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Can risk-neutral skewness and kurtosis subsume the information content of historical jumps?

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ABSTRACT

We examine the relation between jump variations and risk-neutral moments in volatility forecasting. We propose a method that involves no extrapolation in computing the risk-neutral moments of Bakshi et al. (2003) and document that risk-neutral skewness and kurtosis subsume the information content of historical jumps. While historical jumps have significant explanatory power for future volatility and such power is actually not weakened by the inclusion of risk-neutral volatility in models, their predictability does disappear when risk-neutral skewness and kurtosis are included.

1. Introduction

Since the 2007–2009 financial crisis, price jumps have become more and more common and material, in particular, to volatility forecasting. They result in return distributions exhibiting a skewed and fat-tailed shape. In this study, we posit that if implied skewness and kurtosis are measured appropriately, then they could perform better than implied volatility in subsuming information on historical jumps for volatility forecasting. In order to effectively address this issue, we propose a method without involving any extrapolation in computing the risk-neutral moments of Bakshi et al. (2003) on the grounds that extrapolation may potentially distort tail distributions. We propose two hypotheses. The first is that the method which does not involve any extrapolation produces better measures of risk-neutral skewness and kurtosis than the methods that do. The second hypothesis is that risk-neutral skewness and kurtosis perform better than risk-neutral volatility in terms of subsuming information on historical jumps for volatility forecasting.

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Jiang and Tian (2005, 2007) develop two methods that use observed option prices for the computation of their model-free volatility. Given that only limited-range strike prices are traded in the market, a flat extrapolation scheme is adopted in Jiang and Tian (2005). However, in Jiang and Tian (2007), they criticize their earlier method and argue that the flat extrapolation scheme has drawbacks. In this paper, we argue that methods involving extrapolation may distort tail distributions, potentially distorting the information content of implied skewness and kurtosis.

There is evidence in the literature that higher volatility is associated more with negative returns than positive returns.[4] However, the associated jumps are difficult to measure. Thanks to Barndorff-Nielsen and Shephard (2004, 2006) and Andersen et al. (2007), the total variance component attributable to the jump process in prices can now be extracted using high-frequency returns data.

Becker et al. (2009) find that the VIX not only subsumes information on historical jump activity, but also reflects the incremental information pertaining to future jump activity. Busch et al. (2011) discover that implied volatility contains incremental information on future volatility components, relative to past continuous and jump components. Byun and Kim (2013) detect whether risk-neutral skewness and kurtosis have incremental explanatory power on future volatility.

The primary purpose of this paper is to evaluate the performance of the volatility forecasting methods and to improve the accuracy of the forecasts of future volatility. Overall, we find that risk-neutral skewness and kurtosis measures without involving extrapolation subsume the information content of historical jump variations in the heterogeneous autoregressive realized volatility (HAR-RV) model.[5]

Our paper is closely related to Byun and Kim (2013). However, several major differences exist. First, Byun and Kim investigate whether risk-neutral skewness has incremental explanatory power for future volatility in the S&P 500 Index, while we examine the issue on employing extrapolation for computation of risk-neutral moments and investigate the relation between realized jumps and risk-neutral moments using data on the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX).[6] Second, Byun and Kim only find that risk-neutral skewness contains information for future volatility. In contrast, we find that both risk-neutral skewness and kurtosis subsume the information of historical jump variations and contain incremental information. Third, we also find that jumps have statistically significant forecasting ability; however, the predictive power of historical jumps is sharply weakened by both risk-neutral skewness and kurtosis.

The rest of the paper proceeds as follows. In Section 2, we discuss volatility components and risk-neutral moments. Section 3 presents our empirical design and data. Section 4 presents the results and the final section concludes the paper.

2. Volatility components and risk-neutral moments

2.1. Volatility components

It is well known that realized volatility is obtained by:

$$RV_{t-1,t} = \sum_{i=1}^{M} r_{t,i}^2, \tag{1}$$

where $r_{t,i}$ is the ith intraday return on day t; $M = 1/\Delta$ indicates the number of intraday returns in a day; and Δ is the time interval. We follow Bollerslev et al. (2009) and Andersen et al. (2011) to use the 5-min returns of the TAIEX to obtain the realized volatility and the continuous and jump variation estimators; thus, $M = 54$. We compute the realized volatility for our assessment of the performance of the various models for daily, weekly, and monthly forecasting horizons, which is expressed as:

$$RV_{t-h,t} = \left(RV_{t-h,t-h+1} + RV_{t-h+1,t-h+2} + \cdots + RV_{t-1,t}\right) / h,$$

where $h = 1$, 5 and 22. All of the volatility measures are annualized.

Andersen et al. (2003) show that $RV_{t-1,t}$ converges in probability to the quadratic variation as the time interval between the observations becomes smaller; therefore, $RV_{t-1,t}$ captures both the continuous and discontinuous ("jump") components of volatility. We adopt three definitions for our measurement of the jump component in total variance. The first was defined by Andersen et al. (2007) and denoted as $J_{t\text{-}1,t}$, the second by Andersen et al. (2012) and denoted as $MJ_{t\text{-}1,t}$, and the third by

[4] Examples include Bollerslev et al. (1994), Bollerslev et al. (2006), and Andersen et al. (2006). Patton and Sheppard (2015) use a new estimator, realized semi-variance, proposed by Barndorff-Nielsen et al. (2010), as the means of decomposing the usual realized variance into two components, one (the other) relating to positive (negative) high-frequency returns.

[5] We are grateful to an anonymous referee for drawing our attention to accounting for the autoregressive structure of volatility. The HAR-RV model was proposed by Corsi (2009). The main motivation for using this model is that investors with different time horizons perceive, react to, and cause different types of volatility components. Corsi identified three primary volatility components: (1) short-term traders with daily or higher trading frequency (i.e., past daily realized volatility); (2) medium-term investors who typically rebalance their positions weekly (i.e., past weekly realized volatility); and (3) long-term agents with a characteristic time of one month or more (i.e., past monthly realized volatility).

[6] Note that the TAIEX, unlike the S&P 500, is characterized by the high individual participation.

Patton and Sheppard (2015) and denoted as $SJ_{t\text{-}1,t}$. Their continuous components of volatility are denoted as $C_{t\text{-}1,t}$, $MC_{t\text{-}1,t}$, and $BV_{t\text{-}1,t}$, respectively. The definitions of these volatility components can be found in the papers mentioned above.[7]

2.2. Risk-neutral moments

The Britten-Jones and Neuberger (2000) estimator is often used as a proxy for implied volatility. However, Du and Kapadia (2013) conduct an extensive simulation exercise to assess the impact of jumps on implied volatility and recommend the Bakshi et al. (2003) estimator as a jump-robust estimator. In addition, Bakshi et al. also develop estimators of skewness and kurtosis. Interested readers can refer to Bakshi et al. (2003) for the definitions of risk-neutral volatility, skewness, and kurtosis.

The risk-neutral moments are defined as an integral of the option prices over an infinite range of strike prices; however, only a limited range and sparse set of discrete strike prices are listed for trading in the marketplace, which may lead to inaccuracies in the calculation of the risk-neutral volatility. Jiang and Tian (2005, 2007) argue that truncation errors arise if only the limited range of strike prices is used in the calculation, and therefore propose the use of an extrapolation scheme. However, the use of extrapolation may potentially distort tail distributions and thus the information content of risk-neutral skewness and kurtosis. In this study, we refer to our method without involving any extrapolation as the “naïve” method. As for methods developed by Jiang and Tian (2005, 2007), we refer those as the JT05 and JT07 method, respectively.[8]

In order to compute the risk-neutral moments, we first need to fit the Black-Scholes (1973) implied volatility curve. To back-solve the Black-Scholes implied volatilities, we apply put-call parity to create the “implied TAIEX” (referred to as the ex-dividend TAIEX) so as to avoid any estimation errors in the dividend rate. We adopt the TAIEX based upon 1-min intervals to define moneyness, and retain the final trade prices of the out-of-the-money options with trading times prior to 1:30 p.m. (UTC+8). We then apply cubic splines to connect discrete points for the fitting of a smoothing implied volatility curve. In order to eliminate the effects of the differences in time to maturity, we need two smooth implied volatility curves with different time-to-maturity periods, which are referred to as τ_1 and τ_2; we can then apply linear interpolation to construct a 30-calendar-day implied volatility curve.

