positive when θ is small enough, meaning that a positive subsidy will induce an increase in quality differentiation. Therefore, when $\theta = 0$, a negative subsidy (i.e., taxation) can induce lower quality differentiation.

We then solve for the optimal θ and s for the government. Unfortunately, θ is a corner solution instead of an interior one. Therefore, we cannot use a traditional first-order condition approach to solve for the optimal θ, but we can temporarily take θ as exogenous, and then compare the social welfare levels under different values of θ. First, given a $\theta \in [0, 1]$, plugging $(q_1(\theta, s), q_2(\theta, s))$ into the social welfare function yields:

$$\begin{aligned} &W(q_1(\theta, s), q_2(\theta, s), \theta) \\ &= -c + v + \frac{(4\theta + 1)(\Delta q)^2}{4t(2\theta + 1)^2} + \frac{q_1^* + q_2^*}{2} - \frac{t}{12} - K(q_1^*) - K(q_2^*). \end{aligned} \tag{19}$$

Differentiating W with respect to s yields:

$$\frac{dW}{ds} = \left(-\frac{(4\theta + 1)\Delta q}{2t(2\theta + 1)^2} + \frac{1}{2} - K'(q_1^*)\right)\frac{\partial q_1^*}{\partial s} + \left(\frac{(4\theta + 1)\Delta q}{2t(2\theta + 1)^2} + \frac{1}{2} - K'(q_2^*)\right)\frac{\partial q_2^*}{\partial s} = 0. \tag{20}$$

Let $s(\theta)$ satisfy (20) and the second-order condition is assumed to be satisfied:[13] and plugging (14) and (15) into (20) to delete $K'(q_1^*)$ and $K'(q_2^*)$, then we have:

[13] The second-order condition can be expressed as:

$$\begin{aligned} \frac{d^2W}{ds^2} =& \frac{4\theta + 1}{2t(2\theta + 1)^2}\left(\frac{\partial(\Delta q)}{\partial s}\right)^2 + \left(-\frac{(4\theta + 1)\Delta q}{2t(2\theta + 1)^2} + \frac{1}{2} - K'(q_1^*)\right)\frac{\partial^2 q_1^*}{\partial s^2} \\ &+ \left(\frac{(4\theta + 1)\Delta q}{2t(2\theta + 1)^2} + \frac{1}{2} - K'(q_2^*)\right)\frac{\partial^2 q_2^*}{\partial s^2} - K''(q_1^*)\left(\frac{\partial q_1^*}{\partial s}\right)^2 - K''(q_2^*)\left(\frac{\partial q_2^*}{\partial s}\right)^2 \le 0. \end{aligned}$$

$$s(\theta) = \frac{\left(-\frac{3\theta\Delta q}{2t(2\theta+1)^2} + \frac{\theta}{2(2\theta+1)}\right)\frac{\partial q_1^*}{\partial s} + \left(\frac{(4\theta-1)\Delta q}{2t(2\theta+1)^2} + \frac{2\theta-1}{2(2\theta+1)}\right)\frac{\partial q_2^*}{\partial s}}{\theta\frac{\partial q_1^*}{\partial s} + \frac{\partial q_2^*}{\partial s}}. \tag{21}$$

Let $SW(\theta) \equiv W(q_1(\theta, s(\theta)), q_2(\theta, s(\theta)), \theta)$. Obviously, the optimal subsidy is related to the degree of privatization (θ), as shown in (21). However, a complicated result of $\mathrm{d}s(\theta)/\mathrm{d}\theta$ is unmanageable.[14] However, it is easy to have $s(0) = -\frac{1}{2}$ and $s(1) = \frac{1}{6}$, and both cases yield the first-best quality (see Appendix 1). However, when $0 < \theta < 1$, it is shown that the first-best quality allocation is unattainable (see Appendix 2). This implies that for $0 < \theta < 1$, we have $SW(\theta) < SW(0) = SW(1)$; that is, partial privatization will never be chosen by the government.[15] Therefore, either $\theta = 0$ or $\theta = 1$ is the best policy for privatization when θ is endogenous. We can summarize the above results as the following propositions.

