國立政治大學經濟學系博士學位論文

內生化策略性貿易、出口學習效果以及對外直接投資：國際貿易政策與理論論文三篇

Endogenous Strategic Trade Policy, Learning-by-Exporting and Foreign Direct Investment: Three Essays in International Trade Theory and Policy

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中華民國 109 年 8 月

DOI:10.6814/NCCU202100569

致謝

時光飛逝，轉眼間已經是博士六年級了。去年暑假我通過了博士論文口試，而今年即將從政大經濟系博士班畢業。讀博士的過程雖然不輕鬆，但卻是一段非常愉悅的時光，我會永遠記得這段求學的日子。

在當博士生的這段期間裡，我很幸運的碰到了許多很好的同學以及學長和學弟妹，在我碰到困難的時候給予幫助，或是平時閒聊中我們互相勉勵，讓彼此在這個漫長的求學過程中都更有繼續向前的動力。在此我要感謝我的同學博揚，我們一起修課也一起準備資格考試，在這段壓力最大的時候一起成功熬了過來，之後學業上有什麼問題也經常一起討論，你也每週都從中研院來政大一次跟我見面吃飯，很幸運能有你當我的同學。我也要感謝同為翁老師指導的兆璿以及學宏兩位學長，在我剛入博士班的時候很多事情不了解，學長們都很有耐心的教導與指點，除此之外在考資格考的時候學長們也給了我一些寶貴的意見，非常謝謝這些日子來的幫忙。再來是榮澤學妹，跟妳聊天總讓人覺得很輕鬆愉快，妳在準備資格考的時候問了我很多的問題，我也盡力的為妳解答，很高興後來妳能順利通過博士資格考試，也預祝妳將來可以順利完成論文畢業。

最後我要特別感謝我的父親蔡攀龍先生、母親陳彧夏女士以及我的指導教授翁永和老師。家人的支持是我堅持下去走這條路的最大動力，而這條路上受到了許多人的幫助，特別是翁永和老師。從碩士班您就是我的指導教授，一直到現在博士班即將畢業，我們共相處了八年至九年。在這期間除了學術方面的討論之外，老師的樂觀心態更是讓我印象深刻，這讓我學習到有時候別把自己逼得太緊，適時的放鬆調整心情是很重要的，如果想要走得長遠，健康的身體及心靈才是最重要的事。感謝老師在學術上的指導，以及在這段期間大小事情的幫忙，我很高興能有您當我的指導教授。

蔡守容 謹致

DOI:10.6814/NCCU202100569

摘要

本篇論文利用三個章節探討三個與國際貿易相關的主題。第一個主題是策略性貿易政策的相關議題，第二個主題探討出口學習效果對廠商出口行為的影響，第三個主題研究出口學習效果與對外直接投資學習效果如何影響廠商的貿易決策。

本文第二章討論在經典的策略性貿易政策模型架構之下，當廠商可以自行選擇要用數量還是價格做策略性變數時，政府的最適貿易政策為何。 我們得到兩個主要結論: (1) 在符合一些合理的假設之下，兩家廠商皆會選擇數量作為其策略性變數在第三國市場競爭。(2) 因為兩國皆選擇數量為策略性變數，故廠商的競爭型態為庫諾競爭，此時最適的貿易政策為政府補貼本國的出口廠商。

本文第三章討論在異質廠商的架構之下，出口學習效果如何影響廠商的出口決策。主要的結論有: (1) 比起沒有出口學習效果，廠商更有意願出口，因為透過出口廠商可以提升自身的生產力。有些廠商甚至會在出口利潤為負的情況下選擇出口。(2) 出口廠商以及非出口廠商之間存在著一段生產力的差距，亦即最差的出口廠商其生產力仍高於最好的非出口廠商一段距離。

本文第四章延伸第三章的模型，加入了廠商可以選擇對外直接投資的選項，且對外直接投資與出口皆存在著學習效果。本章的主要結論為: (1) 當兩者的學習效果大小相異時，廠商的出口和對外投資決策存在著微弱的互補關係。(2) 當兩者的學習效果相等時，廠商只會選擇出口或是對外直接投資其中一種方式來將其產品賣至國外市場。

關鍵詞: 內生化策略性貿易政策; 異質廠商; 出口學習效果; 出口與對外直接投資

DOI:10.6814/NCCU202100569

Abstract

This thesis explores three topics of international trade in three chapters. One is about strategic trade policy and the other two analyze in a firm's behavior given the existence of technological learning effect of exporting and making FDI.

Chapter 2 discusses the home government's optimal export policy when a firm can endogenously choose its strategic variable in a classic strategic trade policy model structure. Two conclusions are obtained. First, under some moderate, reasonable constraints, two firms (domestic and foreign firms) will compete in quantity in the third country. Second, since a Cournot results appear, the optimal export policy for the home government is to subsidy its export firm.

Chapter 3 constructs a model that captures the features of learning-by-exporting to analyze a firm's export decision based on a heterogeneous firms structure. The main results are: First, compared with the case of no learning effect, a firm is more willing to export since it can upgrade its productivity. Moreover, a firm might export even it has negative export profit. Second, a productivity gap emerges between exporters and non-exporters. The least productive exporters are still more productive than the non-exporters.

Chapter 4 extends the model in Chapter 3. A Firm can use two ways to serve the foreign countries, exporting or making foreign direct investment. There is a learning effect (productivity improvement) on both activities; the magnitude of the learning

DOI:10.6814/NCCU202100569

effect can be symmetric or asymmetric. The main result is that a weakly complementary relationship between export and foreign direct investment may emerge in the case when the learning effect is asymmetric. Conversely, in the case of symmetric learning effect, a firm will use export or foreign direct investment to serve all foreign markets, but not both.

Keywords: endogenous strategic trade policy, heterogeneous firms, learning-by-exporting, export and foreign direct investment

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Chapter 1 Introduction

The pioneering paper of the strategic trade policy, Brander and Spencer (1985) shows that it is possible for a government to increase home country's welfare by taking some action. In the setting of Cournot competition, the home government should subsidize its export firm to increase domestic welfare. However, the robustness of the conclusion is quickly challenged by Eaton and Grossman (1986), which allows firm to compete in price (Bertrand competition) and obtains a completely different conclusion. The home government should tax its export firm. These results suggest that the home government's optimal trade policy is heavily affected by firms' competition mode (compete in price or quantity). Although mountains of researches related to the strategic trade policy have been studied, virtually all of them have assumed that the competition mode of firms is exogenously given and do not focus on the relationship between the optimal policy and the firms' strategic variable. Therefore, the first topic of this thesis is to investigate the relationship between the firms' competition mode and the government's optimal policy, which is studied in chapter 2. Differing from earlier research, the model in Chapter 2 assumes the strategic variable of a firm are endogenously determined, and thus generates a three-stage non-cooperative game. In the first stage, the home government sets an optimal export subsidy (or tax) that maximizes its social welfare; in the second stage, knowing the government's action in the first stage, two firms choose their strategic variables simultaneously; in the third stage, the firms compete in the third market. Using the method of backward induction, we obtained the following results: Both firms will compete in quantity and the home government will set an optimal export subsidy.

DOI:10.6814/NCCU202100569

With an improvement in the availability of the firm- level data after the 1980s, economists have distilled some stylized facts through carefully studying the firm- level data. One of the most salient facts is that export firms are, on average, more productive than their non-export counterpart. Bernard and Jensen (1995) first use data form U.S. manufacturer sector to arrive at this finding. Lots of following researches use data from other countries reach the same conclusion. Although export firms are more productive, the reason for this fact remains unsettled. There are two main theories that explain why exporters perform better than the non-exporters. The first explanation, which is called the self-selection theory, suggests that better firms self-select themselves into the foreign markets. Since to sell product overseas need additional costs, only firms that can bear the costs become exporters. Melitz (2003) provide a rigorous theoretical model for the self-selection behavior. The second explanation is called learning-by-exporting theory. Different from the self-select theory, the learning-by-exporting theory suggests that a firm improves its performance after export due to the foreign demand or obtains advance technology from abroad. The evidence of the learning-by-exporting theory has been found in several empirical researches, its theoretical part is relative underdeveloped. As a result, the second topic of this thesis is to construct a model that describe the learning-by-exporting theory, which is our chapter 3.

The model in chapter 3 is based on Melitz (2003), but with two major modifications. First, all firms are identical (all of them have same productivity level) when they enter the domestic market. Second, productivity heterogeneity appears in the open economy. In the open economy a firm draws a productivity level from a productivity distribution. After knowing the productivity it has drawn, the firm makes its export decision. The firm can achieve the productivity which has drawn from the

DOI:10.6814/NCCU202100569

productivity distribution only if it exports to at least one country; otherwise, the productivity remains at the original level (the productivity as it enters the domestic market). Different from the self-selection model that all firms are heterogeneous originally, a firm can engage in a productivity upgrade only at the cost of selling abroad. This capture the phenomenon of learning-by-exporting. Two main differences compared with the self-selection model arise from our analysis. First, a firm is more willing to export in the learning-by-exporting structure. It may decide to export even the export profit is negative, which is not possible in the self-selection structure. Second, there is a striking productivity gap between exporters and non-exporters. That is, the least productive exporters are still more productive than the non-exporters.

The third topic in chapter 4 of this thesis is to investigate the relationship (substitute or complement) between a firm's decision of export and making foreign direct investment (FDI). In theory, they are alternative strategy from a firm's point of view if the product is narrowly defined. The most productive firms serve foreign market by making FDI while the less productive firms export their product, as obtained in Helpman *et al.* (2004). But the empirical literature provides strong evidence in the complementary relationship of the two. One of the possible explanations of the inconsistence between the theory and the empirical evidence is that there may be some unobserved variables that affect the decision of export and FDI simultaneously. The model in chapter 4 extends the model in chapter 3, attempting to explain the complementary relationship between the two options by capturing a factor that affects a firm's export and FDI decision. In this model, we allow firms to serve the foreign market through export or making FDI. Furthermore, assume both activities can induce a productivity improvement in a symmetric or asymmetric magnitude. The main result is that there may be a weakly complementary relationship of the two actions even all

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the foreign countries are identical in trade cost, given export and FDI have asymmetric impacts on productivity upgrade.

References

Bernard, A. B. and J. B. Jensen, (1995). “Exporters, Jobs, and Wages in U.S. Manufacturing: 1975-1987.” Brookings Papers on Economic Activity: Microeconomics. 67-119.

Brander, J.A. and B.J. Spencer, (1985). “Export Subsidies and International Market Share Rivalry”, Journal of International Economics 18: 83-100.

Eaton, J. and G.M. Grossman, (1986). “Optimal Trade and Industrial Policy under Oligopoly”, Quarterly Journal of Economics 101: 383-406.

Helpman, E., Melitz, M. J., and Yeaple, S. R., (2004). “Export Versus FDI with Heterogeneous Firms.” American Economic Review, 94(1), 300-316.

Melitz, M. J., (2003). “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity.” Econometrica, 71, 1695-1725.

DOI:10.6814/NCCU202100569

Chapter 2 Endogenous Strategic Trade Policy: The case of the Third Market Model[1]

1. Introduction

The seminal papers of Brander and Spencer (1985) as well as Eaton and Grossman (1986) have ushered in a spate of research of strategic trade policy in the context of the "third market model" in the following two decades (Brander, 1995; Feenstra and Rose, 2000). These works, however, are incomplete in the sense that they all assume that the two firms adopt the same strategic variable.[2] The defect has been fixed recently by Schroeder and Tremblay (2015) and Tsai et al. (2016) by allowing the two firms to choose different strategic variables. Both papers arrive at the same conclusion that the optimal trade policy of the home country depends exclusively on what the strategic variable the foreign firm chooses rather than whether the two firms compete in a Cournot or a Bertrand game. This is a very simple, general rule that subsumes the results of Brander and Spencer (1985) as well as Eaton and Grossman (1986). Interestingly, the improvement brings forth another obvious shortcoming in this line of research: the mode of competition between the firms is given exogenously, which can hardly be justified both from the theoretical and from the real world points of view.

As noted by Tsai et al. (2016), the essence of the strategic trade policy is that the home government can take the first-mover advantage vis-à-vis the firms to maximize domestic welfare. However, as followers, it is nothing but natural for each of the firms to choose the best strategic variable after observing the trade policy adopted by the home government. In other words, it makes more sense to treat the mode of competition

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[1] This chapter has been published as: Tsai, Shoou-Rong, Pan-Long Tsai and Yungho Weng, 2018, "Endogenous Strategic Trade Policy: The Case of the Third Market Model,"International Review of Economics and Finance, Vol.58, November, 676-682.

[2] The argument is applicable to the case of multiple firms. However, for the sake of clarity, we will use the duopoly model in our discussion.

DOI:10.6814/NCCU202100569

as endogenous than exogenous, as Singh and Vives (1984), Maggi (1996), Basak and Wang (2016), and Choi et al. (2016) have done. Conversely, knowing that the firms will choose their strategic variable contingent upon its trade policy, the home government would attempt to set appropriate export subsidies (taxes) to induce the firms to a specific mode of competition so that the domestic welfare is maximized. However, with the exception of Choi et al. (2016), none of the existing studies in the context of strategic trade policy has dealt with this endogeneity problem. The main purpose of the paper is thus to fill this knowledge gap in this branch of literature.[3] Our paper also serves as an extension of Schroeder and Tremblay (2015) and Tsai et al. (2016), and gives a quick response to the recent call by Head and Spencer (2017) that "renewed incorporation of oligopolistic firms in international trade is warranted".

2. The Models

Our model follows all the basic assumptions of Brander and Spencer (1985), but with two notable differences. First, the two firms produce differentiated products. Second, instead of given exogenously, the firms can choose endogenously quantity or price as their strategic variable. Specifically, the model is a three-stage non-cooperative game. In the first stage, the home government decides the optimal export subsidy or tax to maximize social welfare; in the second stage, the home and the foreign firms simultaneously choose price or quantity as their strategic variable; the firms then compete with the committed strategic variables in the third stage to maximize their profits. Focusing on the subgame perfect equilibria, we solve the game through

[3] Choi et al. (2016) also deal with the endogenous choice of strategic variables, but they deviate from the original Brander-Spencer model in allowing both governments to set simultaneously their optimal trade policies. As a result, the present study can be regarded as a complement to Choi et al. (2016). We are grateful to an anonymous referee for bringing to our attention this important contribution.

DOI:10.6814/NCCU202100569

backward induction.

2.1 Equilibrium profits

Suppose a home firm (firm h) and a foreign firm (firm f) produce differentiated products, q_h and q_f, and export all their outputs to a third country where the prices are p_h and p_f, respectively. To obtain closed form solutions we follow Singh and Vives (1984) and assume that the linear (inverse) demand functions faced by firm h and firm f are:

$$p_h = a - q_h - bq_f, \tag{2.1}$$

$$p_f = a - q_f - bq_h, \tag{2.2}$$

where $0 < b < 1$ indicates the degree of product differentiation. To focus on the firms' endogenous choice of strategic variables, we assume further that firms have no fixed cost of production, have no capacity limits,[4] and the marginal costs of both firms are equal to a constant c, with $a > c > 0$.

Since each firm has two strategic variables, quantity (C) and price (B), there are four possible modes of competition: (C,C), (C,B), (B,C) and (B,B). The vector (j,k) stands for the mode that firm h (firm f) chooses the strategy variable j (k), j, $k = C, B$. Denote the profit function of firm i under competition mode (j,k) as $\pi_i^{(j,k)}$ and its corresponding profits at the Nash equilibrium as $\pi_i^{(j,k)}*$ $(i = h, f; j, k = C, B)$. Take the competition mode (C,B) as an example. In this case, the demand functions (2.1) and (2.2) faced by firm h and firm f can be rewritten as:

[4] The assumption on capacity is in line with Singh and Vives (1984), Basak and Wang (2016), and Choi et al. (2016), but in stark contrast to Maggi (1996).