3. Empirical design and data

3.1. Empirical design

We use the HAR-RV model as the base model, which is expressed as:

$$RV_{t,t+h} = \beta_0 + \beta_D RV_{t-1,\,t} + \beta_W RV_{t-5,\,t} + \beta_M RV_{t-22,\,t} + \varepsilon_{t,t+h}. \tag{2}$$

We follow Andersen et al. (2007) and employ a model that is capable of detecting which estimator of the jump component has predictive power on total realized volatility; the model is expressed as:

$$RV_{t,t+h} = \beta_0 + \beta_C C_{t-1,\,t} + \beta_J J_{t-1,\,t} + \beta_W RV_{t-5,\,t} + \beta_M RV_{t-22,\,t} + \varepsilon_{t,t+h}, \tag{3}$$

where C and J, respectively, denote the continuous and jump components of the total variance.

Once we have confirmed the predictive power of the jump components, the risk-neutral moments are then added into the model to examine whether the predictive power of the jump variation is changed. The model is expressed as:

$$RV_{t,t+h} = \beta_0 + \beta_C C_{t-1,\,t} + \beta_J J_{t-1,\,t} + \beta_W RV_{t-5,\,t} + \beta_M RV_{t-22,\,t} + \beta_V Var_{\,t} + \beta_S Skew_{\,t} + \beta_K Kurt_{\,t} + \varepsilon_{t,t+h}. \tag{4}$$

In order to assess the role of each risk-neutral moment in capturing the predictive power of jump variations, we sequentially introduce risk-neutral volatility ($Var_{\,t}$), skewness ($Skew_{\,t}$), and kurtosis ($Kurt_{\,t}$) into the model.

Because the risk-neutral skewness and kurtosis computed by the naïve method do not only subsume the historical jump information, but also reflect the incremental information on future volatility, we propose the following model for volatility forecasting:

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[7] To facilitate a comparison with the jump estimators proposed by Andersen et al. (2007) and Andersen et al. (2012), as opposed to the average of the skip-0 to skip-4 estimators developed by Patton and Sheppard (2015), we use the following bipower variation estimator for our empirical analysis: $BV_{t-1,t} = \mu_1^{-2}(\mathrm{M}/(\mathrm{M\text{-}2}))\sum_{i=1}^{M-2}|r_{t,i}||r_{t,i+2}|$, where $\mu_1 = \sqrt{2/\pi}$.

[8] Let K_l (K_u) denote the minimum (maximum) strike price in the marketplace. The interval [K_{min}, K_{max}] defines the range of strike prices that are used to compute the risk-neutral moments. Jiang and Tian (2005) demonstrate that if the left and right truncation points, K_{min}, K_{max}, are three multiples of standard deviations from the initial underlying asset price, then the truncation error will be negligible. They employ a flat extrapolation scheme when $K_{min} < K_l$ or $K_{max} > K_u$, assuming that the implied volatility function is flat beyond the listed strike prices. Jiang and Tian (2007) adjust the slope of the extrapolation segment on both sides in order to match the corresponding slope of the interior segment at K_l or K_u. We use the daily average implied Black-Scholes volatility to be the estimator of standard deviation for TAIEX's returns when computing the risk-neutral moments by the JT05 and JT07 methods.

Table 1

Summary statistics on daily realized volatility including the continuous and discontinuous components

This table reports the summary statistics of daily realized volatility in the TAIEX, including its continuous and discontinuous parts, for a sample period running from January 3, 2005 to December 31, 2019, which provides a total of 3707 daily observations. *RV* denotes daily realized volatility; *C*(*MC*) and *J*(*MJ*) are the respective continuous and discontinuous components, as defined by Andersen et al. (2007, 2012); *BV* refers to the daily bipower variation. *SJ* is the daily signed jump variation; *JP* (*JN*) indicates the positive (negative) jumps, where all are as defined by Patton and Sheppard (2015). The LB_6 column provides the Ljung-Box test statistic for up to sixth-order serial correlation.

Variables	Mean	Median	S.D.	Skewness	Kurtosis	Min.	Max.	LB_6
$RV_{t-1,t}$	0.0305	0.0135	0.0642	7.98	87.50	0.0017	1.149	3166
$C_{t-1,t}$	0.0174	0.0092	0.0249	4.45	28.56	0.0006	0.288	8128
$J_{t-1,t}$	0.0132	0.0000	0.0521	10.25	140.95	0.0000	1.123	549
$MC_{t-1,t}$	0.0301	0.0130	0.0643	7.98	87.44	0.0014	1.149	3117
$MJ_{t-1,t}$	0.0004	0.0000	0.0024	11.05	169.54	0.0000	0.055	1.88
$BV_{t-1,t}$	0.0154	0.0081	0.0222	4.73	33.41	0.0006	0.286	8712
$SJ_{t-1,t}$	−0.0008	0.0008	0.0580	−3.47	111.94	−1.1063	0.841	51
$JP_{t-1,t}$	0.0086	0.0008	0.0337	13.69	256.93	0.0000	0.841	165
$JN_{t-1,t}$	−0.0094	0.0000	0.0455	−12.51	208.48	−1.1063	0.000	317

$$RV_{t,t+h} = \beta_0 + \beta_D C_{t-1,\,t} + \beta_W C_{t-5,\,t} + \beta_M C_{t-22,\,t} + \beta_V Var_{\,t} + \beta_S Skew_{\,t} + \beta_K Kurt_{\,t} + \varepsilon_{t,t+h}. \tag{5}$$

3.2. Data

We employ two data sets: intraday data on the TAIEX and intraday data on TAIEX options. The sample period is January 3, 2005 to December 31, 2019.[9] The risk-free interest rate is proxied by the simple average of the one-month time deposit interest rates of the five major banks in Taiwan, whilst the data on the one-month time deposit rates are obtained from the website of the Central Bank of the Republic of China.[10]

The data on TAIEX options, obtained from the *Taiwan Economic Journal* (TEJ), includes the expiration month, strike price, trading volume, transaction price, trading date, and trading time, with all of the transactions being time-stamped to the second. TAIEX options are European-style options, with the TAIEX as the underlying index, and expiration dates of three near-term months followed by two additional months from the March quarterly cycle (March, June, September, and December).

In order to avoid microstructure-related bias, options with maturities of less than one week are excluded from the sample, which means that we selected the two near-month options with at least eight days until maturity for the computation of the risk-neutral moments. The operating hours in the Taiwan Futures Exchange (TAIFEX) are 8:45 a.m. to 1:45 p.m. (UTC+8). Although the TAIFEX launched an after-hours trading system on May 15, 2016, we exclude all transaction data during the after-hours trading period.

3.3. Summary statistics

Table 1 shows that realized variance ($RV_{t-1,\,t}$) has right-skewed and fat-tailed distribution, with a mean of 0.0305, while the standard Ljung-Box statistic, for up to sixth-order serial correlation (LB_6), is 3,166, which means that the realized volatility exhibits a high degree of inherent serial correlation. When we decompose the realized volatility into a continuous component and a jump component, based upon the definition of Andersen et al. (2007) and denoted as $C_{t-1,\,t}$ and $J_{t-1,\,t}$, they have respective LB_6 statistics of 8128 and 549.[11] The mean of the $J_{t-1,\,t}$ series accounts for 43% of the mean of $RV_{t-1,\,t}$, implying that jumps are important. The continuous and jump components of Andersen et al. (2012) are denoted as $MC_{t-1,\,t}$ and $MJ_{t-1,\,t}$.[12] The jumps here are the least important, with the mean of the $MJ_{t-1,\,t}$ series accounting for just 1% of the mean of $RV_{t-1,\,t}$. If the jump component is measured using the Patton and Sheppard (2015) approach, denoted here as $SJ_{t-1,\,t}$, we find that the

[9] The choice of this sample period is essentially due to the intraday data on the TAIEX offered on the website of the Taiwan Stock Exchange (TWSE) becoming available from October 15, 2004 onwards. The data were initially published every minute until January 16, 2011 when the frequency was changed to every 15 s; this was changed to every 10 s on February 23, 2014, and to every 5 s on December 29, 2014, where it currently remains. The operating hours of the TWSE run from 9:00 a.m. to 1:30 p.m. (UTC+8).