Proposition 1 *The optimal subsidy rate does depend on the degree of privatization, which is contrary to the previous studies such as White* (1996) *and Kato and Tomaru* (2007). *Specifically, the optimal policy is either taxation* ($s(0) = -1/2 < 0$) *when the public firm is fully owned by the government, or subsidy* ($s(1) = 1/6 > 0$) *when it is fully privatized.*

Proposition 2 *The social welfare levels depend on the degree of privatization. That is, the privatization neutrality theorem is not valid in our study. Specifically, the optimal privatization is either zero privatization or full privatization.*

The intuition of Proposition 1 is simple. When there is no quality subsidy policy, as shown in the Proposition 1 in Ishibashi and Kaneko (2008), when $\theta = 0$ the product quality of the public firm will be lower than the social optimal level, while the product quality of the private firm will be higher than the social optimum. According to Eq. (18), a subsidy will enlarge the difference of product quality when θ is small. Therefore, when $\theta = 0$, the optimal corrective policy should be taxation, instead of subsidy, to reduce the difference in product quality. In contrast, the product quality of these two firms is lower than the first best when $\theta = 1$. Therefore, it is proper to use a positive subsidy on product quality to raise their quality.

For $0 < \theta < 1$, although $s(\theta)$ is complicated, as shown in Eq. (21), a numerical analysis in Sect. 2.2 demonstrates that $s(\theta)$ is monotonically increasing in θ (see Fig. 1), given that the cost function on quality is quadratic in quality [see later in Eq. (22)]. In other words, this suggests taxation for a small θ and a subsidy for a large θ.

To the best of our knowledge, this is the first paper showing that both full privatization and zero-privatization cases yield the first-best outcome, but partial privatization

[14] In Sect. 2.2, we assume quadratic cost functions on quality and find that $s(\theta)$ is a monotonically increasing function: $ds(\theta)/d\theta > 0$.

[15] If the government uses discriminatory subsidies, the first-best quality for firms can always be reached by proper subsidies, no matter what θ is. Therefore, the social welfare of the first-best outcome is unrelated to the degree of privatization. However, the optimal subsidy rates are indeed correlated with the degree of privatization. The detailed proof is available upon request.

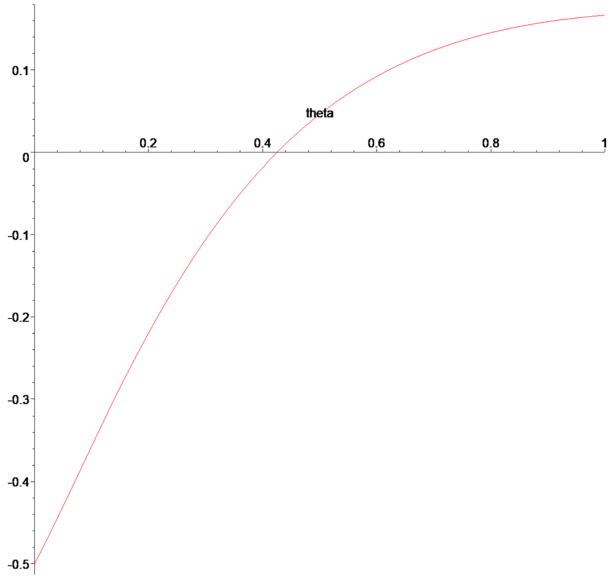

Fig. 1 The relationship between $s(\theta)$ and θ

does not, and the social welfare level under the optimal subsidy depends on the degree of privatization, which is contrary to the traditional wisdom of the neutrality of privatization (White 1996; Kato and Tomaru 2007). The reason behind Proposition 2 is that our model is a multi-stage strategic commitment game for firms, and they have different objectives, which induce different price strategies in the price stage, and these strategies will spill over to the quality stage. Specifically, when the government has only one policy tool (subsidy on quality), while firms have two-stage competition strategies, the optimal subsidy rate and social welfare depend on the degree of privatization, due to asymmetric strategic consideration by firms.[16] In other words, if there exists price differentiation in equilibrium under partial privatization, then the spillover effect will result in quality subsidy being unable to attain the first-best outcome. In contrast, when $\theta = 0$ or $\theta = 1$, price differentiation vanishes, and thus, the spillover effect also disappears; therefore, quality subsidy can obtain the first-best outcome. However, when $0 < \theta < 1$ (existing price spillover effect), the equilibrium prices are not identical, as shown in (11), bringing asymmetry to our model, and it is impossible to reach the first-best quality as per (14) and (15) (see Appendix 2). Therefore, the optimal privatization is either zero privatization or full privatization.