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$$p_h(q_h, p_f) = a(1-b) - (1-b^2)q_h + bp_f \text{,} \tag{2.1'}$$

$$q_f(q_h, p_f) = a - bq_h - p_f \text{.} \tag{2.2'}$$

With the home government's specific export subsidies (taxes) equal to $s > 0$ $(s < 0)$, the profit functions of the two firms are:

$$\pi_h^{(C,B)} = (p_h - c + s)q_h = [(a(1-b) - c + s) - (1-b^2)q_h + bp_f]q_h \text{,} \tag{2.3}$$

$$\pi_f^{(C,B)} = (p_f - c)q_f = (p_f - c)(a - bq_h - p_f) \text{.} \tag{2.4}$$

Solving the first-order conditions for profit maximization, $\partial\pi_h^{(C,B)} / \partial q_h = 0$ and $\partial\pi_f^{(C,B)} / \partial p_f = 0$, gives us the equilibrium profits:

$$\pi_h^{(C,B)*} = \frac{(1-b^2)[(a-c)(2-b)+2s]^2}{(4-3b^2)^2} \text{,} \tag{2.5}$$

$$\pi_f^{(C,B)*} = \frac{[(a-c)(2-b-b^2)-bs]^2}{(4-3b^2)^2} \tag{2.6}$$

Similarly, we can obtain the equilibrium profits of the other three modes of competition, which are summarized in the payoff matrix of Table 2-1.

2.2 Firms' optimal strategies

We now analyze how each firm chooses the strategic variable that brings it the highest profits. For that purpose, we treat Table 2-1 as a normal form game and find its Nash equilibrium. Define $\pi_h^{(.,k)*} = \pi_h^{(C,k)*} - \pi_h^{(B,k)*}$ and $\pi_f^{(j,.)*} = \pi_f^{(j,C)*} - \pi_f^{(j,B)*}$, $j,k = C,B$.

That is, $\pi_h^{(.,k)*}$ is the difference between firm h's realized profits if it chooses quantity and those of choosing price, given firm f chooses k. Likewise, $\pi_f^{(j,.)*}$ is the

DOI:10.6814/NCCU202100569

difference between firm f 's realized profits if it chooses quantity and those of choosing price, given firm h chooses j. As such, the Nash equilibrium of the game can be determined by the signs of $\pi_h^{(.,k)*}$ and $\pi_f^{(j,.)*}$. The results are shown in Figure 2.1 where the signs depend on the value of s, the home government's export policy. As an illustration, let us look at case (1) $\pi_h^{(.,C)*}$ of the figure. The horizontal line is the real line measuring s, with positive (negative) value to the right (left) of 0. When s has the value s_1 or s_2, $\pi_h^{(.,C)*} = \pi_h^{(C,C)*} - \pi_h^{(B,C)*} = 0$, while $\pi_h^{(.,C)*} > 0$ if $s_2 < s < s_1$, and $\pi_h^{(.,C)*} < 0$ if $s < s_2$ or $s > s_1$. The remaining three cases in Figure 2.1 can be interpreted in the same way.[5]

After somewhat tedious calculations we are able to get the following relationship: $s_6 < s_2 < s_4 < s_8 < 0 < s_3 < s_7 < s_5 < s_1$.[6] Now using the upper edge of Table 2-2 as the real line measuring s, we can divide it into 10 intervals, from I_1 to I_{10}, according to the above relationship. Depending on where the value of s lies, we can map the signs of all the four cases in Figure 2-1 onto Table 2-2 The pattern of the signs in Table 2-2 in turn enables us to find the Nash equilibrium in each interval as shown in Table 2-3. Two salient features stand out and can be summarized as:

Proposition 1*: Under the assumption that the two goods are substitutes and the firms can choose their strategic variables endogenously,*

(i) none of the equilibrium involves (C,B) *or* (B,C) *mode of competition,*

[5] Namely, $\pi_h^{(.,B)*} = 0$ if $s = s_3$ or $s = s_4$; $\pi_f^{(C,.)*} = 0$ if $s = s_5$ or $s = s_6$; $\pi_f^{(B,.)*} = 0$ if $s = s_7$ or $s = s_8$.

[6] The values of s_l, $l = 1,\ldots,8$, are shown in Appendix 1.

DOI:10.6814/NCCU202100569

(ii) all the intervals except I_3 *and* I_8 *have just one Nash equilibrium,*

It is noteworthy that result (*i*) is the same as what obtained in Singh and Vives (1984) and Maggi (1996) when $s = 0$. Recall that Bertrand competition is always more competitive than Cournot competition under free trade, thus both firms would have higher profits in the latter mode of competition than the former one. The situation will not change if the home government gives (imposes) a small amount of specific subsidy (tax), which explains why (C, C) is the equilibrium mode of competition in the vicinity of $s = 0$, namely I_4 - I_7.

Another interesting finding from Table 2-3 is that, starting from free trade the equilibrium transits from a Cournot equilibrium to a Bertrand equilibrium as the home country's export subsidies/taxes increase. However, that the mode (B, B) appears as a Nash equilibrium in intervals other than I_4 - I_7 seems counterintuitive, as it implies that the two firms will be more profitable to compete in prices when they are more asymmetry in costs.[7] On a closer examination, the result turns out to be a pure mathematical illusion. Specifically, the higher profits of the higher cost firm due to larger export subsidies (taxes) come from the product of a negative price-marginal cost margin and the negative quantity it produces, which is of course economically nonviable. Once we introduce into our model the modest constraint that both firms' outputs and profits are nonnegative, the (B, B) mode of competition can never be a Nash equilibrium. Under the circumstances, the subsidized firm h will drive firm f out of the third market if the subsidy is sufficiently large (namely, $s > s_1$) and becomes

[7] Because equations (1) and (2) have the same vertical intercept, there is no demand asymmetry *a la* Zanchettin (2006). In fact, the index of asymmetry defined in Zanchettin (2006) reduces to $c_f - (c_h - s) = c - (c - s) = s$ in our model.

DOI:10.6814/NCCU202100569

a *de facto* monopoly without bearing any competitive pressure from firm f regardless of the mode of competition.[8] For a subsidy $s_3 < s < s_1$, firm f would have zero (positive) profits when the two firms engage in Bertrand (Cournot) competition, implying that firm f will always prefer quantity as its strategic variable. Given firm f chooses to compete in quantity, the home firm's best strategy is also to compete in quantity, leading to a unique Cournot equilibrium in intervals $I_7 \sim I_9$.[9] These results are all in line with what obtained in Zanchettin (2006), and are presented in Table 2-4. To sum up, we have:

Proposition 2: *In the third market model, if (i) the goods produced by the home and the foreign firms are substitutes, and (ii) both firms' outputs and profits are nonnegative, then, with home government's export subsidies, there is a unique Cournot equilibrium in intervals* $I_6 \sim I_9$ *while the home firm becomes a monopoly in interval* I_{10}.

Since we are interested only in duopoly competition, the monopoly and limit-pricing equilibria are irrelevant in the analysis below.

2.3 Home government's optimal policy

In the first stage of the game, the home government determines the value of s to maximize home social welfare, knowing that the firms will react endogenously to its

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[8] We do not deal with the case of export taxes because the home government's optimal policy cannot be an export tax as to be discussed in Section 2.3.

[9] As shown in Zanchettin (2006), different from the case $s > s_1$, when $s_3 < s < s_1$ firm h still faces a competition pressure from firm f even if it drives out firm f under Bertrand competition. In other words, this is a limit-pricing equilibrium where the home firm produces more than the monopoly output and charges a lower price. Moreover, given that the index of asymmetry is equal to s in our model from footnote 6, proofs of these results are similar to those in Zanchettin (2006). As such, we do not repeat them here to save space.

DOI:10.6814/NCCU202100569

policy choice. More precisely, the home government must choose the subsidy (tax) that induces the firms to compete in a specific mode in the second stage so that the social welfare $W_h(s) = \pi_h(s) - sq_h(s)$ is maximized. Given that both firms will choose the same strategic variable, we denote the home social welfare function as $W_h^{(C,C)}(s)$ [$W_h^{(B,B)}(s)$] if the mode of competition is (C,C) [(B,B)].

Let $s^{(C,C)*} = \arg\max W_h^{(C,C)}(s)$ [$s^{(B,B)*} = \arg\max W_h^{(B,B)}(s)$] be the optimal s that gives rise to the highest home welfare under the mode of competition (C,C) [(B,B)]. It is easy to show that $W_h^{(C,C)}(s^{(C,C)*}) > W_h^{(B,B)}(s^{(B,B)*})$ for all $b \in (0,1)$.[10] This implies that the maximum home welfare can be achieved only if the two firms compete in quantity in the second stage and the home government sets $s = s^{(C,C)*}$ in the first stage. The problem is how to ensure that $s = s^{(C,C)*}$ would indeed induce both firms to compete in output. The analysis of firms' behavior in last section indicates that it requires $s^{(C,C)*}$ lie in one of the intervals which supports a Nash equilibrium with competition mode (C,C) in Table 2-4. After some calculations we have found that $s^{(C,C)*} = [(a-c)(2-b)b^2]/[4(2-b^2)]$. Since $(a-c)$ is nothing but a scale multiplicatively affecting s_l, $l = 1,\dots,8$ in the same way, the exact location of $s = s^{(C,C)*}$ depends only on the value of b. The fact that $s^{(C,C)*} = 0$ when $b = 0$, $s^{(C,C)*} = s_5$ when $b = 1$, and $ds^{(C,C)*}/db > 0$ gives us: $s^{(C,C)*} \in I_6$ if $0 < b < 0.93$, $s^{(C,C)*} \in I_7$ if $0.93 < b < 0.96$, and $s^{(C,C)*} \in I_8$ if $0.96 < b < 1$.[11] We thus establish:

Proposition 3*: In the third market model, if (i) the goods produced by the home and the foreign firms are substitutes, and (ii) both firms' outputs and profits are nonnegative,*

[10] Derivation of the result is presented in Appendix 2.

[11] $ds^{(C,C)*}/db = [b(a-c)/4][((2-b^2)(4-3b) + 2b^2(2-b))/(2-b^2)^2] > 0$.

DOI:10.6814/NCCU202100569

then the two firms will choose to compete in quantity and the home government can always maximize the home country's welfare by the specific export subsidy $s^{(C,C)*}$.

The proposition suggests that, at least under the linear (inverse) demand functions (2.1) and (2.2), the results of Brander and Spencer (1985) are more relevant than those of Eaton and Grossman (1986) if the firms can and do choose their strategic variables endogenously after the home government made its export policy choice. Similarly, the concern put forward by Maggi (1996): "If a government lacks information about the exact nature of competition, trade policies based on the wrong beliefs can be harmful for the home country…" becomes less a problem when the choice of strategic variables by the firms is endogenous.

The economic rationale behind the result is straightforward. First, proposition 1 excludes any mixed duopoly as a possible mode of competition. Second, the two firms' profits are higher under (C,C) than under (B,B) since the latter is always more competitive than the former. Finally, in the third market model the home government can maximize social welfare only by helping the home firm maximize its profits. Therefore, the home government's policy is incentive-compatible with the home firm's objective. Brander and Spencer's policy suggestion of export subsidies to the home firm is strengthened as long as the firms choose their strategic variables endogenously under some rather moderate conditions. To the extent that the two firms play (C,C) this striking result can be carried directly over to Brander and Spencer's original model with homogeneous products or $b=1$ in our model.

Before concluding the paper it deserves to clarify why Choi et al. (2016) arrives at the result that the mode of competition (B,B), instead of(C,C), prevails when the firms choose the strategic variables endogenously. To be sure, they have essentially the same third market model as we have here, but with two critical differences: (1) the firms

DOI:10.6814/NCCU202100569

choose the mode of competition in the first stage; (2) both the home and the foreign governments set optimal trade policies simultaneously in the second stage. Since the firms choose their strategic variables at the first stage, the mode of competition becomes exogenous from the governments' point of view. Therefore, the rule obtained by Schroeder and Tremblay (2015) and Tsai et al. (2016) that the home country's optimal policy is an export subsidy (tax) to the home firm when the foreign firm chooses quantity (price) as strategic variable is applicable.

Now consider the home firm's choice of strategic variable. There are two cases depending on the foreign firm's choice. (A) If the foreign firm chooses quantity, then the home government will subsidize the home firm's export. In this case, if the home firm chooses quantity, the foreign government will optimally subsidize the foreign firm, leading to a Cournot equilibrium. However, the home firm can improve over that by choosing price since the foreign government will now tax the foreign firm and a (home-subsidized, foreign-taxed) equilibrium emerges. (B) If the foreign firm chooses price, then the home government will tax the home firm's export. In this case, if the home firm chooses quantity, the foreign government will provide optimal subsidy to foreign firm, leading to worst (home-taxed, foreign-subsidized) equilibrium from the home firm's point of view. The home firm can improve over that by choosing price since the foreign government will now tax the foreign firm. (A) and (B) imply that choosing price is the dominant strategy for the home firm. By the same logic, the dominant strategy for the foreign firm is to choose price as strategic variable, and the result of Choi et al. (2016) is obtained. Aside from the order of moves, it is clear that both governments can set their export policies is the key assumption in driving the conclusion of Choi et al. (2016).

DOI:10.6814/NCCU202100569

3. Concluding Remarks

We have shown that in the well-known third market strategic trade policy model the kind of "mixed duopoly" *a la* Singh and Vives (1984) cannot be supported if the two firms are allowed to choose strategic variables endogenously. More importantly, knowing that the firms will react to its policy in choosing their strategic variables, the home government can indeed provide export subsidies to the home firm to maximize the home social welfare if some moderate, reasonable constraints are satisfied. In this sense, Brander and Spencer's policy suggestion of export subsidies to the home firm appears to be more relevant in the real world. While demonstrated using simple linear demand functions, we believe that this paper has contributed usefully to our understanding of the strategic trade policy in an important, yet somewhat neglected, aspect. It is admitted that our results are based on a very simple duopoly model. The robustness of the conclusion needs more nuanced framework and analysis. As such, an extension of the model to include both governments as Choi et al. (2016) or multiple firms as Tsai et al. (2016) constitutes a promising direction for future research.

DOI:10.6814/NCCU202100569

References

Basak, D. and L.F.S. Wang, (2016). "Endogenous Choice of Price or Qnantity Contract and the Implications of Two-Part-Tariff in a Vertical Structure", *Economic Letters* 138: 53-56.

Brander, J.A., (1995). "Strategic Trade Policy", in: G. Grossman and K. Rogoff eds., *Handbook of International Economics* (Elsevier science B.V., Amsterdam, The Netherland): 1397-1455.

Brander, J.A. and B.J. Spencer, (1985). "Export Subsidies and International Market Share Rivalry", *Journal of International Economics* 18: 83-100.

Choi, K., K.D. Lee, and S. Lim, (2016). "Strategic Trade Policies in International Rivalry when Competition Mode is Endogenous", *Hitotsubashi Journal of Economics* 57(2): 223-241.

Eaton, J. and G.M. Grossman, (1986). "Optimal Trade and Industrial Policy under Oligopoly", *Quarterly Journal of Economics* 101: 383-406.

Feenstra, R.C. and A.K. Rose, (2000). "The Journal of International Economics at Fifty: A Retrospective", *Journal of International Economics* 50: 3-15.

Head, K. and B.J. Spencer, (2017). "Oligopoly in International Trade: Rise, Fall and Resurgence," NBER Working Paper 23720.