[10] The website of the Central Bank of the Republic of China is available at: http://www.cbc.gov.tw.

[11] Compared to the dynamic dependence in the continuous sample path price movements, there is significantly less inherent dynamic dependence in the proportion of the overall quadratic variation originating from the discontinuous sample path price process.

[12] The LB_6 statistics for $MC_{t-1,\,t}$ and $MJ_{t-1,\,t}$ are 3117 and 1.88, respectively, which are markedly lower than those of $C_{t-1,\,t}$ and $J_{t-1,\,t}$. It is worthwhile to note that the Ljung-Box statistic of $MJ_{t-1,\,t}$, at 1.88, indicates that the null hypothesis of no serial correlation cannot be rejected.

Table 2
Summary statistics on risk-neutral moments
This table reports the summary statistics on risk-neutral volatility, skewness and kurtosis in TAIEX options for a sample period running from January 3, 2005 to December 31, 2019, which provides a total of 3707 daily observations. The risk-neutral volatility, skewness and kurtosis in TAIEX options are as developed by Bakshi et al. (2003) and are implemented using the naïve, JT05, and JT07 methods. No extrapolation is used in the naïve method, a flat extrapolation scheme is used in the JT05 method and a liner extrapolation scheme is used in the JT07 method.

Variables	Mean	Median	S.D.	Skewness	Kurtosis	Min.	Max.
Panel A: Risk-neutral volatility							
Naïve	0.043	0.027	0.043	3.038	12.973	0.0073	0.403
JT05	0.046	0.027	0.054	3.705	19.596	0.0074	0.584
JT07	0.049	0.027	0.139	45.944	2524.265	0.0074	7.725
Panel B: Risk-neutral Skewness							
Naïve	−0.717	−0.698	0.534	1.096	10.218	−5.0083	4.865
JT05	−0.793	−0.716	0.404	−0.665	0.260	−2.5639	0.215
JT07	−0.807	−0.741	0.412	−0.069	4.281	−2.3433	3.665
Panel C: Risk-neutral Kurtosis							
Naïve	5.062	4.530	2.362	1.457	3.357	1.3295	24.611
JT05	5.436	4.773	2.143	1.634	3.011	2.2422	18.844
JT07	5.594	4.949	2.209	2.300	11.916	0.9637	34.268

distribution is left-skewed and fat-tailed, with a mean of −0.0008, thereby revealing that negative jumps dominate positive jumps.[13] Although the Ljung-Box statistic for $SJ_{t-1,t}$ is significant at conventional levels, it is markedly lower than that for $RV_{t-1,t}$ and $BV_{t-1,t}$. This again confirms that the jump variation is much less persistent than the continuous variation.[14]

Table 2 presents the summary statistics on the risk-neutral moments using the naïve, JT05, and JT07 methods. For the risk-neutral volatility, all of the methods produce mean (median) values that are higher than those for realized volatility, with the naïve method having the lowest mean value at 0.043, essentially because this method does not include the use of extrapolation, and thus, the deep-out-of-the-moneyness options have assumed zero values. The JT07 method produces the most right-skewed distribution (45.944) with extremely high excess kurtosis (2524.265), which is extraordinarily different from the distribution produced by the JT05 method, where the corresponding values are 3.705 and 19.596. Because a volatility smile or skew is commonly observed, the JT07 method has the highest risk-neutral volatility, and thus, produces more upwardly biased forecasting of realized volatility. However, Fig. 1 shows that the three frequency distributions look almost the same in terms of risk-neutral volatility.[15]

The means (medians) of risk-neutral skewness are all negative, indicating that the risk-neutral distribution of TAIEX's return is left-skewed. When examining the shape of the distributions for risk-neutral skewness, we find that the naïve method has right-skewed distribution, at 1.096; however, the JT05 and JT07 methods exhibit left-skewed distributions, at −0.665 and −0.069, respectively. Although all of the methods have fat-tailed distributions, their excess kurtosis values actually differ markedly.[16] Fig. 2 shows that the frequency distribution of the naïve method is really different from those of the JT05 and JT07 methods.

The means (medians) of risk-neutral kurtosis are all greater than 3, indicating that the risk-neutral distributions of TAIEX's return are leptokurtic. For the shape of the distributions for risk-neutral kurtosis, all of the methods have right-skewed and fat-tailed distributions.[17] Fig. 3 shows that the naïve distribution is really different from those of JT05 and JT07 methods.

Table 3 presents the Pearson correlation coefficients among the risk-neutral moments.[18] Risk-neutral volatility is significantly positively related to risk-neutral skewness for all methods at the 0.01 level, but negatively related to risk-neutral kurtosis. The correlations between risk-neutral skewness and kurtosis are all negative. Out of the three methods, risk-neutral volatility and skewness (kurtosis) are most related to each other with a correlation of 0.298 (−0.451) under the naïve method. Collectively, the distribution shapes of and relations among risk-neutral moments vary with the extrapolation methods used.

[13] The Ljung-Box statistics for $BV_{t-1,t}$ and $SJ_{t-1,t}$ are 8712 and 51, respectively, with the serial correlation on $BV_{t-1,t}$ being markedly higher than that of the correlations on $RV_{t-1,t}$, $C_{t-1,t}$ and $MC_{t-1,t}$.

[14] The signed jump variation is further decomposed into positive and negative jump variations, denoted as $JP_{t-1,t}$ and $JN_{t-1,t}$ respectively. Their Ljung-Box statistics are 165 and 317, respectively, which are higher than that for $SJ_{t-1,t}$ and thereby indicate that the negative jump variation has higher serial correlation than the positive jump variation.

[15] In order to meaningfully depict and compare all three distributions within a figure, we remove some outliers. This approach also applies to Figs. 2 and 3.

[16] The naïve method has the highest excess kurtosis, at 10.218, while the JT05 method has the lowest excess kurtosis, at 0.26.

[17] The naïve method has the least right-skewed (1.457) and fat-tailed (3.357) distribution, while the JT05 (JT07) method has a right-skewness value of 1. 634 (2.300) and a fat-tailed distribution of 3.011 (11.916).

[18] We are grateful to an anonymous referee for drawing our attention to the issue on correlations among risk-neutral moments.

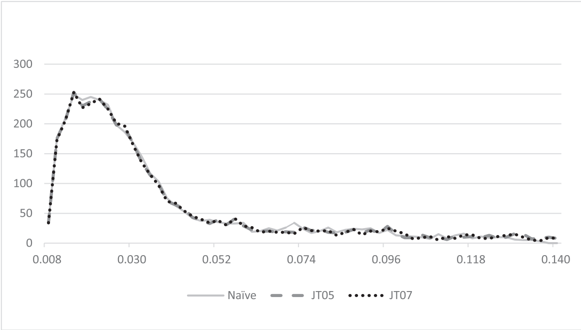

Fig. 1. The risk-neutral volatility
This figure displays the distributions of the risk-neutral volatility computed by the naïve, JT05, and JT07 methods. In order to meaningfully depict and compare all three distributions within a figure, we remove some outliers. The remaining observations account for 95.55%, 94.71%, and 94.2% of a total of 3707 daily observations for the naïve, JT05, and JT07 methods, respectively.

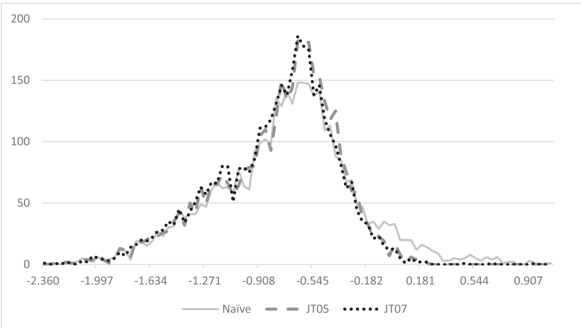

Fig. 2. The risk-neutral skewness
This figure displays the distributions of the risk-neutral skewness computed by the naïve, JT05, and JT07 methods. In order to meaningfully depict and compare all the three distributions within a figure, we remove some outliers. The remaining observations account for 99.19%, 99.97%, and 99.89% of a total of 3707 daily observations for the naïve, JT05, and JT07 methods, respectively.