2.2 Quadratic cost functions on quality

We see that $s(\theta)$ is complicated in (21), which results from a general cost function $K(q_i)$. To have an explicit solution of $s(\theta)$, suppose that the cost function on quality is specific for both firms, such that

[16] We thank one of the anonymous referees for offering this explanation to us.

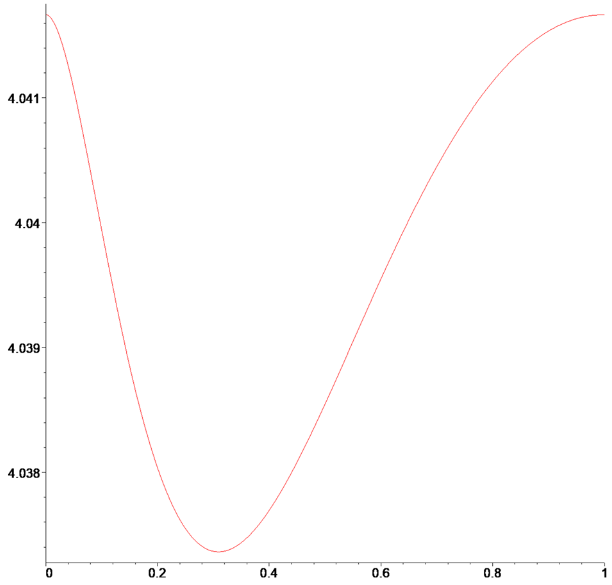

Fig. 2 The relationship between $SW(\theta)$ and θ

$$K(q_i) = \alpha q_i^2. \tag{22}$$

Then, all endogenous variables can be explicitly solved. However, the solution is still complicated. Therefore, to focus on the relationships between subsidy (s) and the degree of privatization (θ), and between social welfare (SW) and θ, we let $\alpha = 1$, $t = 1$, $c = 1$, and $v = 5$ in this subsection. After some calculations similar to (19) and (20), we obtain the optimal subsidy:

$$s(\theta) = \frac{128\theta^6 + 528\theta^5 + 424\theta^4 + 34\theta^3 - 63\theta^2 - 24\theta - 3}{2(1 + 2\theta)(128\theta^6 + 224\theta^5 + 296\theta^4 + 240\theta^3 + 107\theta^2 + 26\theta + 3)},$$

which is drawn in Fig. 1. Note that $s(0) = -1/2$, $s(1) = 1/6$ and $s(\theta)$ is monotonically increasing in $\theta \in [0, 1]$. Proposition 1 can be verified in this figure. The social welfare under the optimal subsidy is;

$$\mathrm{SW}(\theta) = \frac{291 + 2522\theta + 10355\theta^2 + 23232\theta^3 + 28688\theta^4 + 22016\theta^5 + 12224\theta^6}{24(128\theta^6 + 224\theta^5 + 296\theta^4 + 240\theta^3 + 107\theta^2 + 26\theta + 3)},$$

which is drawn in Fig. 2. In words, the optimal subsidy is monotonically increasing in the degree of privatization. Specifically, when θ is small, a negative subsidy (i.e., taxation) is necessary for maximizing social welfare. In contrast, when θ is large, a positive subsidy is required to obtain the maximal social welfare. Moreover, $\mathrm{SW}(\theta)$ has no monotonic relationship in θ, and $\mathrm{SW}(\theta) < \mathrm{SW}(0) = \mathrm{SW}(1)$, $\forall\ \theta \in (0, 1)$ as shown in Proposition 2.

3 An alternative setting and discussion

In this section, we will highlight that the multi-stage setting is crucial for the validity of privatization neutrality.[17] Suppose that the product prices are regulated by the government, such that $p_1 = p_2 = \bar{p}$, and other assumptions are kept. Therefore, firms can only choose their quality. In the first stage, the government announces $\bar{p}$ and s. In the second stage, firms choose q_1 and q_2 simultaneously. The utilities for a consumer x purchasing a unit of product from firms 1 and 2 are as follows (this scenario is denoted by an upper bar):

$$\overline{U}_1 = v + q_1 - t(x - x_1)^2 - \bar{p},$$
$$\overline{U}_2 = v + q_2 - t(x - x_2)^2 - \bar{p}.$$