DOI:10.6814/NCCU202100569

Maggi, G., (1996). "Strategic Trade Policies with Endogenous Mode of Competition", *American Economic Review* 86: 237-258.

Schroeder, E. and V.J. Tremblay, (2015). "A Reappraisal of Strategic Trade Policy", *Journal of Industry, Competition and Trade* 15: 435-442.

Singh, N. and X. Vives, (1984). "Price and Quantity Competition in a Differentiated Duopoly", *Rand Journal of Economics* 15: 546-554.

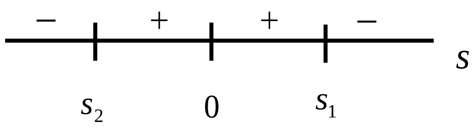
Tsai, S.R., P.L. Tsai and Y. Weng, (2016). "Cournot-Bertrand Competition: A Revisit of Strategic Trade Policy in the Third-Market Model", *Journal of Economic Studies* 43: 475 - 487.

Zanchettin, P., (2006). "Differentiated Duopoly with Asymmetric Costs", *Journal of Economics and Management Strategy* 15: 999-1015.

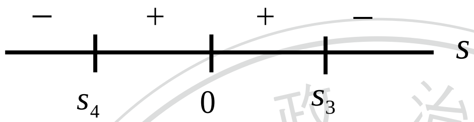
DOI:10.6814/NCCU202100569

Figure 2-1 The signs of $\pi_h^{(.,k)*}$ and $\pi_f^{(j,.)*}$

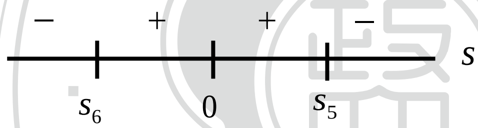
(1) $\pi_h^{(.,C)*}$



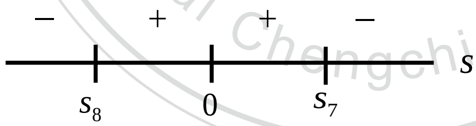
(2) $\pi_h^{(.,B)*}$



(3) $\pi_f^{(C,.)*}$



(4) $\pi_f^{(B,.)*}$



DOI:10.6814/NCCU202100569

Table 2-1 The payoff matrix of different competition modes for given s

Firm h \ Firm f	C	B
C	$(\pi_h^{(C,C)*}, \pi_f^{(C,C)*})$	$(\pi_h^{(C,B)*}, \pi_f^{(C,B)*})$
B	$(\pi_h^{(B,C)*}, \pi_f^{(B,C)*})$	$(\pi_h^{(B,B)*}, \pi_f^{(B,B)*})$

$$\pi_h^{(C,C)*} = [(a-c)(2-b)+2s]^2 / (2-b)^2(2+b)^2$$

$$\pi_f^{(C,C)*} = [(a-c)(2-b)-bs]^2 / (2-b)^2(2+b)^2$$

$$\pi_h^{(C,B)*} = (1-b^2)[(a-c)(2-b)+2s]^2 / (4-3b^2)^2$$

$$\pi_f^{(C,B)*} = [(a-c)(2-b-b^2)-bs]^2 / (4-3b^2)^2$$

$$\pi_h^{(B,C)*} = [(a-c)(2-b-b^2)+(2-b^2)s]^2 / (4-3b^2)^2$$

$$\pi_f^{(B,C)*} = (1-b^2)[(a-c)(2-b)-bs]^2 / (4-3b^2)^2$$

$$\pi_h^{(B,B)*} = [(a-c)(2-b-b^2)+(2-b^2)s]^2 / (1-b^2)(4-b^2)^2$$

$$\pi_f^{(B,B)*} = [(a-c)(2-b-b^2)-bs]^2 / (1-b^2)(4-b^2)^2$$

DOI:10.6814/NCCU202100569

Table 2-2 Signs of $\pi_h^{(.,C)*}$, $\pi_h^{(.,B)*}$, $\pi_f^{(C,.)*}$ and $\pi_f^{(B,.)*}$ in each interval of s

s_6 s_2 s_4 s_8 0 s_3 s_7 s_5 s_1

Sign	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
$\pi_h^{(.,C)*}$	-	-	+	+	+	+	+	+	+	-
$\pi_h^{(.,B)*}$	-	-	-	+	+	+	-	-	-	-
$\pi_f^{(C,.)*}$	-	+	+	+	+	+	+	+	-	-
$\pi_f^{(B,.)*}$	-	-	-	-	+	+	+	-	-	-

Table 2-3 Mode of competition and Nash equilibrium (NE) in each interval of s

s_6 s_2 s_4 s_8 0 s_3 s_7 s_5 s_1

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
NE	(B,B)	(B,B)	(B,B) or (C,C)	(C,C)	(C,C)	(C,C)	(C,C)	(B,B) or (C,C)	(B,B)	(B,B)

DOI:10.6814/NCCU202100569

Table 2-4 Types of equilibrium with nonnegative outputs and profits constraints under export subsidies

0 s_3 s_7 s_5 s_1

<table>
<tr><td>I_6</td><td>I_7</td><td>I_8</td><td>I_9</td><td>I_{10}</td></tr>
<tr><td rowspan="2">(C,C)</td><td>(C,C)</td><td>(C,C)</td><td>(C,C)</td><td rowspan="2">monopoly equilibrium</td></tr>
<tr><td colspan="3">Limit-pricing equilibrium (if the two firms playing (B,B))</td></tr>
</table>

Appendix 1. The value of s_l, $l = 1,\ldots,8$, in Figure 2.1

$s_1 = (a-c)(2-b)/b > 0$

$s_2 = -(a-c)(2-b)(8-6b^2-b^3)/[(4-b^2)^2-4b^2] < 0$

$s_3 = (a-c)(1-b)(2+b)/b > 0$

$s_4 = -(a-c)(1-b)(2+b)(8-6b^2+b^3)/(16-20b^2+5b^4) < 0$

$s_5 = (a-c)(2-b)(8-6b^2-b^3)/4b(2-b^2) > 0$

$s_6 = -(a-c)(2-b)/2 < 0$

$s_7 = (a-c)(1-b)(2+b)(8-6b^2+b^3)/b(8-8b^2+b^4) > 0$

$s_8 = -(a-c)(1-b)(2+b)/(2-b^2) < 0$

DOI:10.6814/NCCU202100569

Appendix 2. Derivation of $W_h^{(C,C)}(s^{(C,C)*}) > W_h^{(B,B)}(s^{(B,B)*})$

Plugging the optimal values of the strategic variables obtained in Section 2.1 under the four modes of competition into the home country's welfare function $W_h(s) = \pi_h(s) - sq_h(s)$ and maximizing it with respect to s gives us

$$s^{(C,C)*} = (a-c)(2-b)b^2 \big/ 4(2-b^2) > 0$$

$$s^{(B,B)*} = -(a-c)(1-b)(2+b)b^2 \big/ 4(2-b^2) < 0$$

As a result, we have

$$W_h^{(C,C)}(s^{(C,C)*}) = (a-c)^2(2-b)^2 \big/ 8(2-b^2),$$

$$W_h^{(B,B)}(s^{(B,B)*}) = (a-c)^2(1-b)(2+b)^2 \big/ 8(1+b)(2-b^2),$$

and

$$W_h^{(C,C)}(s^{(C,C)*}) - W_h^{(B,B)}(s^{(B,B)*}) = (a-c)^2 b^3 \big/ 4(1+b)(2-b^2) > 0.$$

DOI:10.6814/NCCU202100569

Chapter 3 Learning-by-Exporting and Firm Heterogeneity

1. Introduction.

With an improvement in the availability and quality of the firm- level data economists have done lots of research to investigate the relation between export behavior and the performance of firms. One of the most robust results is that export firms, on average, are more productive than the non-export firms. Using data from the U.S. manufacture sector, Bernard and Jensen (1995) find evidence that export firms are more productive than its non-export counterpart. Mountains of researches using firm-level data from different countries arrive at the same result. This gives a strong empirical regularity that exporters are more productive than the non-exporters.

However, the interesting question is: what causes export firms to be more productive? Bernard and Jensen (1999) have suggested two possible explanations: the self-selection effect and the learning-by-exporting effect[1]. Many researches give support to the self-selection effect, such as Bernard and Jensen (1999) (for US), Temouri, Vogel and Wagner (2013) (for UK, Germany and France), and Arnold and Hussinger (2004) (for Germany). As for the learning-by-exporting effect, although not as strong as self-selection effect, it has been found in several studies. De Loecker (2007) has found the evidence of learning-by-exporting for Slovenia. Furthermore, it is found that the magnitude of the learning effect is correlated with the export destination. De Loecker (2013) also suggests that the lack of evidence of the learning by exporting effect might be caused by the biasedness of the test method. Bai et al. (2017) investigate the learning effect and firms' export mode (direct or indirect exporter) for Chinese

[1] The two effects are not mutually exclusive events, it is possible for them to exist simultaneously.

DOI:10.6814/NCCU202100569

firms. They have found that the direct exporters learn more than the indirect ones. Crespi et al. (2008) (for UK) find that firms that exported in the past learn from their clients, resulting in a faster labor productivity growth. Baldwin and Gu (2003), using the data of the Canadian manufacturing firms, find that the two effects exist simultaneously, which gives an evidence of the non-mutually exclusive feature of self-selection and learning-by-exporting. These researches all provide evidence supporting learning-by-exporting, although the mechanism for productivity growth might be different.

Despite many studies on firm performance and the export (import) activity, most of them are empirical papers, the theoretical part is relatively scant. Melitz (2003) has provided an excellent theoretical model to explain how high productive firms self-select themselves into the export activity. However, to my knowledge, no theoretical model that focuses on the learning-by-exporting aspect has been developed. Consequently, the aim of this paper is to construct a theoretical model for the learning-by-exporting effect, based on the basic structure of Melitz (2003). Since whether there is a learning effect on export is an empirically problem, instead of designing a mechanism of productivity growth or justifying the existence of the phenomenon, we are interested in what features may appear if there is a learning effect on export which cannot be captured by the self-selection model.

We assume all the incumbent firms compete in a monopolistic competition market with identical productivity (marginal cost) in the close economy as Krugman (1979), but with an entry cost when a firm decides to enter the domestic market as Melitz (2003). Heterogeneity arises in the open economy. Where each firm can draw a productivity from a productivity distribution. To acquire the productivity it had drawn, a firm needs to export to at least one country. There exists a learning-by-exporting effect without any uncertainty. A firm's productivity changes only if it exports, whereas

DOI:10.6814/NCCU202100569

firms which do not export remain at the original productivity level. This setting split the two effects completely. All firms are identical without export so that there are no better firms self-selecting into the export market in the open economy. On the contrary, what determines the export decision is that a firm expects to become a better one after export. Although the difference in productivity growth is affected by various factors in reality (such as export destination, foreign clients), in our model the magnitude of the productivity growth for all the identical firms depends purely on luck. It is determined by the productivity drawing from a productivity distribution. This setting simplifies our analysis and captures the phenomenon that productivity growth can be different for firms which are initially identical.

As Melitz (2003), we find gains from trade due to increases in product varieties in a country and reallocation of the resources toward the more productive firms. Furthermore, in our model increases in productivity of the export firms also lead to a welfare gain. We also find two phenomena that do not appear in the self-selection structure: The first one is that export improves firms' productivity and influences profits earned from the domestic sales, and that firm is more willing to export compared with the self-selection case (no learning effect). In our model, it is possible for a firm to choose export even the profit from the export sales is negative since the loss in the export sales can be compensated by the increase in the profit of the domestic sales. The second one is that we find a productivity gap between the exporters and the non-exporters. Specifically, the least productivity exporters is more productive than the firms who serve the domestic market only. There is a productivity region that segments the exporters and non-exporters with no firms in it. This is due to the fact that the growth in productivity in this region after export is too small, a firm would give up acquiring higher productivity through export. They continue to serve the domestic market only and remain at the lowest productivity level. The main difference between

DOI:10.6814/NCCU202100569

the firms with and without the learning effect is the change in their profits due to productivity changes. This has a crucial effect on firms' export decision. A firm might sacrifice some of the profits to acquire a higher productivity (some of the firms get a productivity growth at the cost of negative export profit). This result is useful in discussing various issues, such as the export and foreign direct investment decision for a firm.

Aside from the introduction, the remainder of this paper is organized as follows. Section 2 sets up the model, describing consumer preferences and firms' technology. Section 3 discusses a firm's entry into and exit of the market. Section 4 presents the closed economy equilibrium. Section 5 elaborates the open economy model. Section 6 analyzes the export decision of the firms in the open economy. Section 7 and Section 8 discuss the zero cutoff point condition and the free entry condition under open economy, respectively. Section 9 presents the open economy equilibrium. Section 10 investigates the effects of trade. Section 11 concludes the paper.

2. The Model

Our model follows the basic structure of Melitz (2003) but with a crucial different set-up in firms' marginal costs. At the beginning, there are a large number of potential entrants and all of them have the same constant marginal cost, as Krugman (1979). In the closed economy equilibrium, all firms share identical profits and market share since all of them have an identical cost function. However, our model assumes that a firm can improve its productivity (lower its marginal cost) through export. In the open economy, each firm draws a productivity from a productivity distribution. After recognizing the productivity it has drawn, the firm decides whether to export or not. The productivity of the firm will grow to the level it has drawn from the distribution if export, otherwise

DOI:10.6814/NCCU202100569

the productivity remains at the original level. There is no uncertainty with the productivity growth in our model. In the case of open economy, some of the firms choose to export and improve their productivity while the others choose to sell domestically only and remain at the original productivity level.

The aim of this model is to investigate the effect of learning-by-exporting. As such, we assume the same original productivity for all firms, and the productivity can change only through export. Although the model is very simple in the assumption of the productivity growth mechanism, it captures some crucial features which do not exist in the self-selection model. We start at the closed economy and then move toward to the open economy cases.

2.1. Closed economy

The demand side of our model follows all the assumptions of Melitz (2003), which are summarized as follows:

2.1.1. Demand

The consumers have CES preferences, which is:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega\right]^{\frac{1}{\rho}},\ \rho \in (0,1). \tag{3.1}$$

Ω represents the set of goods that are available in a country, and ω denotes variety of good q.

DOI:10.6814/NCCU202100569

The price index is:

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}, \tag{3.2}$$

where $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution of any two varieties. The demand and expenditure on variety ω are:

$$q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\sigma}, \tag{3.3}$$

$$r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma}, \tag{3.4}$$

where $Q \equiv U$ according to Dixit and Stiglitz (1977). $R = PQ = \int_{\omega\in\Omega} r(\omega) d\omega$ is the aggregate expenditure.

In the production side, all firms have the same technology with a constant marginal cost. The features of the production side are summarized as follows:

2.1.2. Production

The cost function is: $l = f + \frac{q}{\varphi_c}$, where $\frac{1}{\varphi_c}$ is the marginal cost of all firms and c denotes closed economy. The pricing rule is $p(\varphi_c) = \frac{w}{\rho\varphi_c}$, which becomes $p(\varphi_c) = \frac{1}{\rho\varphi_c}$ when w is normalized to one. The output function, revenue function and profit function of a firm are, respectively:

DOI:10.6814/NCCU202100569

$$q(\varphi_c) = Q\left[\frac{1}{P\rho\varphi_c}\right]^{-\sigma} = Q(P\rho\varphi_c)^{\sigma}, \tag{3.5}$$

$$r(\varphi_c) = R\left[\frac{1}{P\rho\varphi_c}\right]^{1-\sigma} = R(P\rho\varphi_c)^{\sigma-1}, \tag{3.6}$$

$$\pi(\varphi_c) = \frac{r(\varphi_c)}{\sigma} - f = \frac{R(P\rho\varphi_c)^{\sigma-1}}{\sigma} - f \ . \tag{3.7}$$

2.1.3 Aggregation

The price index in the equilibrium with a mass of incumbent firms M (M goods) with a productivity φ_c is given by:

$$P = \left[\int_{\omega \in M} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} = [p(1)^{1-\sigma} + p(2)^{1-\sigma} + \ldots + p(M)^{1-\sigma}]^{\frac{1}{1-\sigma}} =$$

$$M^{\frac{1}{1-\sigma}}[E(p(\varphi_c))^{1-\sigma}]^{\frac{1}{1-\sigma}} = M^{\frac{1}{1-\sigma}}p(\varphi_c). \tag{3.8}$$

Notice that $p(1) = p(2) = \ldots = p(M) = p(\varphi_c)$.