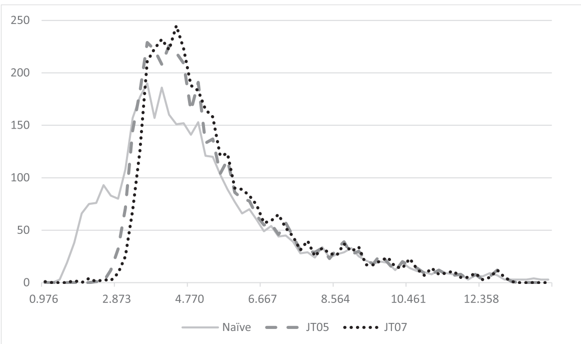

Fig. 3. The risk-neutral kurtosis
This figure displays the distributions of the risk-neutral kurtosis computed by the naïve, JT05, and JT07 methods. In order to meaningfully depict and compare all the three distributions within a figure, we remove some outliers. The remaining observations account for 99.76%, 99.24%, and 99.03% of a total of 3707 daily observations for the naïve, JT05, and JT07 methods, respectively.

Table 3
The Pearson correlation coefficient matrix of the risk-neutral moments
This table reports the correlations among risk-neutral moments for a sample period running from January 3, 2005 to December 31, 2019, which provides a total of 3707 daily observations. The risk-neutral volatility, skewness and kurtosis in TAIEX options are as developed by Bakshi et al. (2003) and are implemented using the naïve, JT05 and JT07 methods. No extrapolation is used in the naïve method, a flat extrapolation scheme is used in the JT05 method and a liner extrapolation scheme is used in the JT07 method. The correlations reported in this table are all statistically significant at the 0.01 level.

Method	Naïve		JT05		JT07	
Risk-neutral	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis
Volatility	0.298	−0.451	0.176	−0.384	0.016	−0.154
Skewness	–	−0.679	–	−0.778	–	−0.632

Table 4
The contemporaneous relation between jump variations and risk-neutral moments
This table reports the contemporaneous regression results on the assessment of whether risk-neutral moments can effectively capture information similar to daily jump variations for a sample period running from January 3, 2005 to December 31, 2019. The *t*-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5.

Variables	Var_t		$Skew_t$		$Kurt_t$		Adj. R^2 (%)
	Coeff.	*t*-stat.	Coeff.	*t*-stat.	Coeff.	*t*-stat.	
Panel A: Naïve Method							
$J_{t-1,t}$	0.395	7.43	0.284	4.01	0.324	4.76	14.4
$MJ_{t-1,t}$	0.037	1.65	−0.004	−0.28	−0.003	−0.21	0.1
$SJ_{t-1,t}$	−0.126	−2.47	−0.373	−4.57	−0.279	−3.57	8.2
$JP_{t-1,t}$	0.297	4.74	−0.056	−1.51	0.065	1.35	6.6
$JN_{t-1,t}$	−0.311	−6.53	−0.404	−4.42	−0.361	−4.20	14.6
Panel B: JT05 Method							
$J_{t-1,t}$	0.377	6.93	0.073	2.39	0.116	3.21	11.4
$MJ_{t-1,t}$	0.031	1.43	0.006	0.25	−0.003	−0.13	0.0
$SJ_{t-1,t}$	−0.125	−2.29	−0.053	−1.66	−0.043	−1.16	1.4
$JP_{t-1,t}$	0.283	5.76	0.036	1.12	0.086	2.16	6.3
$JN_{t-1,t}$	−0.302	−5.32	−0.082	−2.68	−0.097	−2.76	7.5
Panel C: JT07 Method							
$J_{t-1,t}$	0.377	6.98	−0.008	−0.40	0.064	2.92	12.4
$MJ_{t-1,t}$	0.026	1.36	0.009	0.61	−0.011	−0.86	0.1
$SJ_{t-1,t}$	−0.136	−2.40	0.006	0.35	−0.005	−0.29	1.7
$JP_{t-1,t}$	0.274	6.11	0.004	0.15	0.062	2.31	6.3
$JN_{t-1,t}$	−0.309	−5.11	0.005	0.35	−0.040	−2.45	8.5

Table 5
The predictive power of the risk-neutral moments on future jump forecasting computed by the naïve method
This table reports the regression results on the assessment of whether daily jumps are predictable using the three lag terms of the jumps or the previous-day risk-neutral volatility, skewness, and kurtosis, computed by the naïve method, for a sample period running from January 3, 2005 to December 31, 2019. Panel A shows the results based upon the use of the three jump lag terms as explanatory variables along with corresponding adjusted R^2 values, where *Y* is the dependent variable (i.e., the various types of jumps). Panel B shows the results based upon the use of the previous-day risk-neutral volatility, skewness, and kurtosis as explanatory variables along with their corresponding adjusted R^2 values. The t-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5.

Variables	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Adj. R^2 (%)
Panel A:	$Lag1(Y)$		$Lag2(Y)$		$Lag3(Y)$		
$J_{t-1,t}$	0.187	3.16	0.159	3.42	0.031	1.06	7.9
$MJ_{t-1,t}$	−0.009	−1.35	0.013	0.82	0.002	0.16	−0.1
$SJ_{t-1,t}$	0.071	1.29	0.037	0.87	−0.025	−1.11	0.6
$JP_{t-1,t}$	0.157	1.78	0.079	2.31	0.014	0.47	3.6
$JN_{t-1,t}$	0.117	2.03	0.106	2.29	0.028	1.74	3.0
Panel B:	Var_{t-1}		$Skew_{t-1}$		$Kurt_{t-1}$		
$J_{t-1,t}$	0.365	7.16	0.291	3.22	0.312	3.40	13.1
$MJ_{t-1,t}$	0.037	1.60	−0.004	−0.21	−0.007	−0.48	0.1
$SJ_{t-1,t}$	−0.082	−1.40	−0.172	−1.24	−0.162	−1.11	1.8
$JP_{t-1,t}$	0.306	6.15	0.135	2.12	0.155	2.60	8.0
$JN_{t-1,t}$	−0.264	−4.54	−0.274	−1.96	−0.274	−1.85	8.2

Table 6

The predictive power of risk-neutral moments on future jump forecasting computed by the JT05 and JT07 methods

This table reports the regression results on the assessment of whether daily jumps are predictable using the previous-day risk-neutral volatility, skewness and kurtosis, computed by the JT05 and JT07 method, for a sample period running from January 3, 2005 to December 31, 2019. Panel A (Panel B) shows the results based upon the use of the JT05 (JT07) method, followed by their corresponding adjusted R^2 values. The t-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5.

Variables	Var_{t-1}		$Skew_{t-1}$		$Kurt_{t-1}$		Adj. R^2 (%)
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	
Panel A: JT05 Method							
$J_{t-1,t}$	0.338	6.56	0.052	1.68	0.081	2.40	9.5
$MJ_{t-1,t}$	0.033	1.43	−0.001	−0.03	−0.008	−0.36	0.1
$SJ_{t-1,t}$	−0.062	−1.28	−0.018	−0.62	−0.023	−0.73	0.2
$JP_{t-1,t}$	0.302	6.45	0.049	1.77	0.068	2.22	7.8
$JN_{t-1,t}$	−0.239	−4.70	−0.048	−1.57	−0.064	−1.94	4.7
Panel B: JT07 Method							
$J_{t-1,t}$	0.340	6.64	0.019	0.80	0.056	2.31	10.2
$MJ_{t-1,t}$	0.034	1.55	0.006	0.33	−0.004	−0.25	0.1
$SJ_{t-1,t}$	−0.070	−1.44	−0.005	−0.24	−0.016	−0.82	0.3
$JP_{t-1,t}$	0.294	6.81	0.023	0.87	0.043	1.85	7.7
$JN_{t-1,t}$	−0.243	−4.72	−0.018	−0.94	−0.042	−1.96	5.1

Table 7

Information on future volatility provided by jumps

This table reports the significance of previous-day jumps on future volatility, $RV_{t,t+h}$ (where $h = 1, 5, 22$), for a sample period running from January 3, 2005 to December 31, 2019. The t-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5, 10, and 44, for respective forecasting horizons of 1, 5, and 22 days. Standard adjusted R^2 values are reported for each regression, with the second adjusted R^2 value, Adj. R^2 (2), reporting the values for the regressions without jump variance.