Solving $\overline{U}_1 = \overline{U}_2$ yields the indifferent consumer $\bar{x}$:

$$\bar{x} = \frac{t - \Delta q}{2t}.$$

Then, p_1 and p_2 in (4) and (5) are replaced with $\bar{p}$, and $\Delta p = 0$ is substituted into (6). After some calculations, we have:

$$\begin{aligned}\overline{\Omega} &= (1-\theta)\overline{W} + \theta\overline{\pi}_1 \\ &= (1-\theta)\left(v - c + \frac{(q_2 - q_1)^2}{4t} + \frac{q_1 + q_2}{2} - \frac{t}{12} - K(q_1) - K(q_2)\right) \\ &\quad + \theta\left((\bar{p} - c)\frac{(q_1 - q_2 + t)}{2t} - K(q_1) + sq_1\right), \\ \overline{\pi}_2 &= (\overline{p} - c)\left(1 - \frac{q_1 - q_2 + t}{2t}\right) - K(q_2) + sq_2.\end{aligned}$$

The first-order conditions are:

$$\overline{\Omega}_1 \equiv \frac{\partial\overline{\Omega}}{\partial q_1} = \frac{1}{2t}\big((1-\theta)(t - \Delta q) + \theta(\overline{p} - c + 2ts)\big) - K'(q_1) = 0, \tag{23}$$

$$\overline{\pi}_2^2 = \frac{\partial\overline{\pi}_2}{\partial q_2} = \frac{1}{2t}\big(\overline{p} - c + 2st\big) - K'(q_2) = 0. \tag{24}$$

Simultaneously, solving (23) and (24) yields the optimal quality levels $q_1(\theta, s) \equiv \bar{q}_1$ and $q_2(\theta, s) \equiv \bar{q}_2$. The comparative statics are as follows:

[17] The setting in Sect. 2 is multi-staged in a quality-then-price framework. Section 3 will provide a scenario where competition is purely on quality, and prices are regulated by the government, such as the university competition in Taiwan, and will highlight the importance of the multi-stage setting.

$$\frac{\partial \bar{q}_1}{\partial s} = \frac{-\theta \overline{\pi}_{22}^2 + \overline{\Omega}_{12}}{\overline{D}}, \tag{25}$$

$$\frac{\partial \bar{q}_2}{\partial s} = \frac{-\overline{\Omega}_{11} + \theta \overline{\pi}_{21}^2}{\overline{D}} > 0, \tag{26}$$

where $\overline{\Omega}_{12} = \frac{\theta-1}{2t} < 0$, $\overline{\pi}_{21}^2 = 0$, $\overline{\Omega}_{11} = \frac{1-\theta}{2t} - K''(q_1)$, $\overline{\pi}_{22}^2 = -K''(q_2) < 0$, and $\overline{D} = \begin{vmatrix} \overline{\Omega}_{11} & \overline{\Omega}_{12} \\ \overline{\pi}_{21}^2 & \overline{\pi}_{22}^2 \end{vmatrix} = \overline{\Omega}_{11} < 0$ if the second-order condition for firm 1 is imposed. The sign of (25) is ambiguous, unless θ is small enough. In this section, (26) is always positive, because $\overline{\pi}_{21}^2 = 0$. Plugging $\bar{q}_1$ and $\bar{q}_2$ into the social welfare function yields $W(\bar{q}_1, \bar{q}_2, \theta)$. Differentiating W with respect to s yields:

$$\frac{d\overline{W}}{ds} = \frac{\partial \overline{W}}{\partial \overline{q}_1} \cdot \frac{\partial \overline{q}_1}{\partial s} + \frac{\partial \overline{W}}{\partial \overline{q}_2} \cdot \frac{\partial \overline{q}_2}{\partial s} = 0, \tag{27}$$

where

$$\frac{\partial \overline{W}}{\partial \overline{q}_1} = -\frac{\overline{q}_2 - \overline{q}_1}{2t} + \frac{1}{2} - K'(\overline{q}_1),$$
$$\frac{\partial \overline{W}}{\partial \overline{q}_2} = \frac{\overline{q}_2 - \overline{q}_1}{2t} + \frac{1}{2} - K'(\overline{q}_2).$$

The optimal subsidy s^o must satisfy (27), which includes two general terms, $K''(\overline{q}_1)$ and $K''(\overline{q}_2)$, and thus, we cannot obtain the optimal subsidy directly from (27). However, we can let $K'(q_1) = K'(q_2) = 1/2$ in (23) and (24) as per Ishibashi and Kaneko (2008, pp. 218) to solve the optimal subsidy $s^o = 1/2 + \frac{c-\bar{p}}{2t}$, and thus, $W(\overline{q}_1, \overline{q}_2, \theta) = v - c - \frac{t}{12} + \overline{q} - 2K(\overline{q})$, where $\overline{q} = \overline{q}_1 = \overline{q}_2$ and $K'(\overline{q}) = 1/2$ represents the first-best quality, and the social welfare level here is independent of θ, which is identical to the privatization neutrality theorem in the traditional studies such as White (1996) and Kato and Tomaru (2007).[18]