All aggregate variables can be characterized by the equilibrium mass of firms and the average productivity as below:

Aggregate price: $P = M^{\frac{1}{1-\sigma}}p(\varphi_c)$,

Aggregate expenditure: $R = PQ = \int_{\omega \in M} r(\omega)d\omega = r(\omega_1) + \ldots + r(\omega_M) = Mr(\varphi_c)$,

(3.9)

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Aggregate profit: $\Pi = \int_{\omega \in M} \pi(\omega) d\omega = \pi(\omega_1) + \ldots + \pi(\omega_M) = M\pi(\varphi_c)$. (3.10)

By the definition of $Q \equiv U$, we get:

$$Q \equiv U = \left[\int_{\omega \in M} q(\omega)^{\rho} d\omega\right]^{\frac{1}{\rho}} = [q(\omega_1)^{\rho} + \ldots + q(\omega_M)^{\rho}]^{\frac{1}{\rho}} = M^{\frac{1}{\rho}} q(\varphi_c). \tag{3.11}$$

Since all firms have the same productivity, we have $r(\omega_i) = r(\omega_j)$, $\pi(\omega_i) = \pi(\omega_j)$ and $q(\omega_i) = q(\omega_j)$ for all $i, j \in M$.

3. Firm Entry and Exit

Differing from Melitz (2003), in our model all firms have the same productivity when they enter the domestic market. There is an entry cost f_e, and there is a probability $\delta \in (0,1)$ for an existing firm to meet a bad shock and exits the market.

For any firm, its value function is given by $v(\varphi) = max\left\{0, \frac{1}{\delta}\pi(\varphi)\right\}$. This means that a firm will earn the sum of its expected profit in every period if the profit in every period is positive; otherwise, it will quit the market. Since a firm's profit is increasing in the productivity level[2], we can define a cutoff productivity φ_a^* which makes a firm earn zero profit. Namely, the cutoff productivity in the closed economy is $\varphi_a^* = \{\varphi : \pi(\varphi_a^*) = 0\}$.

3.1. ZCP condition

[2] $\pi(\varphi) = \frac{r(\varphi)}{\sigma} - f = \frac{R(P\rho\varphi)^{\sigma-1}}{\sigma} - f$, which is increasing in φ.

DOI:10.6814/NCCU202100569

Using the properties of the revenue and the profit functions, we can write the average profit function as a function of cutoff productivity φ_a^*, which is called the zero cutoff point curve (ZCP curve).

Using the revenue function (3.6) of the firm, we can derive the relative revenue between φ_c and φ_a^* as $\frac{r(\varphi_c)}{r(\varphi_a^*)} = \frac{R(P\rho\varphi_c)^{\sigma-1}}{R(P\rho\varphi_a^*)^{\sigma-1}} = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1}$. Thus the average revenue of the surviving firm is $\bar{r} = r(\varphi_c) = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} r(\varphi_a^*)$. Since $\pi(\varphi_a^*) = \frac{r(\varphi_a^*)}{\sigma} - f = 0$, we have $r(\varphi_a^*) = \sigma f$. Thus, $\bar{r} = r(\varphi_c) = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} \sigma f$. (Recall that all the firms have the same productivity level φ_c). Substituting $r(\varphi_c) = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} \sigma f$ into equation (3.7) gives us the ZCP curve:

ZCP curve: $\bar{\pi} = \pi(\varphi_c) = \frac{\bar{r}}{\sigma} - f = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} f - f = f\left[\left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} - 1\right]$. (3.12)

The ZCP curve shows the average profit for all existing firms when the cutoff productivity is φ_a^*. Since φ_c is exogenously given, it is easy to verify that the ZCP curve is decreasing and convex in φ_a^*. Moreover, it intersects the horizontal-axis at φ_c, as shown in Figure 3-1.

3.2. FE condition

The free entry condition (FE condition) characterizes the relationship between the least expected profit a firm required to enter the market and the cutoff productivity φ_a^*. Let p_{in} be the probability for a successful entry (which means a firm can earn positive

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profit after entry), then the probability for successful entry times the expected profit after entry must be greater or equal to the entry cost a firm has paid. As a result, for a firm to entry, the least requirement is to recover its entry cost, which is $p_{in}\bar{v} = f_e$, where $\bar{v} = \frac{\bar{\pi}}{\delta}$. Since all firms have the same productivity φ_c, the probability for a successful entry equals to one if $\varphi_a^* \leq \varphi_c$ (namely, all firms earn nonnegative profit in every period), and equals to zero if $\varphi_a^* > \varphi_c$. As the result, the FE curve $\bar{\pi} = \frac{\delta f_e}{P_{in}}$ is flat and with the height equals δf_e in the region $\varphi_a^* \leq \varphi_c$ and is infinitive in the region that $\varphi_a^* > \varphi_c$. In fact, we always have $\varphi_a^* < \varphi_c$ in the closed economy equilibrium. (Figure 3.1)

FE curve: $\bar{\pi} = \frac{\delta f_e}{P_{in}}$ (where $p_{in} = 1$ if $\varphi_a^* \leq \varphi_c$; $p_{in} = 0$ if $\varphi_a^* > \varphi_c$). (3.13)

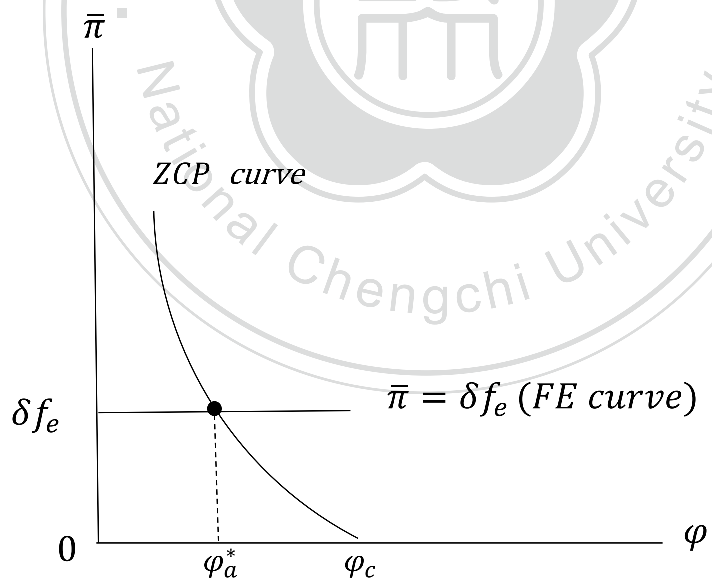

Figure 3-1 Determination of the equilibrium average profit and cutoff productivity (φ_a^*) in the closed economy.

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4. Closed Economy Equilibrium

Since the FE curve represents the relationships between the least expected profit for a firm to enter and the cutoff productivity, and the ZCP condition links the average profit of the existing firms with the cutoff productivity, the intersection of the two curves gives the cutoff productivity level that makes the average profit and the least required profit equal, and thus determines the closed economy equilibrium. If the average profit of the existing firms is greater than the least expected profit a firm required to enter, more new firms will enter the market; conversely, if the average profit is lower than the least expected profit, some firms will exit until the average profit equals to the least expected profit.

It is obvious that the equilibrium profit is δf_e for all firms since the FE curve is horizontal with the height δf_e, and all firms are symmetric in the closed economy. (see Figure 3-1)

$$\pi(\varphi_c) = \frac{r(\varphi_c)}{\sigma} - f = \delta f_e. \tag{3.14}$$

Using (3.14) we have the revenue in equilibrium for all firms:

$$r(\varphi_c) = \sigma(f + \delta f_e) = R(P\rho\varphi_c)^{\sigma-1}. \tag{3.15}$$

Since the productivity level for all firms φ_c is exogenously given, we can solve the equilibrium price index P from (3.15):

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$$r(\varphi_c) = \sigma(f + \delta f_e) = R(P\rho\varphi_c)^{\sigma-1} \;\Rightarrow\; P = \frac{1}{\rho\varphi_c}\left[\frac{\sigma}{R}(f + \delta f_e)\right]^{\frac{1}{\sigma-1}}. \tag{3.16}$$

Using $P = M^{\frac{1}{1-\sigma}}p(\varphi_c) = M^{\frac{1}{1-\sigma}}\frac{1}{\rho\varphi_c}$ and equation (3.16), we can solve the equilibrium mass of firms:

$$M = \frac{R}{\sigma(f+\delta f_e)}. \tag{3.17}$$

The equilibrium cutoff productivity can be solved by the FE and the ZCP curves:

$$\delta f_e = f\left[\left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} - 1\right] \;\Rightarrow\; \varphi_a^* = \left(\frac{f}{f+\delta f_e}\right)^{\frac{1}{\sigma-1}}\varphi_c \;\; (< \varphi_c). \tag{3.18}$$

Notice that φ_a^* is increasing in φ_c, implying the cutoff productivity is higher if all firms are more productive, which leads to a more intense competition.

Welfare per worker is:

$$W = \frac{R}{L}P^{-1} = P^{-1} = M^{\frac{1}{\sigma-1}}\rho\varphi_c.^{3} \tag{3.19}$$

5. Open Economy

Following the basic assumptions of Melitz (2003), we assume that firms can trade with $n \geq 1$ identical countries. To capture the effect of learning-by-exporting, we assume a

[3] As explained by Melitz (2003), it can be shown that $L = R$ in a stationary equilibrium due to the model's property.

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firm's productivity will grow if and only if it chooses to export. Each firm draws a productivity from a continuous productivity distribution in the open economy, and then decides whether to export or not. If a firm chooses not to export, its productivity remains at the original level φ_c; however, if the firm chooses to export, its productivity will grow to the level that it has drawn from the productivity distribution. For simplicity, we assume that there is no uncertainty about the growth of the productivity.

5.1. Assumptions in the open economy

I. An export firm has to pay a fixed export cost f_{ex} and an iceberg cost, $\tau > 1$.

Fixed export cost: Let a one-time investment fixed cost be $f_{ex} > 0$, or an amortized per-period fixed cost $f_x = \delta f_{ex}$.[4] We assume that export has greater per period fixed cost than domestic sales, that is $f_x > f$.

Iceberg cost: $\tau > 1$, meaning that τ units of good need to be shipped for one unit of good to arrive in the foreign country.

II. Firm can trade with $n \geq 1$ identical countries.

III. Productivity grows if the firm decides to export.

1. In the open economy, a firm has one chance to draw a productivity φ from

[4] In other words, $(\sum_{t=0}^{\infty}(1-\delta)^t f_x = f_{ex},\ f_x = \delta f_{ex})$, where δ is the probability of bad shock.

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a productivity distribution with PDF $g(\varphi)$, which has a support $(0, \infty)$ and a CDF $G(\varphi)$.

2. Denote φ_x as the productivity that a firm has drawn from $g(\varphi)$, then the productivity of firm will grow to $Max[\varphi_c, \varphi_x]$ if the firm export. The productivity remains at φ_c (back to φ_c) if the firm does not export to any country (stopped exporting). The productivity growth is assumed to be independent of the number of export countries

5.2. A firm's total revenue under open economy

Pricing rule for foreign markets reflects the increase in marginal cost. The price firms charge in the foreign markets is:

$$p_x(\varphi) = \frac{\tau}{\rho\varphi} = \tau p_d(\varphi). \tag{3.20}$$

Where $p_x(p_d)$ is the price a firm charges in the foreign (domestic) market.

Using the price for foreign country and the revenue function (3.6), we have a firm's revenue from export sales:

$$r_x(\varphi) = R\left[\frac{p_x(\varphi)}{P}\right]^{1-\sigma} = \tau^{1-\sigma}R(P\rho\varphi)^{\sigma-1} = \tau^{1-\sigma}r_d(\varphi). \tag{3.21}$$

The notations $r_x(\varphi)$ and $r_d(\varphi)$ represent the revenue from foreign and domestic market, respectively.

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Combining domestic sales and export sales, the total revenue of a firm in the open economy is:

$r(\varphi) = r_d(\varphi_c)$, if the firm sells at home country only. (3.22)

$r(\varphi) = r_d(\varphi_x) + r_x(\varphi_x) = r_d(\varphi_x)[1 + \tau^{1-\sigma}]$, if the firm exports to "one" foreign country. (3.23)

$r(\varphi) = r_d(\varphi_x) + nr_x(\varphi_x) = r_d(\varphi_x)[1 + n\tau^{1-\sigma}]$, if the firm exports to "n" foreign countries. (3.24)

If a firm decides to export (to either one or n countries), its productivity will grow to the value it had drawn from the productivity distribution $g(\varphi)$, denoting as φ_x in the rest of the paper. Otherwise its productivity remains at the original level φ_c.

6. The Decision for Export.

A firm will decide to export if its profits are greater with export under the productivity level φ_x it has drawn. The export decision in our model has a critical difference from that of Melitz (2003). This is because export changes the productivity, and thus changes the profits from domestic sales. A firm will export if the combined profits of export and domestic sales are greater than sales at domestic market only. Since a firm's productivity changes after export, the profits it earns from the domestic sales also change after export. Therefore, the export decision here considers not just whether the export profits are greater than zero. Instead, a firm has to compare the combined domestic and export profits after the productivity change with the profits if it does not

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export. In fact, under our assumption of the cost, it is possible for a firm to choose to export even it has a negative export profits since the firm can earn more from the domestic sales due to the productivity growth.

The cutoff productivity level, φ_x^s, is defined as the productivity level that makes a firm indifferent between exporting (to "one" country, since export to "$n > 1$" countries doesn't make productivity grow further) and selling in the domestic market only. In notations, we have $\pi_d(\varphi_x^s) + \pi_x(\varphi_x^s) = \pi_d(\varphi_c)$, or

$$\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right]. \tag{3.25}$$

$\pi_d(\varphi)$ and $\pi_x(\varphi)$ are the firm's profit from domestic, and export sales, respectively.

The LHS of (3.25) is the total profit of the firm if it exports to "one" country, which is an increasing function of φ; the RHS is the profit if the firm sells in the domestic market only. As a result, a firm will export if it has drawn a productivity level greater than φ_x^s.

It can be shown that φ_x^s is a function of open economy equilibrium cutoff productivity φ^*, and other exogenous variables. Defined φ^* as the cutoff productivity for a firm selling domestically to make zero profit under open economy equilibrium. It can be determined endogenously as follow:

$$\pi_d(\varphi^*) = 0 \;\Leftrightarrow\; r_d(\varphi^*) = \sigma f. \tag{3.26}$$

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Using the definition of φ_x^s and φ^*, it can be shown that

$$\varphi_x^s = \varphi^*(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left[\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} + \frac{f_x}{f}\right]^{\frac{1}{\sigma-1}}. \quad [5] \tag{3.27}$$

Consequently, φ_x^s is an increasing function of φ^* and the exogenous variables that represent trade cost (τ, f_x).