Variables	Horizons					
	1 Day		5 Days		22 Days	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Panel A: Historical daily, weekly and monthly RV						
$RV_{t-1,t}$	0.070	3.17	0.130	7.37	0.135	8.07
$RV_{t-5,t}$	0.332	9.80	0.314	7.96	0.257	6.30
$RV_{t-22,t}$	0.338	11.51	0.408	10.11	0.449	8.54
Adj.R^2 (%)	49.6		64.7		62.7	
Panel B: $RV_{t-1,t}$ replaced by $C_{t-1,t}$ and $J_{t-1,t}$						
$C_{t-1,t}$	0.231	11.08	0.249	10.07	0.241	7.55
$J_{t-1,t}$	0.016	0.94	0.015	1.37	0.017	0.86
$RV_{t-5,t}$	0.244	8.33	0.262	6.90	0.211	5.27
$RV_{t-22,t}$	0.289	10.35	0.358	8.80	0.402	7.36
Adj.R^2 (%)	51.4		66.3		64.2	
Adj.R^2 (2)(%)	51.4		66.3		64.2	
Panel C: $RV_{t-1,t}$ replaced by $MC_{t-1,t}$ and $MJ_{t-1,t}$						
$MC_{t-1,t}$	0.076	3.45	0.132	7.51	0.131	8.15
$MJ_{t-1,t}$	0.018	1.78	0.010	0.88	0.018	1.37
$RV_{t-5,t}$	0.328	9.73	0.313	7.90	0.260	6.38
$RV_{t-22,t}$	0.337	11.51	0.408	10.10	0.449	8.54
Adj.R^2 (%)	49.7		64.7		62.7	
Adj.R^2 (2)(%)	49.6		64.7		62.7	
Panel D: $RV_{t-1,t}$ replaced by $BV_{t-1,t}$ and $SJ_{t-1,t}$						
$BV_{t-1,t}$	0.249	11.21	0.272	10.20	0.264	7.12
$SJ_{t-1,t}$	−0.046	−2.84	−0.042	−3.65	−0.029	−2.84
$RV_{t-5,t}$	0.227	7.75	0.241	6.42	0.194	4.44
$RV_{t-22,t}$	0.288	10.66	0.357	8.96	0.400	7.47
Adj.R^2 (%)	51.9		66.8		64.6	
Adj.R^2 (2)(%)	51.7		66.6		64.5	
Panel E: $RV_{t-1,t}$ replaced by $BV_{t-1,t}$, $JP_{t-1,t}$ and $JN_{t-1,t}$						
$BV_{t-1,t}$	0.248	11.18	0.271	10.17	0.264	7.18
$JP_{t-1,t}$	−0.035	−1.28	−0.031	−1.91	−0.016	−1.07
$JN_{t-1,t}$	−0.031	−2.15	−0.029	−2.63	−0.024	−1.55
$RV_{t-5,t}$	0.234	8.00	0.247	6.49	0.195	4.73
$RV_{t-22,t}$	0.287	10.58	0.356	8.91	0.400	7.44
Adj.R^2 (%)	51.9		66.8		64.6	
Adj.R^2 (2)(%)	51.7		66.6		64.5	

Table 8
The effects of risk-neutral moments on the information of the signed jumps for future volatility
This table reports the effects of risk-neutral moments on the information of the signed jump for future volatility, $RV_{t,\ t+h}$ (where $h = 1, 5, 22$), for a sample period running from January 3, 2005 to December 31, 2019. The respective results on the risk-neutral moments computed by the naïve, JT05, and JT07 methods are presented in Panels A, B, and C, with Adjusted R^2 values being reported for each regression. The t-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5, 10, and 44, for respective forecasting horizons of 1, 5 and 22 days.

Variables	Horizons					
	1 Day		5 Days		22 Days	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Panel A: Naïve Method						
$BV_{t-1,\ t}$	0.167	7.72	0.175	7.41	0.163	5.06
$SJ_{t-1,\ t}$	−0.020	−1.23	−0.011	−0.93	0.006	0.47
$RV_{t-5,\ t}$	0.165	5.57	0.166	4.35	0.114	2.95
$RV_{t-22,\ t}$	0.040	1.28	0.046	0.99	0.071	0.95
Var_t	0.384	11.54	0.485	10.02	0.521	6.75
$Skew_t$	0.145	8.47	0.168	7.66	0.191	5.16
$Kurt_t$	0.074	3.58	0.104	4.28	0.137	2.83
Adj.R^2 (%)	54.2		70.3		68.6	
Panel B: JT05 Method						
$BV_{t-1,\ t}$	0.167	7.30	0.170	6.95	0.148	4.65
$SJ_{t-1,\ t}$	−0.036	−2.46	−0.030	−2.97	−0.016	−1.71
$RV_{t-5,\ t}$	0.170	5.89	0.175	4.54	0.129	3.13
$RV_{t-22,\ t}$	0.023	0.74	0.044	0.94	0.083	1.14
Va_{tt}	0.402	11.55	0.498	9.97	0.537	6.75
$Skew_t$	0.062	3.25	0.114	4.24	0.188	3.79
$Kurt_t$	0.012	0.59	0.075	2.50	0.157	2.40
Adj.R^2 (%)	54.1		70.1		68.6	
Panel C: JT07 Method						
$BV_{t-1,\ t}$	0.171	7.52	0.180	7.23	0.165	4.91
$SJ_{t-1,\ t}$	−0.030	−2.26	−0.024	−2.35	−0.011	−1.11
$RV_{t-5,\ t}$	0.171	5.92	0.177	4.59	0.131	3.09
$RV_{t-22,\ t}$	0.023	0.73	0.048	1.02	0.085	1.17
Var_t	0.398	11.79	0.474	9.64	0.499	6.42
$Skew_t$	0.039	2.25	0.062	2.27	0.098	2.28
$Kurt_t$	−0.009	−0.50	0.021	0.66	0.063	1.10
Adj.R^2 (%)	54.1		69.8		68.0	

4. Results

4.1. Relations between jump variations and risk-neutral moments

4.1.1. Contemporaneous relationships

Table 4 reveals that the risk-neutral moments computed using the naïve method are capable of explaining a higher proportion of jump variations than either the JT05 or JT07 method in terms of their adjusted R^2 values. Given that the risk-neutral moments are computed using the naïve method, the effects of risk-neutral skewness and kurtosis on $J_{t-1,\ t}$, $SJ_{t-1,\ t}$, and $JN_{t-1,\ t}$ are all statistically significant, while their corresponding coefficients are also higher than that of risk-neutral volatility except $J_{t-1,\ t}$. These findings indicate that both risk-neutral skewness and kurtosis have higher explanatory power than risk-neutral volatility on jump variations. However, this is not necessarily the case when the JT05 or JT07 methods are used.

4.1.2. Is jump variation predictable?

The regression results of the assessment of the predictability of daily jumps using the three lag terms are reported in Panel A of Table 5, with Panel B reporting the alternative results using the previous-day risk-neutral moments, computed by the naïve method. In terms of their adjusted R^2 values, Table 5 shows that historical jumps have only weak predictive power on future jumps; however, if the historical jumps are replaced by the risk-neutral moments computed using the naïve method, there is an approximate two-fold increase in predictive power. The results based upon the use of the JT05 and JT07 methods are presented in Table 6. The results show that while both methods have higher predictive power than that of historical jumps, the predictive power of the naïve method is superior to both the JT05 and JT07 methods.