Finally, our model structure is mathematically equivalent to that in Matsumura and Matsushima (2004), who discussed cost-reducing R&D investment. If our subsidy (or taxation) scenario is applied to Matsumura and Matsushima (2004), then it would suggest a tax ($s < 0$) on R&D to correct over-investment, and a subsidy ($s > 0$) in case of two private firms ($\theta = 1$), and the social welfare will depend on the degree of privatization.[19]

[18] It is worth noting that the symmetry setting is crucial in our model. If the regulated prices are $\overline{p}_1 \neq \overline{p}_2$, then the optimal subsidy will depend on θ and so will W. Therefore, the privatization neutrality theorem is not satisfied, implying that the theorem is very sensitive to symmetry.

[19] We thank one of the anonymous referees for pointing out this comparison.

4 Conclusion

We set up a mixed oligopoly model with quality competition, price competition, and a subsidy on firms' quality. A multi-stage game (quality then price) for mixed oligopoly firms is employed to discuss the neutrality of privatization, as well as the optimal subsidy policy. It is shown that the setting of multiple stages is crucial for the validity of privatization neutrality. In contrast to the one-stage game, a policy tool can solve for distortion and reach the first-best outcome, while in an asymmetric multi-stage game for mixed oligopoly firms (one public and one private), if there exists a price spillover effect on quality competition, then one policy tool cannot totally eliminate the overall distortion, and the first-best outcome cannot be reached, except when the public firm is fully owned by the government or fully privatized. Under optimal subsidies, it is shown that the levels of social welfare will depend on the degree of privatization, and thus, the neutrality of privatization is not valid. Finally, a more general multi-stage setting is worth exploring in the future.

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Appendix

Appendix 1: The proof of the first-best quality allocation can be reached by a subsidy when $\theta = 0$ or $\theta = 1$.

Let $q_1(\theta, s(\theta)) \equiv q_1^*$, $q_2(\theta, s(\theta)) \equiv q_2^*$.

(i) When $\theta = 0$, from (14) and (15),

$$\frac{1}{2} - \frac{q_2^* - q_1^*}{2t} - K'(q_1^*) = 0, \tag{A1}$$

$$1 - \frac{q_2^* - q_1^*}{t} - K'(q_2^*) + s(0) = 0. \tag{A2}$$

It is easy to show that if we let $s(0) = -\frac{1}{2}$, then (q_1^*, q_2^*) will satisfy the first-best quality allocation $K'(q_1^*) = K'(q_2^*) = \frac{1}{2}$ (see Ishibashi and Kaneko (2008), pp. 218 for details).

(ii) When $\theta = 1$, from (14) and (15):

$$\frac{1}{3} + \frac{q_2^* - q_1^*}{9t} - K'(q_1^*) + s(1) = 0, \tag{A3}$$

$$\frac{1}{3}+\frac{q_2^*-q_1^*}{9t}-K'(q_2^*)+s(1)=0. \tag{A4}$$

Thus, if we let $s(1)=1/6$, then (q_1^*,q_2^*) will also satisfy $K'(q_1^*)=K'(q_2^*)=1/2$. □

Appendix 2: The proof that the first-best solution cannot be reached by a subsidy when $0<\theta<1$.

Given $0<\theta<1$, assume that the first-best solution (q_1^*,q_2^*) can be reached under a subsidy policy. That is, $K'(q_1^*)=K'(q_2^*)=\frac{1}{2}$. From (14) and (15), we have:

$$\frac{\theta+1}{2(2\theta+1)}-K'(q_1^*)+s\theta=0, \tag{A5}$$

$$\frac{1}{(2\theta+1)}-K'(q_2^*)+s=0. \tag{A6}$$

Simultaneously, solving (A5) and (A6) and deleting s yield $(1-\theta)(\frac{1}{2(2\theta+1)}-K'(q_1^*))=0$, which implies $K'(q_1^*)\neq 1/2$, unless $\theta=0$ or $\theta=1$, a contradiction. □

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