To guarantee $\varphi_x^s > \varphi_c$, we assume that the original productivity level is not productive enough for a firm to make positive export profit under the closed economy equilibrium price index. That is, we block the case that all firms can export under the open economy. By using the equilibrium profit in the closed economy, we can show that $f_x > \tau^{1-\sigma}(f + \delta f_e)$.[6] Under the closed economy equilibrium, we have:

$$\pi_d(\varphi_c) = \frac{r_d(\varphi_c)}{\sigma} - f = \delta f_e.$$

The assumption that φ_c is not high enough to make positive export profit under the closed economy equilibrium price index implies: $\frac{\tau^{1-\sigma} r_d(\varphi_c)}{\sigma} - f_x < 0$. Combining $\frac{r_d(\varphi_c)}{\sigma} - f = \delta f_e$ and $\frac{\tau^{1-\sigma} r_d(\varphi_c)}{\sigma} - f_x < 0$ gives us: $f_x > \tau^{1-\sigma}(f + \delta f_e)$.[7]

The assumption $f_x > f$ guarantees $\pi_d(\varphi) > \pi_x(\varphi)$. As to be discussed later, this is a key feature for a firm's export decision. That $f_x > f$ guarantees $\pi_d(\varphi) >$

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[5] See appendix 1 for derivation.
[6] That $f_x > \tau^{1-\sigma}(f + \delta f_e)$ guarantees $\varphi_x^s > \varphi_c$ will hold only if the equilibrium price index is lower under the open economy equilibrium. If we have a higher equilibrium price index in the open economy equilibrium, the guarantee faisl.
[7] See appendix 2 for detailed discussions.

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$\pi_x(\varphi)$ can be shown as follow:

$$\pi_d(\varphi) - \pi_x(\varphi) = \left[\frac{R(P\rho\varphi)^{\sigma-1}}{\sigma} - f\right] - \left[\frac{R\tau^{1-\sigma}(P\rho\varphi)^{\sigma-1}}{\sigma} - f_x\right]$$

$= R(P\rho\varphi)^{\sigma-1}\left[\frac{1}{\sigma}(1-\tau^{1-\sigma})\right] + [f_x - f]$. Thus, $f_x > f \Rightarrow \pi_d(\varphi) - \pi_x(\varphi) > 0$ (the converse is not true).

The assumptions for export costs are summarized as follow:

(i) $f_x > \tau^{1-\sigma}(f + \delta f_e)$. (for $\varphi_x^s > \varphi_c$) (3.28)

(ii) $f_x > f$. (for $\pi_d(\varphi) > \pi_x(\varphi)$) (3.29)

Under the assumptions of export costs, we now turn to discuss some different situations of equation (3.25).

DOI:10.6814/NCCU202100569

6.1. The sign of each term in equation (3.25)

	$\pi_d(\varphi_x^s) = \left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right]$	$\pi_x(\varphi_x^s) = \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right]$	$\pi_d(\varphi_c) = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right]$
1.	+	-	-
2.	+	-	+
3.	-	-	-
4.	+	+	-
5.	-	+	+
6.	-	+	-
7.	-	-	+
8.	+	+	+

Table 3-1: The sign of each term in equation (3.25)

It is clear from table 3-1 that only the cases 1~3 of the eight cases are possible:

(i) Case 4 and case 7 are not possible since the equal sign is not satisfied.

(ii) Case 5 and case 6 cannot be true since $\pi_x(\varphi_x^s) > \pi_d(\varphi_x^s)$, contradicting with the assumptions of (3.29).

(iii) Case 8 is not possible since it can be true only if $\varphi_c > \varphi_x^s$, contradicting with (3.28).

6.2. Discussions of case 1~case 3

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6.2.1. Case 1: $\boldsymbol{\varphi_x^s > \varphi^* > \varphi_c}$

In case 1 we have $\pi_d(\varphi_x^s) + \pi_x(\varphi_x^s) = \pi_d(\varphi_c) < 0$. The result implies $\varphi_x^s > \varphi^* > \varphi_c$ since $\pi_d(\varphi_x^s) > 0$, $\pi_d(\varphi^*) = 0$ and $\pi_d(\varphi_c) < 0$. Under the situation, neither the firm selling domestically nor the exporting firm with $\varphi = \varphi_x^s$ can survive (since it has negative combined profits). The surviving firm in case 1 must have drawn a productivity level greater than φ_x^s. Define a productivity level $\varphi_x^{s'}$ that makes the firm's combined profits equal to zero, that is,

$$\left[\frac{r_d(\varphi_x^{s'})}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^{s'})}{\sigma} - f_x\right] = 0. \tag{3.30}$$

Note that $\varphi_x^{s'} > \varphi_x^s$ in case 1. Since $\pi_d(\varphi) > \pi_x(\varphi)$, we must have $\left[\frac{r_d(\varphi_x^{s'})}{\sigma} - f\right] > 0$ and $\left[\frac{r_x(\varphi_x^{s'})}{\sigma} - f_x\right] < 0$ when the equal sign holds.

As such, a firm can survive under case 1 only if it has drawn a productivity level $\varphi \geq \varphi_x^{s'}$ and decides to export.

6.2.2. The decision making of exporting to $\boldsymbol{one}$ or $\boldsymbol{n > 1}$ countries[8]

Given that a firm decides to export, the firm will export to n countries only if its export profit, $\pi_x(\varphi) \geq 0$. An export firm with a negative export profit ($\pi_x(\varphi) < 0$) will export to only one country. The reason why a firm might export even it has a negative export profit is that export improves productivity, leading to an increase in the

[8] In the following discussion, n always means $n > 1$.

DOI:10.6814/NCCU202100569

profit in the domestic market. It is perfectly likely that the increase in the domestic profit dominates the loss from export sales. Define a productivity level φ_x^* that makes zero export profit, that is:

$$\pi_x(\varphi_x^*) = 0 \quad \Leftrightarrow \quad \left[\frac{r_x(\varphi_x^*)}{\sigma} - f_x\right] = 0 \ \Leftrightarrow \ r_x(\varphi_x^*) = \sigma f_x. \tag{3.31}$$

Using $r_x(\varphi_x^*) = \sigma f_x = R(P\rho\varphi_x^*)^{\sigma-1}\tau^{1-\sigma}$ and $r_d(\varphi^*) = \sigma f = R(P\rho\varphi^*)^{\sigma-1}$ we get:

$$\varphi_x^* = \varphi^* \tau (\frac{f_x}{f})^{\frac{1}{\sigma-1}}. \tag{3.32}$$

Since $f_x > f \ \Rightarrow \ \varphi_x^* > \varphi^*$. A firm will export to $\boldsymbol{n}$ countries if it has drawn a productivity level $\varphi \geq \varphi_x^*$. In case 1, the surviving firm with $\varphi \in [\varphi_x^{s'}, \varphi_x^*)$ will export to only one country, while the firm with $\varphi \in [\varphi_x^*, \infty]$ will export to n countries.[9] We can summarize the results as the following proposition:

Proposition 1.

In case 1,

(i) we have $\varphi_a^* < \varphi_c < \varphi^* < \varphi_x^s < \varphi_x^{s'} < \varphi_x^*$;

(ii) only export firms with $\varphi \geq \varphi_x^{s'}$ *can survive*;

(iii) firms with the productivity $\varphi \in [\varphi_x^{s'}, \varphi_x^*)$ *export to one country; firms with* $\varphi \in$

[9] $(\varphi_x^* > \varphi_x^{s'}$ since $\pi_x(\varphi_x^*) = 0,\ \pi_x(\varphi_x^{s}{}') < 0)$

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$[\varphi_x^*, \infty]$ *export to* n *countries.*

It is worth noting that firms can incur a productivity growth only if it exports. The productivity of a firm remains at φ_c if it doesn't export, and φ_c is lower than the equilibrium cutoff productivity φ^* in case 1. Therefore, all non-export firms will suffer an economic loss and exit the market. Also notice that a firm with $\varphi \in [\varphi_x^s, \varphi_x^{s'})$ cannot survive. Although the firm is better off if it chooses to export, its combined profit is still negative.

To be clearer, the notations for different productivity levels are summarized below:

φ_a^*: The equilibrium cutoff productivity under autarky, defined as $\pi(\varphi_a^*) = 0$

φ_c: The original productivity level for all firms.

φ_x^s: The productivity level that makes a firm indifferent between export and selling in the domestic market only, defined as $\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right]$.

$\varphi_x^{s'}$: The productivity level that makes a firm earn zero combined profit if it exports, defined as $\left[\frac{r_d(\varphi_x^{s'})}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^{s'})}{\sigma} - f_x\right] = 0$.

φ^*: The equilibrium cutoff productivity under open economy, defined as $\pi_d(\varphi^*) = 0$.

φ_x^*: The productivity level that makes zero export profit, defined as $\pi_x(\varphi_x^*) = 0$.

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6.2.3. Case 2: $\boldsymbol{\varphi_x^s > \varphi_c > \varphi^*}$

Case 2 is the case that $\pi_d(\varphi_x^s) > 0, \pi_x(\varphi_x^s) < 0$ and $\pi_d(\varphi_x^s) + \pi_x(\varphi_x^s) = \pi_d(\varphi_c) > 0$. This means that a firm can survive even it doesn't export, that is, $\varphi_c > \varphi^*$. In this case a firm will choose to export if it has drawn a productivity level greater than or equal to φ_x^s; otherwise, it will sell at the domestic market only and make a positive profit. There will be a productivity gap between exporters and non-exporters. There are no firms with productivity in the region (φ_c, φ_x^s) since the domestic firms all share the same productivity level φ_c, whereas the least productive exporting firm has the productivity level φ_x^s. This productivity gap exists since it is costly for a firm to export. Only the firms with productivity level sufficiently higher than φ_c can better off through export. Another noteworthy point is that we have $\varphi_x^s > \varphi_x^{s'}$ in case 2. This means the productivity level (φ_x^s) that makes a firm indifferent between export and selling in the domestic market can also make a positive combined profit. A result differs from case 1. We therefore have:

Proposition 2.

In case 2,

(i) *we have* $\varphi_a^* < \varphi^* < \varphi_c < \varphi_x^s < \varphi_x^*$;

(ii) *non-export firms survive and have the original productivity level* φ_c;

(iii) *no firms with productivity* φ *in the region* (φ_c, φ_x^s);

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(iv) a firm will export to 1 (n) country if it has a productivity $\varphi \in [\varphi_x^s, \varphi_x^*)$ *(* $\varphi \in [\varphi_x^*, \infty]$*).*

6.2.4. Case 3: $\boldsymbol{\varphi^* > \varphi_x^s > \varphi_c}$

The analysis of case 3 is almost the same as that of case 1. The only difference is $\pi_d(\varphi_x^s) < 0$ in case 3. As a result, we have $\varphi^* > \varphi_x^s$. All other features are similar to those of case 1.

6.2.5. The relation between $\boldsymbol{\varphi_x^{s'}}$, $\boldsymbol{\varphi_x^s}$ and $\boldsymbol{\varphi^*}$

We can solve the export cutoff productivity $\varphi_x^{s'}$ (φ_x^s) in case 1 and case 3 (case2) as a function of the equilibrium cutoff productivity φ^*. The definition of $\varphi_x^{s'}$ is:

$$\left[\frac{r_d(\varphi_x^{s'})}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^{s'})}{\sigma} - f_x\right] = 0.$$

$\Rightarrow \quad r_d(\varphi_x^{s'}) + r_x(\varphi_x^{s'}) = \sigma(f + f_x) = R(P\rho\varphi_x^{s'})^{\sigma-1}(1 + \tau^{1-\sigma})$. Using $r_d(\varphi^*) = \sigma f = R(P\rho\varphi^*)^{\sigma-1}$,

$$\Rightarrow \frac{r_d(\varphi_x^{s'}) + r_x(\varphi_x^{s'})}{r_d(\varphi^*)} = \frac{R(P\rho\varphi_x^{s'})^{\sigma-1}(1+\tau^{1-\sigma})}{R(P\rho\varphi^*)^{\sigma-1}} = \frac{\sigma(f+f_x)}{\sigma f}$$

$$\Rightarrow \left(\frac{\varphi_x^{s'}}{\varphi^*}\right)^{\sigma-1}(1 + \tau^{1-\sigma}) = \left(1 + \frac{f_x}{f}\right)$$

$$\Rightarrow \varphi_x^{s'} = \varphi^*(1 + \tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left(1 + \frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}. \tag{3.33}$$

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Obviously $\varphi_x^{s'}$ is an increasing function of φ^*.

Recall that $\varphi_x^s = \varphi^*(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left[\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} + \frac{f_x}{f}\right]^{\frac{1}{\sigma-1}}$, and notice that $\varphi^* > \varphi_c$ in cases 1 and 3 and $\varphi^* < \varphi_c$ in case 2. Thus, $\varphi_x^{s'} > \varphi_x^s$ ($\varphi_x^{s'} < \varphi_x^s$) if $\varphi^* > \varphi_c$ ($\varphi^* < \varphi_c$) .

7. The ZCP Curve in the Open Economy

Like the case of closed economy, we can link the relation between cutoff productivity φ^* and the average profit for the incumbent firms in the open economy (ZCP). Consider a continuous productivity distribution with probability density function $g(\varphi)$ and cumulative distribution function $G(\varphi)$, and define a weighted average productivity level for firms in the region $\varphi \in [\varphi_a, \varphi_b]$ as:

$$\tilde{\varphi}(\varphi_a) = \left[\frac{1}{G(\varphi_b)-G(\varphi_a)}\int_{\varphi_a}^{\varphi_b} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}. \tag{3.34}$$

Let $\bar{\pi}$ be the overall average of all firms' profits from domestic sales, export sales to one country, and export sales to n countries. We will discuss two cases.

7.1. $\varphi^* > \varphi_c$ (case 1 and case 3)

In this case, the ZCP curve is:

$$\bar{\pi}_{1,3} = \pi_d\left(\tilde{\varphi}\left(\varphi_x^{s'}\right)\right) + \frac{\left[G(\varphi_x^*) - G\left(\varphi_x^{s'}\right)\right]}{1-G\left(\varphi_x^{s'}\right)}\pi_{x1}\left(\tilde{\varphi}_{x1}\left(\varphi_x^{s'}\right)\right) + \frac{1-G(\varphi_x^*)}{1-G\left(\varphi_x^{s'}\right)} n\pi_{xn}\left(\tilde{\varphi}_{xn}(\varphi_x^*)\right) =$$

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$$f\left[\left(\frac{\tilde{\varphi}(\varphi_x^{s'})}{\varphi^*}\right)^{\sigma-1}-1\right]+\frac{[G(\varphi_x^*)-G(\varphi_x^{s'})]}{1-G(\varphi_x^{s'})}f_x\left[\left(\frac{\tilde{\varphi}_{x1}(\varphi_x^{s'})}{\varphi_x^*}\right)^{\sigma-1}-1\right]+$$

$$\frac{1-G(\varphi_x^*)}{1-G(\varphi_x^{s'})}nf_x\left[\left(\frac{\tilde{\varphi}_{xn}(\varphi_x^*)}{\varphi_x^*}\right)^{\sigma-1}-1\right]. \tag{3.35}$$

Recall that $\varphi_x^{s'}$ and φ_x^* are functions of φ^*. The meanings of each term are:

(i) The coefficient of the term $\pi_d(\tilde{\varphi}(\varphi_x^{s'}))$ equals to one, meaning that the probability of a surviving firms selling in the domestic market equals to one.