Table 9
The effects of risk-neutral moments on the information of the positive and negative jumps for future volatility
This table reports the effects of risk-neutral moments on the information of the positive and negative jumps for future volatility, $RV_{t,\,t+h}$ (where $h = 1, 5, 22$), for a sample period running from January 3, 2005 to December 31, 2019. The respective results on the risk-neutral moments computed by the naïve, JT05 and JT07 methods are presented in Panels A, B and C, with Adjusted R^2 values being reported for each regression. The t-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5, 10 and 44, for respective forecasting horizons of 1, 5 and 22 days.

Variables	Horizons					
	1 Day		5 Days		22 Days	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Panel A: Naïve Method						
$BV_{t-1,\,t}$	0.167	7.72	0.175	7.41	0.163	5.07
$JP_{t-1,\,t}$	−0.017	−0.59	−0.010	−0.60	0.004	0.30
$JN_{t-1,\,t}$	−0.012	−1.02	−0.006	−0.56	0.004	0.28
$RV_{t-5,\,t}$	0.169	5.85	0.169	4.35	0.113	2.99
$RV_{t-22,\,t}$	0.040	1.29	0.046	1.00	0.071	0.96
Var_t	0.384	11.50	0.485	10.01	0.521	6.75
$Skew_t$	0.146	8.33	0.169	7.61	0.191	5.18
$Kurt_t$	0.076	3.92	0.105	4.33	0.136	2.81
Adj.R^2 (%)	54.2		70.3		68.5	
Panel B: JT05 Method						
$BV_{t-1,\,t}$	0.166	7.29	0.170	6.93	0.148	4.67
$JP_{t-1,\,t}$	−0.026	−1.07	−0.022	−1.76	−0.009	−0.75
$JN_{t-1,\,t}$	−0.025	−1.92	−0.020	−1.97	−0.013	−0.97
$RV_{t-5,\,t}$	0.176	5.99	0.180	4.56	0.130	3.26
$RV_{t-22,\,t}$	0.023	0.73	0.044	0.93	0.083	1.14
Var_t	0.402	11.52	0.498	9.95	0.537	6.75
$Skew_t$	0.063	3.29	0.114	4.25	0.188	3.79
$Kurt_t$	0.014	0.66	0.076	2.52	0.157	2.40
Adj.R^2 (%)	54.1		70.1		68.6	
Panel C: JT07 Method						
$BV_{t-1,\,t}$	0.170	7.48	0.180	7.18	0.165	4.92
$JP_{t-1,\,t}$	−0.027	−1.08	−0.022	−1.68	−0.009	−0.70
$JN_{t-1,\,t}$	−0.018	−1.63	−0.013	−1.20	−0.007	−0.55
$RV_{t-5,\,t}$	0.178	6.07	0.184	4.64	0.133	3.24
$RV_{t-22,\,t}$	0.022	0.70	0.047	1.00	0.085	1.16
Var_t	0.399	11.85	0.475	9.66	0.499	6.42
$Skew_t$	0.039	2.23	0.062	2.26	0.098	2.28
$Kurt_t$	−0.008	−0.43	0.022	0.70	0.063	1.11
Adj.R^2 (%)	54.1		69.8		68.0	

4.2. Selection of jump variations

Panel A of Table 7 shows that the historical daily, weekly, and monthly realized volatilities have significantly positive effects on future variance for all forecasting horizons at conventional levels.[19] As shown in Panels B and C, all of the continuous components are positive and significant, but the jump components developed by Andersen et al. (2007) and Andersen et al. (2012) are all insignificant. Panel D reveals that the signed jump variation has significantly negative effects on future variance, indicating an increase in future variance when negative jumps occur, as compared to a reduction when positive jumps occur. This is again confirmed when the signed jump is decomposed into positive and negative jumps, as reported in Panel E.

Although the signed jumps have statistically significant effects on future variance, their adjusted R^2 values are only slightly higher than those of the jumps developed by Andersen et al. (2007). A potential reason is that the predictive power of historical jumps is very small in volatility forecasting. To explore this, we remove the daily jump components from the regressions and provide a second set of adjusted R^2 values (adjusted R^2 (2)). These values remain virtually unchanged. Because the jump components developed by Andersen et al. (2007) and Andersen et al. (2012) have no predictive power on future variance, we employ the jump components developed by Patton and Sheppard (2015) in our subsequent analysis.

4.3. Is extrapolation necessary?

We next investigate whether the risk-neutral moments subsume the information on the contribution made by historical jump variations to total variance. As shown in Table 8, the significantly negative effect of $SJ_{t-1,t}$ on future variance disappears when the risk-neutral moments are computed using the naïve method, whereas the effects of $SJ_{t-1,t}$ are still significantly

[19] Their adjusted R^2 values for the daily, weekly, and monthly forecasting horizons are 49.6%, 64.7%, and 62.7%, respectively.

Table 10
The effects of risk-neutral skewness and kurtosis on the information in the signed jumps for future volatility
This table reports the effects of risk-neutral skewness and kurtosis computed using the naïve method on the information of the signed jump for future volatility, $RV_{t,\,t+h}$ (where $h = 1, 5, 22$), for a sample period running from January 3, 2005 to December 31, 2019. The t-statistics are based on standard errors computed using Newey and West (1987) correction for serial correlation of order 5, 10 and 44 for respective forecasting horizons of 1, 5, and 22 days.

Variables	$RV_{t,t+h}$		$RV_{t,t+h}$		$RV_{t,t+h}$		$RV_{t,t+h}$	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Panel A: 1-Day Horizon								
$BV_{t-1,\,t}$	0.249	11.21	0.198	8.78	0.168	7.48	0.167	7.72
$SJ_{t-1,\,t}$	−0.046	−2.84	−0.046	−2.72	−0.028	−1.78	−0.020	−1.23
$RV_{t-5,\,t}$	0.227	7.75	0.190	6.48	0.169	5.63	0.165	5.57
$RV_{t-22,\,t}$	0.288	10.66	0.070	2.14	0.059	1.87	0.040	1.28
Var_t	–	–	0.314	10.00	0.334	10.70	0.384	11.54
$Skew_t$	–	–	–	–	0.101	6.18	0.145	8.47
$Kurt_t$	–	–	–	–	–	–	0.074	3.58
Adj. R^2 (%)	51.9		53.3		54.0		54.2	
Panel B: 5-Day Horizon								
$BV_{t-1,\,t}$	0.272	10.20	0.208	7.93	0.176	6.89	0.175	7.41
$SJ_{t-1,\,t}$	−0.042	−3.65	−0.042	−3.44	−0.024	−2.01	−0.011	−0.93
$RV_{t-5,\,t}$	0.241	6.42	0.195	5.04	0.172	4.46	0.166	4.35
$RV_{t-22,\,t}$	0.357	8.96	0.085	1.74	0.073	1.55	0.046	0.99
Var_t	–	–	0.393	8.51	0.413	9.03	0.485	10.02
$Skew_t$	–	–	–	–	0.106	5.17	0.168	7.66
$Kurt_t$	–	–	–	–	–	–	0.104	4.28
Adj. R^2 (%)	66.8		69.0		69.9		70.3	
Panel C: 22-Day Horizon								
$BV_{t-1,\,t}$	0.264	7.12	0.197	5.57	0.165	4.58	0.163	5.06
$SJ_{t-1,\,t}$	−0.029	−2.84	−0.030	−2.60	−0.011	−0.95	0.006	0.47
$RV_{t-5,\,t}$	0.194	4.44	0.146	3.21	0.123	2.97	0.114	2.95
$RV_{t-22,\,t}$	0.400	7.47	0.119	1.49	0.107	1.41	0.071	0.95
Var_t	–	–	0.406	5.34	0.427	5.56	0.521	6.75
$Skew_t$	–	–	–	–	0.109	3.09	0.191	5.16
$Kurt_t$	–	–	–	–	–	–	0.137	2.83
Adj. R^2 (%)	64.6		67.0		67.9		68.6	

negative for daily and weekly forecasting horizons when the risk-neutral moments are computed using the JT05 and JT07 methods.

Similar results are also obtained for $JP_{t\text{-}1,t}$ and $JN_{t\text{-}1,t}$ in Table 9, indicating that the risk-neutral moments computed by the naïve method subsume the information of historical jump variations. Increases are also found in the size of the adjusted R^2 values, to 54.2%, 70.3%, and 68.6% for the daily, weekly, and monthly forecasting horizons, respectively, which indicate that relative to historical realized volatility and its components, the risk-neutral moments contain incremental information on volatility forecasting.

Note that the addition of the risk-neutral moments leads to a reduction in the impacts of historical realized volatilities, daily continuous components, and jump variations, and also leads to a particularly sharp reduction in historical monthly realized volatilities, with almost all of their coefficients changing from significant to insignificant. These findings imply that the risk-neutral moments subsume the information content of historical monthly realized volatility.

4.4. The roles of risk-neutral skewness and kurtosis

Table 10 shows the size of the coefficient on $SJ_{t\text{-}1,t}$ is almost unchanged when risk-neutral volatility is included, while it is reduced sharply when risk-neutral skewness and kurtosis are sequentially included. It suggests that it is risk-neutral skewness and kurtosis, rather than risk-neutral volatility, taking on the roles of subsuming the information content of historical jumps. Table 11 shows that the size of the coefficient on $JN_{t\text{-}1,t}$ is slightly increased when risk-neutral volatility is included, but the size is reduced sharply when risk-neutral skewness and kurtosis are sequentially included. The finding drawn from Table 10 is confirmed in Table 11.

4.5. Forecasting performance

4.5.1. In-sample forecasting performance

As shown in Table 12, if $RV_{t\text{-}5,t}$ is replaced by $BV_{t\text{-}5,t}$, then there are increases in the adjusted R^2 values, to 54.9%, 70.6%, and 69.3% for the daily, weekly, and monthly forecasting horizons, respectively. However, if $RV_{t\text{-}22,t}$ is sequentially replaced by $BV_{t\text{-}22,t}$, then the adjusted R^2 values remain virtually unchanged. As noted in the above discussion, $RV_{t\text{-}22,t}$ is not found to have any significant predictive power when the risk-neutral moments are included in the HAR-RV model, and indeed, Table 12 shows

Table 11
The effects of risk-neutral skewness and kurtosis on the information in the positive and negative jumps for future volatility
This table reports the effects of risk-neutral skewness and kurtosis computed using the naïve method on the information of the positive and negative jumps for future volatility, $RV_{t,\ t+\ h}$ (where $h = 1, 5, 22$), for a sample period running from January 3, 2005 to December 31, 2019. The t-statistics are based on standard errors computed using Newey and West (1987) correction for serial correlation of order 5, 10 and 44 for respective forecasting horizons of 1, 5 and 22 days.

Variables	$RV_{t,t+h}$		$RV_{t,t+h}$		$RV_{t,t+h}$		$RV_{t,t+h}$	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Panel A: 1-Day Horizon								
$BV_{t-1,t}$	0.248	11.18	0.198	8.79	0.168	7.48	0.167	7.72
$JP_{t-1,t}$	−0.035	−1.28	−0.026	−0.98	−0.015	−0.51	−0.017	−0.59
$JN_{t-1,t}$	−0.031	−2.15	−0.037	−2.39	−0.024	−1.84	−0.012	−1.02
$RV_{t-5,t}$	0.234	8.00	0.193	6.64	0.170	5.92	0.169	5.85
$RV_{t-22,t}$	0.287	10.58	0.071	2.16	0.059	1.88	0.040	1.29
Var_t	–	–	0.313	9.99	0.333	10.84	0.384	11.50
$Skew_t$	–	–	–	–	0.101	6.30	0.146	8.33
$Kurt_t$	–	–	–	–	–	–	0.076	3.92
Adj. R^2 (%)	51.9		53.3		54.0		54.2	
Panel B: 5-Day Horizon								
$BV_{t-1,t}$	0.271	10.17	0.208	7.94	0.176	6.90	0.175	7.41
$JP_{t-1,t}$	−0.031	−1.91	−0.020	−1.28	−0.008	−0.43	−0.010	−0.60
$JN_{t-1,t}$	−0.029	−2.63	−0.036	−3.00	−0.023	−2.19	−0.006	−0.56
$RV_{t-5,t}$	0.247	6.49	0.195	4.95	0.170	4.35	0.169	4.35
$RV_{t-22,t}$	0.356	8.91	0.085	1.74	0.072	1.54	0.046	1.00
Var_t	–	–	0.393	8.47	0.414	9.04	0.485	10.01
$Skew_t$	–	–	–	–	0.106	5.23	0.169	7.61
$Kurt_t$	–	–	–	–	–	–	0.105	4.33
Adj.R^2 (%)	66.8		69.0		69.9		70.3	
Panel C: 22-Day Horizon								
$BV_{t-1,t}$	0.264	7.18	0.198	5.62	0.165	4.63	0.163	5.07
$JP_{t-1,t}$	−0.016	−1.07	−0.005	−0.37	0.008	0.50	0.004	0.30
$JN_{t-1,t}$	−0.024	−1.55	−0.032	−2.15	−0.017	−1.41	0.004	0.28
$RV_{t-5,t}$	0.195	4.73	0.140	3.29	0.115	2.92	0.113	2.99
$RV_{t-22,t}$	0.400	7.44	0.118	1.49	0.106	1.40	0.071	0.96
Var_t	–	–	0.408	5.41	0.429	5.62	0.521	6.75
$Skew_t$	–	–	–	–	0.110	3.13	0.191	5.18
$Kurt_t$	–	–	–	–	–	–	0.136	2.81
Adj. R^2 (%)	64.6		67.0		67.9		68.5	

that when the risk-neutral moments are included, the decomposition of $RV_{t\text{-}22,t}$ into its continuous and jump components does not lead to any enhancement of its predictive power.

4.5.2. Out-of-sample forecasting performance

We consider six models for our out-of-sample forecasting, four of which include both $RV_{t\text{-}5,t}$ and $RV_{t\text{-}22,t}$, but differ in terms of the previous-day information used in the models. The first model, denoted RV and described in equation (2), is the standard HAR-RV model containing $RV_{t\text{-}1,t}$, $RV_{t\text{-}5,t}$, and $RV_{t\text{-}22,t}$, with the remaining models considering the information in jump variations. *SJ* is a specification that includes $BV_{t\text{-}1,t}$ and $SJ_{t\text{-}1,t}$, as described in equation (3); *JN* is a specification that decomposes $SJ_{t\text{-}1,t}$ into its positive and negative components; and *BV* is a model that excludes jump information and includes only recent bipower variations, $BV_{t\text{-}1,t}$. The fifth model, denoted BV3, is the HAR-BV model, which excludes all jump information and thus replaces $RV_{t\text{-}1,t}$, $RV_{t\text{-}5,t}$, and $RV_{t\text{-}22,t}$ with $BV_{t\text{-}1,t}$, $BV_{t\text{-}5,t}$, and $BV_{t\text{-}22,t}$. The sixth model, denoted RNM, is a specification that includes risk-neutral moments in the HAR-BV model.

All of the forecasts are generated using rolling ordinary least squares (OLS) regressions based upon 1000 observations, with the parameter estimates being updated daily. Forecast performance is evaluated using the Diebold and Mariano (1995) test, with negative Gaussian quasi-likelihood (QLIKE) as the loss function:

$$L\left(RV_{t+h,t}, \widehat{RV}_{t+h,t}\right) = \frac{RV_{t+h,t}}{\widehat{RV}_{t+h,t}} - \ln\left(\frac{RV_{t+h,t}}{\widehat{RV}_{t+h,t}}\right) - 1,$$

where $\widehat{RV}_{t+h,t}$ is the $RV_{t+h,\ t}$ forecast. The definition of QLIKE is normalized to yield a distance of zero when $RV_{t+h,t} = \widehat{RV}_{t+h,t}$, with this loss function having been shown by Patton and Sheppard (2009) to have good power properties, and by Patton (2011) to be robust to noise as a proxy for volatility.

Table 12

The effects of excluding jump components on volatility forecasting This table reports the effects of the exclusion of jump components on volatility forecasting when the risk-neutral moments are computed using the naïve method for a sample period running from January 3, 2005 to December 31, 2019. The t-statistics are based upon standard errors computed using Newey and West (1987) correction for serial correlation of order 5, 10 and 44, for respective forecasting horizons of 1, 5, and 22 days.