(ii) $\pi_d(\tilde{\varphi}(\varphi_x^{s'}))$ represents the average profit of all surviving firms on their domestic sales, where $\tilde{\varphi}(\varphi_x^{s'})$ is the average productivity of all surviving firms,

$$\tilde{\varphi}(\varphi_x^{s'})=\left[\frac{1}{1-G(\varphi_x^{s'})}\int_{\varphi_x^{s'}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}. \tag{3.36}$$

(iii) $\frac{[G(\varphi_x^*)-G(\varphi_x^{s'})]}{1-G(\varphi_x^{s'})}$, (3.37)

is the probability of a surviving firm that exports to “one” country in case 1 and case 3.

(iv) $\pi_{x1}(\tilde{\varphi}_{x1}(\varphi_x^{s'}))$ is the average profit on export sales of the surviving firms that export to “one” country. $\tilde{\varphi}_{x1}(\varphi_x^{s'})$ represents the average productivity of the surviving firms that export to one country, that is,

$$\tilde{\varphi}_{x1}(\varphi_x^{s'})=\left[\frac{1}{G(\varphi_x^*)-G(\varphi_x^{s'})}\int_{\varphi_x^{s'}}^{\varphi_x^*}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}. \tag{3.38}$$

(v) $\frac{1-G(\varphi_x^*)}{1-G(\varphi_x^{s'})}$, (3.39)

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is the probability of a surviving firm exporting to "n" countries.

(vi) $\pi_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*))$ is the average profit on export sales for firm that export to n countries. $\tilde{\varphi}_{xn}(\varphi_x^*)$ is the average productivity of surviving firms that export to "n" countries, that is,

$$\tilde{\varphi}_{xn}(\varphi_x^*) = \left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}. \quad (3.40)$$

7.2. $\boldsymbol{\varphi^* \leq \varphi_c}$ (case 2)

In this case, the ZCP curve becomes:

$$\bar{\pi}_2 = \pi_d\big(\tilde{\varphi}(\varphi_c)\big) + [G(\varphi_x^*) - G(\varphi_x^S)]\pi_{x1}\big(\tilde{\varphi}_{x1}(\varphi_x^S)\big) + [1 - G(\varphi_x^*)]n\pi_{xn}\big(\tilde{\varphi}_{xn}(\varphi_x^*)\big) = f\left[\left(\frac{\tilde{\varphi}(\varphi_c)}{\varphi^*}\right)^{\sigma-1} - 1\right] + [G(\varphi_x^*) - G(\varphi_x^S)]f_x\left[\left(\frac{\tilde{\varphi}_{x1}(\varphi_x^S)}{\varphi_x^*}\right)^{\sigma-1} - 1\right] + [1 - G(\varphi_x^*)]nf_x\left[\left(\frac{\tilde{\varphi}_{xn}(\varphi_x^*)}{\varphi_x^*}\right)^{\sigma-1} - 1\right]. \quad (3.41)$$

The meanings of each term are as follows.

(i) The coefficient of the term $\pi_d(\tilde{\varphi}(\varphi_c))$ equals to one, meaning that all surviving firms will sell in the domestic market.

(ii) $\pi_d(\tilde{\varphi}(\varphi_c))$ is the average profit of all surviving firms on their domestic sales, where $\tilde{\varphi}(\varphi_c)$ represents the average productivity of all surviving firms, namely,

$$\tilde{\varphi}(\varphi_c) = \left\{G(\varphi_x^S)\varphi_c^{\sigma-1} + [1 - G(\varphi_x^S)]\frac{1}{1 - G(\varphi_x^S)}\left[\int_{\varphi_x^S}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]\right\}^{\frac{1}{\sigma-1}}$$

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$$= \{G(\varphi_x^s)\varphi_c^{\sigma-1} + [1 - G(\varphi_x^s)]\tilde{\varphi}^{\sigma-1}(\varphi_x^s)\}^{\frac{1}{\sigma-1}}. \quad (3.42)$$

(iii) $[G(\varphi_x^*) - G(\varphi_x^s)]$, (3.43)

is the probability of a surviving firm that exports to "one" country.

(iv) $\pi_{x1}(\tilde{\varphi}_{x1}(\varphi_x^s))$ is the average profit of firms that export to one country; $\tilde{\varphi}_{x1}(\varphi_x^s)$ is the average productivity for firms exporting to one country, or

$$\tilde{\varphi}_{x1}(\varphi_x^s) = \left[\frac{1}{G(\varphi_x^*) - G(\varphi_x^s)} \int_{\varphi_x^s}^{\varphi^*} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}. \quad (3.44)$$

(v) $[1 - G(\varphi_x^*)]$, (3.45)

is the probability of surviving firms that export to n countries.

(vi) $\pi_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*))$ is the average profit from export sales for surviving firms that export to n countries, where $\tilde{\varphi}_{xn}(\varphi_x^*)$ represents their average productivity, or

$$\tilde{\varphi}_{xn}(\varphi_x^*) = \left[\frac{1}{1 - G(\varphi_x^*)} \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}.$$

7.3. Brief summary of the ZCP curve

I. Closed economy:

$$\bar{\pi} = \pi_d(\varphi_c) = f\left[\left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} - 1\right].$$

II. Open economy:

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$$\bar{\pi} = \begin{cases} \bar{\pi}_{1,3} & if\ \varphi^* > \varphi_c \\ \bar{\pi}_2 & if\ \varphi^* \leq \varphi_c \end{cases},$$

where

$$\bar{\pi}_{1,3} = \pi_d(\tilde{\varphi}(\varphi_x^{s'})) + \frac{[G(\varphi_x^*)-G(\varphi_x^{s'})]}{1-G(\varphi_x^{s'})}\pi_{x1}(\tilde{\varphi}_{x1}(\varphi_x^{s'})) + \frac{1-G(\varphi_x^*)}{1-G(\varphi_x^{s'})}n\pi_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*));$$

$$\bar{\pi}_2 = \pi_d(\tilde{\varphi}(\varphi_c)) + [G(\varphi_x^*) - G(\varphi_x^s)]\pi_{x1}(\tilde{\varphi}_{x1}(\varphi_x^s)) + [1 - G(\varphi_x^*)]n\pi_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*)).$$

It is obvious that the ZCP curve of the open economy shifts upward compared with that of the close economy. Moreover, the ZCP curve in the open economy jumps and is discontinuous at $\varphi^* = \varphi_c$ as shown in Figure 3-2 .

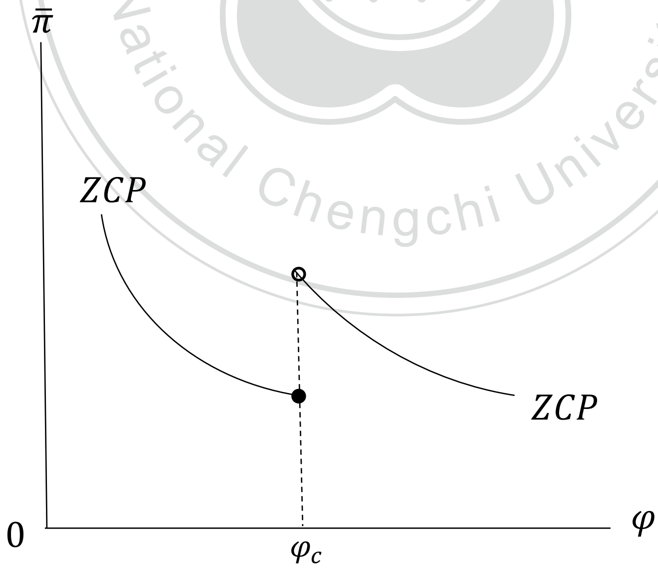

Figure 3-2 The ZCP curve in the open economy

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8. The FE Curve in the Open Economy

The FE condition shows the lowest average profit that a firm required to cover its entry cost under different cutoff levels. Like the close economy case, the FE condition can be expressed follows:

$$\bar{\pi} = \frac{\delta f_e}{P_{in}}, \text{ where } P_{in} = \begin{pmatrix} 1 \\ 1 - G(\varphi_x^{s'}) \end{pmatrix} \text{ if } \begin{pmatrix} \varphi^* \leq \varphi_c \\ \varphi^* > \varphi_c \end{pmatrix}. \text{ (see equation (3.33) for } \varphi_x^{s'}) \quad (3.46)$$

What differs from the closed economy is the probability for the firm to survive under the condition $\varphi^* > \varphi_c$ is $1 - G\left(\varphi_x^{s'}\right) > 0$. This is because a firm can improve its productivity level through exporting. It has a chance to improve its productivity and survive even when $\varphi^* > \varphi_c$.

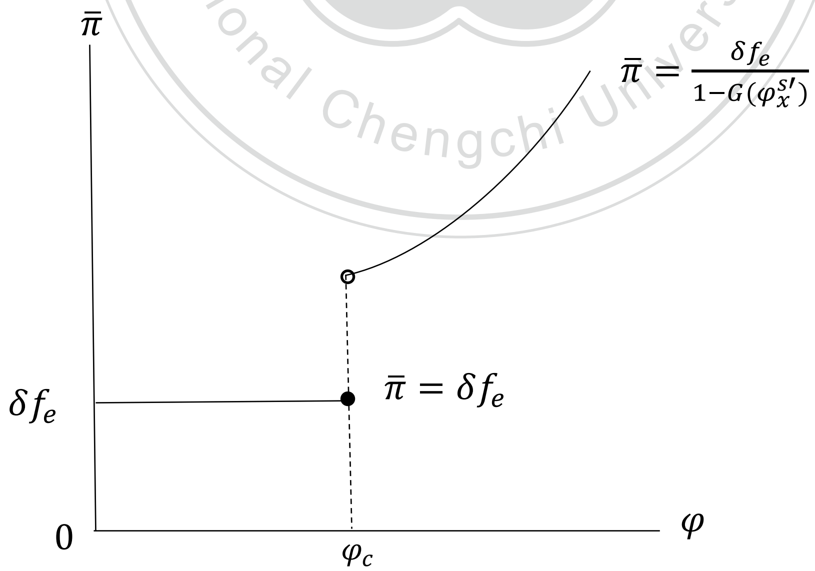

Figure 3-3 The FE curve under the open economy.

DOI:10.6814/NCCU202100569

Figure 3-3 shows that the value of the FE curve jumps at $\varphi^* = \varphi_c$, since the probability for a firm to enter and successfully survive in the market is "one" for all $\varphi^* \le \varphi_c$. Recall that all firms share the same productivity level φ_c when they enter the domestic market. However, if the value of the equilibrium cutoff productivity φ^* is greater than φ_c, a new entrant can survive only if it exports and improves its productivity to the degree that is greater than (or equal to) $\varphi_x^{s'}$. This is the situation of case 1 or case 3 discussed above. In either case, the least productivity for a firm to stay in business is $\varphi_x^{s'}$. As a result, the probability for a new entrant to successfully enter and stay in the market is the probability it draws the productivity level greater than (or equal to) $\varphi_x^{s'}$, or $P_{in} = 1 - G(\varphi_x^{s'})$. Although the FE and the ZCP curves in the open economy are discontinuous at the point $\varphi^* = \varphi_c$, the unique equilibrium exists if $\bar{\pi}_2$ has negative slope and $\bar{\pi}_{1,3}$ has nonpositive slope.[10]

9. Open Economy Equilibrium

Case 1: $\boldsymbol{\varphi_x^s > \varphi^* > \varphi_c}$

In this case, we have:

$\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right] < 0$. However, $\left[\frac{r_d(\varphi_c)}{\sigma} - f\right] = \pi_d(\varphi_c) <$

0

implies $\varphi^* > \varphi_c$. These mean that the ZCP curve intersects with the FE curve in the region where FE is positively sloped in the equilibrium, as shown in Figure 3-4. Recall that $\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] > 0$ and $\left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] < 0$. We thus have the cutoff in case 1 as follows:

[10] See Appendix 3 (A3).

DOI:10.6814/NCCU202100569

$\varphi_a^* < \varphi_c < \varphi^* < \varphi_x^s < \varphi_x^{s'} < \varphi_x^*.$

Case 3: $\boldsymbol{\varphi^* > \varphi_x^s > \varphi_c}$

Like case 1, in case 3 we also have:

$\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right] < 0$ and $\left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] < 0$, The only difference is $\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] < 0$ in case 3. Since $\left[\frac{r_d(\varphi_c)}{\sigma} - f\right] = \pi_d(\varphi_c) < 0$ in case 3 implies $\varphi^* > \varphi_c$ in the equilibrium, the ZCP curve cuts the FE curve in the area where the FE curve is positively sloped, as depicted in Figure 3-5. We summarize the cutoff in case 3 as follows:

$\varphi_a^* < \varphi_c < \varphi_x^s < \varphi^* < \varphi_x^{s'} < \varphi_x^*.$

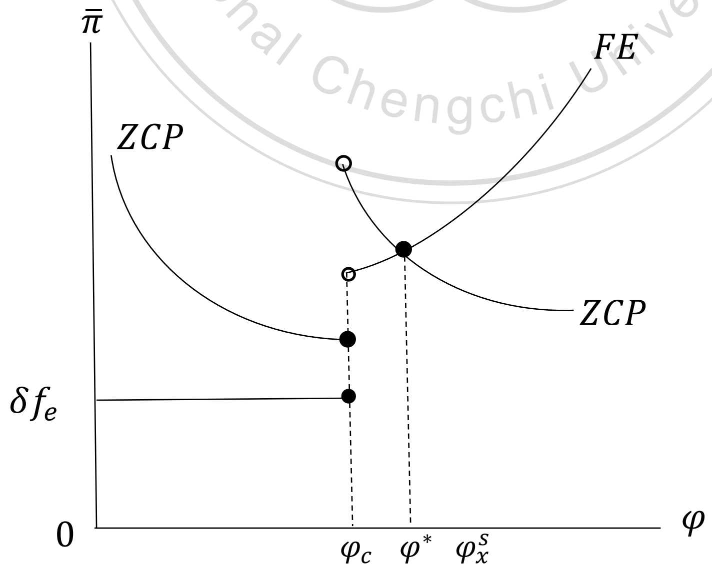

Figure 3-4 Equilibrium cutoff in case 1.

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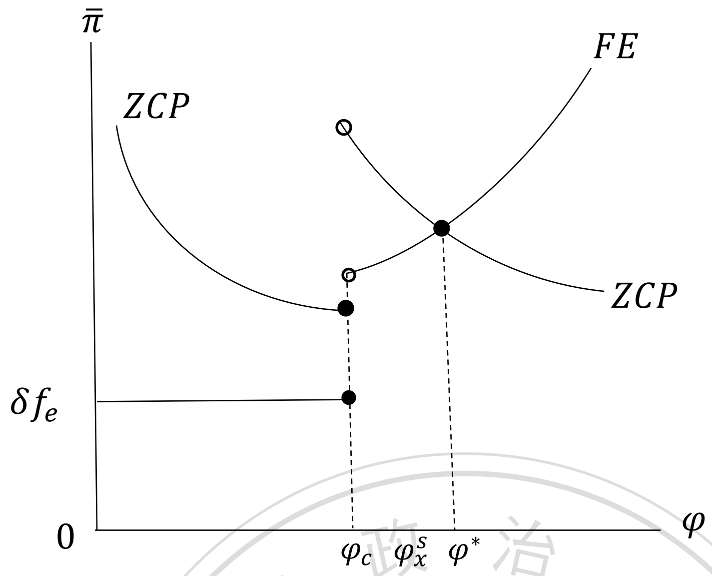

Figure 3-5 Equilibrium cutoff in case 3.