Variables	Horizons					
	1 Day		5 Days		22 Days	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Panel A:						
$BV_{t-1,t}$	0.167	7.75	0.175	7.43	0.162	5.08
$RV_{t-5,t}$	0.165	5.59	0.166	4.34	0.114	2.95
$RV_{t-22,t}$	0.037	1.18	0.044	0.95	0.072	0.97
Var_t	0.389	11.69	0.488	10.12	0.520	6.80
$Skew_t$	0.153	8.60	0.173	8.14	0.189	5.26
$Kurt_t$	0.080	4.12	0.107	4.62	0.135	2.86
Adj.R^2 (%)	54.2		70.3		68.6	
Panel B:						
$BV_{t-1,t}$	0.078	3.11	0.109	5.47	0.074	4.29
$BV_{t-5,t}$	0.318	8.76	0.264	5.52	0.283	4.99
$RV_{t-22,t}$	0.017	0.56	0.042	0.91	0.038	0.50
Var_t	0.351	10.15	0.461	9.03	0.480	6.10
$Skew_t$	0.133	6.63	0.158	6.77	0.169	4.53
$Kurt_t$	0.085	3.99	0.112	4.67	0.138	2.89
Adj.R^2 (%)	54.9		70.6		69.3	
Panel C:						
$BV_{t-1,t}$	0.077	3.06	0.107	5.31	0.076	4.12
$BV_{t-5,t}$	0.328	8.66	0.266	5.12	0.240	4.61
$BV_{t-22,t}$	−0.009	−0.22	0.026	0.47	0.112	1.48
Var_t	0.367	10.72	0.478	9.53	0.452	5.99
$Skew_t$	0.134	6.59	0.159	6.65	0.163	4.24
$Kurt_t$	0.086	4.10	0.116	4.85	0.140	2.98
Adj.R^2 (%)	54.9		70.6		69.4	

Table 13

Out-of-sample forecast comparison of volatility forecasting models

This table reports the test results of the predictive accuracy of volatility forecasting models. The values shown are the difference in the QLIKE between a pair of models, whilst the Diebold-Mariano test statistics (DM. stat.) are based on standard errors computed using Newey and West (1987) correction for serial correlation of order 2, where a negative difference indicates that the model on the row outperforms the model on the column.

Models	Models									
	SJ		JN		BV		BV3		RNM	
	Diff.	DM. stat.	Diff.	DM. stat.	Diff.	DM. stat.	Diff.	DM. stat.	Diff.	DM. stat.
Panel A: 1-Day Horizon										
RV	−0.026	−0.59	−0.061	−2.34	−0.068	−4.55	−0.102	−3.98	−0.153	−5.54
SJ			−0.035	−0.97	−0.042	−1.02	−0.076	−1.58	−0.127	−2.82
JN					−0.007	−0.41	−0.041	−1.36	−0.092	−4.16
BV							−0.034	−1.59	−0.085	−4.18
BV3									−0.051	−2.52
Panel B: 5-Day Horizon										
RV	−0.033	−1.15	−0.045	−2.71	−0.054	−5.10	−0.064	−2.14	−0.146	−5.85
SJ			−0.012	−0.70	−0.021	−0.78	−0.032	−0.84	−0.113	−3.25
JN					−0.009	−0.74	−0.020	−0.68	−0.101	−4.06
BV							−0.010	−0.43	−0.092	−4.40
BV3									−0.082	−4.24
Panel C: 22-Day Horizon										
RV	−0.039	−4.31	−0.037	−3.85	−0.038	−4.61	−0.065	−2.55	−0.127	−5.67
SJ			0.002	0.69	0.000	0.16	−0.027	−1.27	−0.088	−4.82
JN					−0.001	−0.36	−0.028	−1.36	−0.090	−4.86
BV							−0.027	−1.30	−0.089	−4.79
BV3									−0.062	−4.11

Table 14
Out-of-sample R^2 for the alternative models used in the forecast evaluation
The out-of-sample R^2 (%) is computed as 1 minus the ratio of the out-of-sample model-based MSE to the out-of-sample MSE from a forecast that includes only a constant. The largest value in each row is shown in bold text.

Horizons (days)	Models					
	RV	SJ	JN	BV	BV3	RNM
1	39.3	41.5	41.7	41.6	43.0	**44.9**
5	54.2	56.5	56.5	56.5	56.9	**60.9**
22	52.2	54.1	54.0	54.1	55.5	**58.3**

The results of our comparisons between pairs of forecasting models for the various forecasting horizons are reported in Table 13. We find that the JN model is significantly superior to the RV model at conventional levels, indicating that the decomposition of the signed jump variation into positive and negative components does enhance forecasting performance.[20] The BV model, which excludes historical daily jump variations, is significantly superior to the RV models, suggesting that the recent jump information distorts the information content of $RV_{t-1,t}$, thereby resulting in inferior performance of the standard HAR-RV model. The BV3 model significantly outperforms the RV models, which means that not only is the previous-day jump information detrimental to volatility forecasting, but that this is also the case for both the previous-week and previous-month jump information. Finally, the RNM model, that is the HAR-BV model with risk-neutral moments, is significantly superior to all the other models, thereby confirming the excellent role of option-implied information in volatility forecasting.

Following Patton and Sheppard (2015), we compute the out-of-sample R^2 values for the six forecasting models in Table 13. The results are presented in Table 14. First, the SJ, JN, and BV models all provide similar performance; relative to the standard HAR-RV model, these models have out-of-sample R^2 value gains of 2.13%, 2.15%, and 2.16% for the daily, weekly, and monthly forecasting horizons, respectively. Relative to the BV model, the BV3 model (i.e., the HAR-BV model) has out-of-sample R^2 value gains of 1.39%, 0.36%, and 1.42% for the daily, weekly, and monthly forecasting horizons, respectively. Furthermore, relative to the BV3 model, the RNM model (i.e., the HAR-BV model with risk-neutral moments) has out-of-sample R^2 value gains of 1.85%, 4.01%, and 2.82% for the respective forecasting horizons. Taken together, the statistically significant gains reported in Table 13 translate to the economically meaningful improvements shown in Table 14.

5. Conclusion

Using TAIEX and TAIEX option prices, we investigate the relation between jump variations and risk-neutral moments in volatility forecasting. We obtain several interesting results. In the in-sample analysis, our method that does not involve any extrapolation produces better measures of risk-neutral skewness and kurtosis than the other methods that do. Risk-neutral skewness and kurtosis perform better than risk-neutral volatility in subsuming information on historical jumps for volatility forecasting. In the out-of-the sample analysis, the model specification that includes risk-neutral skewness and kurtosis in the HAR-BV model outperforms the other models we examined.

We also find that the predictive power of historical jumps on future jumps is weaker than that of the risk-neutral moments. The statistically significant predictive power of jumps on future volatility disappears when risk-neutral moments are included in the HAR-RV model. The predictive power of historical jumps almost remains unchanged when risk-neutral volatility is added into the HAR-RV model; however, it decreases sharply with the sequential inclusion of risk-neutral skewness and kurtosis.

We make a contribution to the literature by introducing a novel estimator, which does not involve the use of extrapolation, for computing the risk-neutral moments of Bakshi et al. (2003). The benefit of no extrapolation is that it allows the data to tell the story, and avoids any distortion of the tail distribution. To the best of our knowledge, our study is the first to explore the relation between risk-neutral moments and realized jumps in volatility forecasting, from which we find that risk-neutral skewness and kurtosis subsume the information content of historical jump variations. Our findings have an implication for academics and practitioners. The HAR-BV model with risk-neutral moments can be used for volatility forecasting. In the meantime, no extrapolation should be implemented when computing the risk-neutral moments in order to avoid any distortion of the tail information.

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[20] The value in column (1) in Panel A of Table 13, which is the results of a comparison of the loss values of the SJ and RV models, is −0.026, with a corresponding Diebold-Mariano test statistic (DM statistic) of −0.59, which indicates that the SJ model is insignificantly superior to the RV model. The value in column (2) of Panel A, which is the results of a comparison of the loss values of the JN and RV models, is −0.061, with a corresponding DM statistic of −2.34, which indicates that the JN model is significantly superior to the RV model.

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