Figure 3-4 and Figure 3-5 show the open economy equilibrium in case 1 and case 3, respectively. Since we have $\varphi^* > \varphi_c$ in both cases, it must be true that ZCP curve intersects with the FE curve in the region where the FE curve is positively sloped. All non-export firms exit the market in both cases. Although $\varphi^* < \varphi_x^{s'}$ means that the least productivity for a firm to make nonnegative profit on its domestic sales is smaller than the cutoff productivity that makes nonnegative combined profit, the firm can incur a productivity growth only if it exports. Consequently, a firm having drawn a productivity level smaller than $\varphi_x^{s'}$ will not be able to make positive combined profit. As such only the exporting firms with $\varphi \geq \varphi_x^{s'}$ can survive in case 1 and case 3.

Case 2: $\boldsymbol{\varphi_x^s > \varphi_c > \varphi^*}$

In this case, we have:

$\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right] > 0$. Moreover, $\left[\frac{r_d(\varphi_c)}{\sigma} - f\right] > 0$ implies

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$\varphi^* < \varphi_c$. These mean that a firm can make positive profit even it does not serve the foreign market and remain at its original productivity level. The ZCP curve intersects with the FE curve in the region where the FE curve is horizontal, as shown in Figure 3-6. Since $\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] > 0$ and $\left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] < 0$, we summarize the cutoff in case 2 as follows:

$\varphi_a^* < \varphi^* < \varphi_c < \varphi_x^s < \varphi_x^*$.

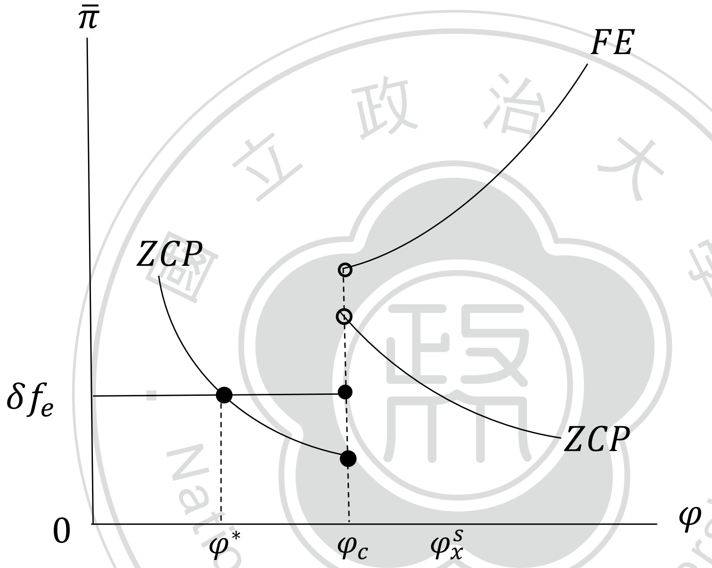

Figure 3-6 Equilibrium cutoff in case 2.

Figure 3-6 shows the open economy equilibrium in case 2. $\varphi_c > \varphi^*$ implies that a firm can survive even it chooses not to export and remains at the original productivity level φ_c. Notice that no firms exist in the region $\varphi \in (\varphi_c, \varphi_x^s)$ since it is better for a firm not to export if it has drawn a productivity level in this region. As a result, there is a productivity gap ($\varphi \in (\varphi_c, \varphi_x^s)$) between exporters and non-exporters. The non-exporting firms all share the same productivity level φ_c, while the least productive exporting firm has a productivity level φ_x^s.

DOI:10.6814/NCCU202100569

Obviously, the point where of the ZCP and the FE curves intersect has a crucial implication on the firms' survival in the equilibrium. Case 1 and case 3 represent an upward shift of the ZCP curve compared with case 2. Case 1 and case 3 may be regarded as more liberalization of trade (such as lower trade cost) compared to case 2 since a lower trade barrier is more likely to shift the ZCP curve upward. In such circumstances, resources are reallocated toward the high productivity firms (exporters) and thus lead to the exit of the low productive firms (non-exporters). On the contrary, if the magnitude of the resources reallocation is weaker, some of the non-exporters can stay in business, which is our case 2.

Aggregation

Like the closed economy, the open economy variables can be expressed as a weighted average productivity and a total mass of varieties available in the economy[11]. The aggregate variables are summarized below.

Price index: $P = {M_t}^{\frac{1}{1-\sigma}} p(\tilde{\varphi}_t) = {M_t}^{\frac{1}{1-\sigma}} \frac{1}{\rho\tilde{\varphi}_t}$. (3.47)

Total expenditure: $R = M_t r_d(\tilde{\varphi}_t)$. (3.48)

Welfare per worker: $W = P^{-1} = {M_t}^{\frac{1}{\sigma-1}} \rho\tilde{\varphi}_t$. (3.49)

The meanings of the notations are:

[11] See appendix 4.

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(i) The weighted average productivity for all surviving firms in the open economy is:

$$\tilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\tilde{\varphi}^{\sigma-1} + M_{x1}(\tau^{-1}\tilde{\varphi}_{x1})^{\sigma-1} + nM_{xn}(\tau^{-1}\tilde{\varphi}_{xn})^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}, \tag{3.50}$$

where:

$$\tilde{\varphi} = \begin{cases} \tilde{\varphi}(\varphi_x^{s'}) \\ \{G(\varphi_x^s)\varphi_c^{\sigma-1} + [1-G(\varphi_x^s)]\tilde{\varphi}_x^{\sigma-1}(\varphi_x^s)\}^{\frac{1}{\sigma-1}} \end{cases} \text{for } \begin{cases} case1, case3 \\ case2 \end{cases},$$

$$\tilde{\varphi}_{x1} = \begin{cases} \tilde{\varphi}_{x1}(\varphi_x^{s'}) \\ \tilde{\varphi}_{x1}(\varphi_x^s) \end{cases} \text{for } \begin{cases} case1, case3 \\ case2 \end{cases},$$

$\tilde{\varphi}_{xn} = \tilde{\varphi}_{xn}(\varphi_x^*)$ for all three cases,[12]

(ii) M denotes the equilibrium mass of incumbent firms,

(iii) $p_{x1} = \begin{cases} \frac{G(\varphi_x^*)-G(\varphi_x^{s'})}{1-G(\varphi_x^{s'})} \\ G(\varphi_x^*) - G(\varphi_x^s) \end{cases}$ for $\begin{cases} case1, case3 \\ case2 \end{cases}$ is the probability of the surviving firms that export to one country,

(iv) $M_{x1} = p_{x1}M$ is the number of firms exporting to one country,

(v) $p_{xn} = \begin{cases} \frac{1-G(\varphi_x^*)}{1-G(\varphi_x^{s'})} \\ 1 - G(\varphi_x^*) \end{cases}$ for $\begin{cases} case1, case3 \\ case2 \end{cases}$ denotes the probability of the surviving

[12] A brief review of the average productivity:

$\tilde{\varphi}$: Average productivity for all incumbent firms.

$\tilde{\varphi}_{x1}$: Average productivity for firms that export to one country.

$\tilde{\varphi}_{xn}$: Average productivity for firms that export to n countries.

DOI:10.6814/NCCU202100569

firms that export to n countries,

(vi) $M_{xn} = p_{xn}M$ is the number of firms exporting to n countries,

(vii) $M_t = M + M_{x1} + nM_{xn}$ is the mass of varieties available to consumers in any country.

10. The Effect of Trade

Like Melitz (2003), heterogeneity of the firms brings about a resource reallocation among firms. Some firms with productivity not high enough will exit from the market, leading to a decrease in the number of the incumbent firms. Resources are reallocated to the more productive firms. All the exporting firms enjoy an increase in the market share, and those with the higher productivity even increase in their profits.

10.1. Decrease in the equilibrium mass of incumbent firms

Denote $p_{x1}^k(p_{xn}^k)$ as the probability for a surviving firm that export to $1(n)$ country (countries) in case k, $k = 1,2,3$.

10.1.1. Case 1 and case 3

$$\bar{\pi} = \left[\frac{r_d(\tilde{\varphi}(\varphi_x^{S'}))}{\sigma} - f\right] + p_{x1}^{1(3)}\left[\frac{r_{x1}(\tilde{\varphi}_{x1}(\varphi_x^{S'}))}{\sigma} - f_x\right] + p_{xn}^{1(3)} n\left[\frac{r_{xn}(\tilde{\varphi}_{xn}(\varphi_x^{*}))}{\sigma} - f_x\right].$$

$$\bar{r} = r_d(\tilde{\varphi}) + p_{x1}^{1(3)} r_{x1}(\tilde{\varphi}_{x1}) + np_{xn}^{1(3)} r_{xn}(\tilde{\varphi}_{xn}) = \sigma(\bar{\pi} + f + p_{x1}^{1(3)} f_x + p_{xn}^{1(3)} n f_x).$$

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$$M_{1(3)} = \frac{R}{\bar{r}} = \frac{R}{\sigma(\bar{\pi}+f+p_{x1}^{1(3)}f_x+p_{xn}^{1(3)}nf_x)} < M_a = \frac{R}{\sigma(\bar{\pi}_a+f)}. \tag{3.51}$$

10.1.2. Case 2

$$\bar{\pi} = \left[\frac{r_d(\tilde{\varphi}(\varphi_c))}{\sigma} - f\right] + p_{x1}^2\left[\frac{r_{x1}(\tilde{\varphi}_{x1}(\varphi_x^S))}{\sigma} - f_x\right] + p_{xn}^2 n\left[\frac{r_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*))}{\sigma} - f_x\right].$$

$$\bar{r} = r_d(\tilde{\varphi}) + p_{x1}^2 r_{x1}(\tilde{\varphi}_{x1}) + np_{xn}^2 r_{xn}(\tilde{\varphi}_{xn}) = \sigma(\bar{\pi} + f + p_{x1}^2 f_x + p_{xn}^2 nf_x).$$

$$M_2 = \frac{R}{\bar{r}} = \frac{R}{\sigma(\bar{\pi}+f+p_{x1}^2 f_x+p_{xn}^2 nf_x)} < M_a = \frac{R}{\sigma(\bar{\pi}_a+f)}. \tag{3.52}$$

It is very likely that we have $M_2 > M_{1(3)}$. For instance, if $p_{x1}^{1(3)} > p_{x1}^2$, then $M_2 > M_{1(3)}$ is guaranteed. The result is intuitive, since case 1 and case 3 represent a more liberalization of trade compared to case 2, which leads to a stronger reallocation of resources toward high productive firms and crowds out more inefficient firms.

10.2. The reallocation of market share ($\frac{r(\varphi)}{R}$)

We will now see how reallocates the market share. Let $r_a(\varphi)$ denote a firm's revenue in the autarky equilibrium and $r(\varphi)$ the revenue of the firm in the open economy equilibrium.

10.2.1. Case 1 and case 3.

(i) Autarky

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$r_a(\varphi_c) = \sigma(f + \delta f_e)$

(ii) Open economy

Non-exporting firms: exit the market.

Firm exporting to one country: $r(\varphi) = r_d(\varphi_x) + r_x(\varphi_x) = r_d(\varphi_x)(1 + \tau^{1-\sigma})$, $\varphi_x \in [\varphi_x^{s'}, \varphi_x^*)$.

Firm exporting to n countries: $r(\varphi) = r_d(\varphi_x) + nr_x(\varphi_x) = r_d(\varphi_x)(1 + n\tau^{1-\sigma})$, $\varphi_x \in [\varphi_x^*, \infty]$.

It is obvious that $r_d(\varphi_x)(1 + n\tau^{1-\sigma})$ decreases as τ increases. Notice that the autarky equilibrium is an equilibrium as $\tau \to \infty$, that is, $r_a(\varphi) = lim_{\tau\to\infty}[\,r_d(\varphi)(1 + n\tau^{1-\sigma})]$. Therefore, $r_d(\varphi)(1 + n\tau^{1-\sigma}) > r_a(\varphi)$ for any finite τ and n. Since an exporting firm incurs a productivity growth, we have $\varphi_x > \varphi_c$ for all exporting firms, and thus:

$$r_a(\varphi_c) < r_d(\varphi_x)(1 + \tau^{1-\sigma}) < r_d(\varphi_x)(1 + n\tau^{1-\sigma}). \tag{3.53}$$

The market share of an exporting firm increases under the open economy equilibrium, whereas all the non-exporting firms exit in case 1 and case 3. At the same time, all surviving firms enjoy a greater market share compared with the autarky equilibrium.

10.2.2. Case 2.

DOI:10.6814/NCCU202100569

(i) Autarky

$$r_a(\varphi_c) = \sigma(f + \delta f_e) = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} \sigma f.$$

(ii) Open economy

Non-exporting firms: $r_d(\varphi_c) = \left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} \sigma f$.

Firms exporting to one country: $r(\varphi) = r_d(\varphi_x) + r_x(\varphi_x) = r_d(\varphi_x)(1 + \tau^{1-\sigma})$, $\varphi_x \in [\varphi_x^s, \varphi_x^*)$.

Firms exporting to n countries: $r(\varphi) = r_d(\varphi_x) + nr_x(\varphi_x) = r_d(\varphi_x)(1 + n\tau^{1-\sigma})$, $\varphi_x \in [\varphi_x^*, \infty]$.

Since $\varphi^* > \varphi_a^*$, we have:

$$r_d(\varphi_c) < r_a(\varphi_c) < r_d(\varphi_x)(1 + \tau^{1-\sigma}) < r_d(\varphi_x)(1 + n\tau^{1-\sigma}). \tag{3.54}$$

For a non-exporting firm, the market share shrinks compared with the case of autarky. On the contrary, the market share for all export firms increases. The increase is more for firms with higher productivity levels and exporting to n countries.

10.3. The change in firms' profits

Firms' profits under case 1 and case 3:

DOI:10.6814/NCCU202100569

(i) Autarky

$$\pi_a(\varphi_c) = \frac{r_a(\varphi_c)}{\sigma} - f = f\left[\left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} - 1\right].$$

(ii) Open economy

Non-exporting firms: exit the market.

Firms exporting to one country:

$$\pi_d^{1(3)}(\varphi_x) + \pi_{x1}^{1(3)}(\varphi_x) = \left[\frac{r_d(\varphi_x)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x)}{\sigma} - f_x\right]$$
$$= f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1} - 1\right] + \left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}\tau^{1-\sigma}f - f_x\right]$$
$$= f\varphi_x^{\sigma-1}\left[\frac{1+\tau^{1-\sigma}}{(\varphi^*)^{\sigma-1}} - \frac{1}{(\varphi_x)^{\sigma-1}}\right] - f_x,\ \ \varphi_x \in [\varphi_x^{s'}, \varphi_x^*).$$

Firms exporting to n countries:

$$\pi_d^{1(3)}(\varphi_x) + n\pi_{xn}^{1(3)}(\varphi_x) = \left[\frac{r_d(\varphi_x)}{\sigma} - f\right] + n\left[\frac{r_x(\varphi_x)}{\sigma} - f_x\right] = f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1} - 1\right] + n\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}\tau^{1-\sigma}f - f_x\right] = f\varphi_x^{\sigma-1}\left[\frac{1+n\tau^{1-\sigma}}{(\varphi^*)^{\sigma-1}} - \frac{1}{(\varphi_x)^{\sigma-1}}\right] - nf_x,\ \ \varphi_x \in [\varphi_x^*, \infty].$$

10.3.1. Change in profits for firms in case 1 and case 3

(i) Change in profit for Non-exporting firms:

DOI:10.6814/NCCU202100569

$$\varDelta\pi = 0 - \pi_a(\varphi_c) = -f\left[\left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} - 1\right] < 0. \tag{3.55}$$

(ii) Change in profits for firms exporting to one country:

$$\begin{aligned}\varDelta\pi &= \pi_d^{1(3)}(\varphi_x) + \pi_{x1}^{1(3)}(\varphi_x) - \pi_a(\varphi_c) \\ &= f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}(1+\tau^{1-\sigma}) - \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1}\right] - f_x.\end{aligned} \tag{3.56}$$

(iii) Change in profits for firms exporting to n countries.

$$\begin{aligned}\varDelta\pi &= \pi_d^{1(3)}(\varphi_x) + n\pi_{xn}^{1(3)}(\varphi_x) - \pi_a(\varphi_c) \\ &= f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}(1+n\tau^{1-\sigma}) - \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1}\right] - nf_x.\end{aligned} \tag{3.57}$$

Notice that, in each case, $\varDelta\pi$ increases in φ_x, but decreases in τ and f_x.

Firms' profits under Case 2:

(i) Autarky

$$\pi_a(\varphi_c) = \frac{r(\varphi_c)}{\sigma} - f = f\left[\left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} - 1\right].$$

(ii) Open economy

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Non-exporting firms: $\pi_d(\varphi_c) = \frac{r_d(\varphi_c)}{\sigma} - f = f\left[\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} - 1\right] > 0$. (since $\varphi_c > \varphi^*$ in case 2)

Firms exporting to one country:

$$\pi_d^{(2)}(\varphi_x) + \pi_{x1}^{(2)}(\varphi_x) = \left[\frac{r_d(\varphi_x)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x)}{\sigma} - f_x\right] = f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1} - 1\right] + \left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}\tau^{1-\sigma}f - f_x\right] = f\varphi_x^{\sigma-1}\left[\frac{1+\tau^{1-\sigma}}{(\varphi^*)^{\sigma-1}} - \frac{1}{(\varphi_x)^{\sigma-1}}\right] - f_x,\ \varphi_x \in [\varphi_x^s, \varphi_x^*).$$

Firms exporting to n countries:

$$\pi_d^{(2)}(\varphi_x) + n\pi_{xn}^{(2)}(\varphi_x) = \left[\frac{r_d(\varphi_x)}{\sigma} - f\right] + n\left[\frac{r_d(\varphi_x)}{\sigma} - f_x\right] = f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1} - 1\right] + n\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}\tau^{1-\sigma}f - f_x\right] = f\varphi_x^{\sigma-1}\left[\frac{1+n\tau^{1-\sigma}}{(\varphi^*)^{\sigma-1}} - \frac{1}{(\varphi_x)^{\sigma-1}}\right] - nf_x,\ \varphi_x \in [\varphi_x^*, \infty].$$

10.3.2. Change in profits for firms in case 2

(i) Change in profits for non-exporting firms:

$$\Delta\pi = \pi_d^{(2)}(\varphi_c) - \pi_a^{(2)}(\varphi_c) = f\left[\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} - \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1}\right] < 0. \tag{3.58}$$

Since $\varphi^* > \varphi_a^*$, (3.58) holds.

(ii) Change in profits for firm exporting to one country.

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$$\Delta\pi = \pi_d^{(2)}(\varphi_x) + \pi_{x1}^{(2)}(\varphi_x) - \pi_a(\varphi_c)$$
$$= f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}(1+\tau^{1-\sigma}) - \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1}\right] - f_x. \tag{3.59}$$

(iii) Change in profits for firms exporting to n countries.

$$\Delta\pi = \pi_d^{(2)}(\varphi_x) + n\pi_{xn}^{(2)}(\varphi_x) - \pi_a(\varphi_c)$$
$$= f\left[\left(\frac{\varphi_x}{\varphi^*}\right)^{\sigma-1}(1+n\tau^{1-\sigma}) - \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1}\right] - nf_x. \tag{3.60}$$

Like case 1 and case 3, $\Delta\pi$ is increasing in φ_x and decreasing in τ and f_x. In all cases, a firm suffer a revenue and profit loss if it doesn't export, while there is an increase in revenue for exporting firms, even the firms exporting to only one country. Furthermore, a firm can increase its profit if its productivity has grown high enough. These results are the same as those obtained in Melitz (2003).

10.4. Change in the welfare per worker

Although trade decreases the number of the incumbent firms, the foreign exporters come and the varieties that domestic consumers consume may increase. At the same time, the surviving firms have a higher average productivity compared with the autarky case. In fact, as Melitz (2003), it can be shown that the welfare per worker $W = P^{-1} = M^{\frac{1}{1-\sigma}}p(\tilde{\varphi})^{-1}$ increases in all the three open economy cases.

(i) Autarky

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$$W_a = M_a^{\frac{1}{\sigma-1}} \rho \varphi_c = \left(\frac{R}{\sigma f}\right)^{\frac{1}{\sigma-1}} \rho \varphi_a^*. \quad (\text{using } \frac{r_a(\varphi_c)}{r_a(\varphi_a^*)} = \left(\frac{\varphi_c}{\varphi_a^*}\right)^{\sigma-1} = \frac{\frac{R}{M_a}}{\sigma f}). \tag{3.61}$$

(ii) Open economy

$$W_t = M_t^{\frac{1}{\sigma-1}} \rho \tilde{\varphi}_t = \left(\frac{R}{\sigma f}\right)^{\frac{1}{\sigma-1}} \rho \varphi^*. \quad (\text{using } \frac{r_d(\tilde{\varphi}_t)}{r_d(\varphi^*)} = \left(\frac{\tilde{\varphi}_t}{\varphi^*}\right)^{\sigma-1} = \frac{\frac{R}{M_t}}{\sigma f}). \tag{3.62}$$

Since $\varphi^* > \varphi_a^*$, we have $W_t > W_a$. The welfare is greater under the open economy equilibrium then that of autarky.

11. Conclusions

Following Melitz (2003), we have constructed a model that describes firms' export decision making if there exists a learning effect on export. The main features of our model are that all firms are identical when they enter the domestic market, however they can improve their productivity through export to the foreign market. The key differences between with and without learning effect is the change in firms' profit from domestic sales, which in turn influence the firms' export decision. The important findings of this paper are: (1) Compared with the case of no learning effect, a firm is more willing to export if it can improve its productivity through export. It is possible for a firm to export even the export profit is negative, since it can cover the loss by the increase in profit from domestic sales. (2) It is possible for all non-exporters to exit the market when the ZCP curve shift rightward a lot, as in our case 1 and case 3. (3) When the ZCP curve doesn't shift rightward a lot, which is our case 2, a pool of the non-exporting firms with the original productivity level survive even if they don't export. In this case, the productivity distribution of the surviving firms is divided into two

DOI:10.6814/NCCU202100569

groups, exporters and non-exporters. For the non-exporters, all firms share the same productivity; for the exporters, the productivity distribution is the distribution for the productivity drawn conditional on successfully exporting. (4) The growth in the productivity brings about the productivity gap between the exporters and the non-exporters in our case 2. Although the non-exporters survive, their productivity is visibly lower than the least productive exporters.

Although the distribution of the surviving firms changes, the impact of trade on the welfare and resources allocation is basically same as that of Melitz (2003). Firms with high productivity (exporting firms) increase their market share, and those with higher productivity increase in profits also. The firms selling domestically decrease in both profits and the market share. They might all exit the market if the liberalization of trade is large enough, as case 1 and case 3 in our model. Gains from trade come from resources reallocation toward more productive firms, from increases in the varieties for consumers in any country, and from the productivity growth due to the learning effect of export.

While we have obtained some interesting results that are not captured by the self-selection model, there are limitations in our model due to the simplified assumptions. Modifying these assumptions may be a direction for future research. For instance, the mechanism of productivity growth after export and the symmetric assumption for all countries prevent us from discussing some interesting issues, such as the choice of export destination. Moreover, the assumption that export to more than one country does not make the productivity growth further plays an important role in our model. It simplifies the calculation but directly affects the number of export countries chosen by firms in the equilibrium. It also deters us from discussing the relationship between the

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number of export countries and the learning effect, or, between the quantity of export and the learning effect. Despite of the shortcomings, we believe that our model makes an important contribution in supplementing selection framework of Melitz (2003) in understating a salient empirical regularity in the trade literature.

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Appendix 1. Derivation of equation (3.27)

From the definition of φ_x^s we have:

$$\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right] \Rightarrow r_d(\varphi_x^s) + r_x(\varphi_x^s) = r_d(\varphi_c) + \sigma f_x. \tag{A1.1}$$

Since $r_d(\varphi^*) = \sigma f$, $\frac{r_d(\varphi_c)}{r_d(\varphi^*)} = \left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} \quad \Rightarrow \quad r_d(\varphi_c) = \sigma f \left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1}$. Substituting $r_d(\varphi_c)$ into (A1.1), we get:

$$r_d(\varphi_x^s) + r_x(\varphi_x^s) = \sigma f \left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} + \sigma f_x = \sigma\left[f\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} + f_x\right]. \tag{A1.2}$$

Dividing (A1.2) by $r_d(\varphi^*)$ gives us:

$$\frac{r_d(\varphi_x^s)+r_x(\varphi_x^s)}{r_d(\varphi^*)} = \frac{R(P\rho)^{\sigma-1}\left[(\varphi_x^s)^{\sigma-1}+\tau^{1-\sigma}(\varphi_x^s)^{\sigma-1}\right]}{R(P\rho\varphi^*)^{\sigma-1}} = \frac{\sigma\left[f\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1}+f_x\right]}{\sigma f}. \tag{A1.3}$$

From the second equal sign of (A1.3), we arrive at:

$$\varphi_x^s = \varphi^*(1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left[\left(\frac{\varphi_c}{\varphi^*}\right)^{\sigma-1} + \frac{f_x}{f}\right]^{\frac{1}{\sigma-1}}.$$

Appendix 2. Derivation of the condition $\varphi_x^s > \varphi_c$

From equation (3.25), $\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] + \left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] = \left[\frac{r_d(\varphi_c)}{\sigma} - f\right]$, one finds that if

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$\varphi_x^s > \varphi_c$ is true, then it must be $\left[\frac{r_d(\varphi_x^s)}{\sigma} - f\right] - \left[\frac{r_d(\varphi_c)}{\sigma} - f\right] = -\left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] > 0$, or $\left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] < 0$. Since $\pi_x(\varphi_x^*) = \left[\frac{r_x(\varphi_x^*)}{\sigma} - f_x\right] = 0$, $\left[\frac{r_x(\varphi_x^s)}{\sigma} - f_x\right] < 0$ implies $\varphi_x^* > \varphi_x^s$.

We now find the condition for $\varphi_x^* > \varphi_x^s$. Use $\frac{r_d(\varphi_c)}{r_x(\varphi_x^*)} = \left(\frac{\varphi_c}{\varphi_x^*}\right)^{\sigma-1} \tau^{\sigma-1}$ to get $r_d(\varphi_c) = \left(\frac{\varphi_c}{\varphi_x^*}\right)^{\sigma-1} \tau^{\sigma-1}\sigma f_x$.

Substituting $r_d(\varphi_c)$ into equation (3.25), we get:

$$\frac{r_d(\varphi_x^s) + r_x(\varphi_x^s)}{r_x(\varphi_x^*)} = \frac{R(P\rho)^{\sigma-1}[(\varphi_x^s)^{\sigma-1} + \tau^{1-\sigma}(\varphi_x^s)^{\sigma-1}]}{R(P\rho\varphi_x^*)^{\sigma-1}\tau^{1-\sigma}} = \frac{\sigma f_x\left[\left(\frac{\varphi_c}{\varphi_x^*}\right)^{\sigma-1}\tau^{\sigma-1} + 1\right]}{\sigma f_x}$$

$\Rightarrow$ $(\varphi_x^s)^{\sigma-1}[\tau^{\sigma-1} + 1] = (\varphi_c)^{\sigma-1}\tau^{\sigma-1} + (\varphi_x^*)^{\sigma-1}$, and thus:

$$\begin{cases} \varphi_c > \varphi_x^* \Rightarrow \varphi_x^s > \varphi_x^* \\ \varphi_c = \varphi_x^* \Rightarrow \varphi_x^s = \varphi_x^* \\ \varphi_c < \varphi_x^* \Rightarrow \varphi_x^s < \varphi_x^* \end{cases} \tag{A2.1}$$

Using $\pi_x(\varphi_x^*) = 0$, we have $\varphi_x^* = \frac{\tau}{P\rho}\left[\frac{\sigma f_x}{R}\right]^{\frac{1}{\sigma-1}}$. P is the equilibrium price index in the open economy equilibrium, which will be endogenously determined by the model. We predict there will be welfare gains after export, which is represented by a lower price index in the open economy equilibrium, like Melitz (2003). If the prediction holds and the productivity level φ_x^* is greater than φ_c under the closed economy price index, then it guarantees that φ_x^* will be greater than φ_c at the open economy equilibrium price index, since φ_x^* is decreasing in P.

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Substitute the closed economy equilibrium price index (3.16) ($P = \frac{1}{\rho\varphi_c}\left[\frac{\sigma}{R}(f + \delta f_e)\right]^{\frac{1}{\sigma-1}}$) into φ_x^* and get $\varphi_x^* = \varphi_c(f + \delta f_e)^{\frac{1}{1-\sigma}}(f_x)^{\frac{1}{\sigma-1}}\tau$. As a result, if $\varphi_x^* > \varphi_c$ under the closed economy price index, we have:

$$(f + \delta f_e)^{\frac{1}{1-\sigma}}(f_x)^{\frac{1}{\sigma-1}}\tau > 1 \Rightarrow f_x > \tau^{1-\sigma}[f + \delta f_e].$$

Of course, the condition fails if we have higher price index in the open economy.

Appendix 3. The existence and uniqueness of equilibrium in the open economy

We demonstrate this by showing that $\bar{\pi}_{1,3} = \frac{\bar{\pi}_2}{1-G(\varphi_x^{s'})}$ at $\varphi^* = \varphi_c$.

From (3.27) and (3.33) we know that $\varphi_x^{s'} = \varphi_x^s$ at $\varphi^* = \varphi_c$. Substitute it into $\bar{\pi}_2$ (equation 3.41) to get:

$$\bar{\pi}_2 = \pi_d(\tilde{\varphi}(\varphi_c)) + [G(\varphi_x^*) - G(\varphi_x^{s'})]\pi_{x1}(\tilde{\varphi}_{x1}(\varphi_x^{s'})) + [1 - G(\varphi_x^*)]n\pi_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*)).$$

Dividing $\bar{\pi}_2$ by $[1 - G(\varphi_x^{s'})]$ gives us:

$$\frac{\bar{\pi}_2}{1-G(\varphi_x^{s'})} = \frac{\pi_d(\tilde{\varphi}(\varphi_c))}{1-G(\varphi_x^{s'})} + \frac{[G(\varphi_x^*)-G(\varphi_x^{s'})]}{1-G(\varphi_x^{s'})}\pi_{x1}(\tilde{\varphi}_{x1}(\varphi_x^{s'}) + \frac{1-G(\varphi_x^*)}{1-G(\varphi_x^{s'})}n\pi_{xn}(\tilde{\varphi}_{xn}(\varphi_x^*)).$$

(A3.1)

The term $\frac{\pi_d(\tilde{\varphi}(\varphi_c))}{1-G(\varphi_x^{s'})}$ can be decomposed as (see (3.42) for $\pi_d(\tilde{\varphi}(\varphi_c))$):

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$$\frac{\pi_d\big(\tilde{\varphi}(\varphi_c)\big)}{1-G\big(\varphi_x^{s'}\big)} = \frac{R(P\rho)^{\sigma-1}}{\sigma}\left[\frac{G\big(\varphi_x^{s'}\big)}{1-G\big(\varphi_x^{s'}\big)}\varphi_c^{\sigma-1} + \tilde{\varphi}^{\sigma-1}\big(\varphi_x^{s'}\big)\right] - \frac{f}{1-G\big(\varphi_x^{s'}\big)}$$

$$= \frac{G\big(\varphi_x^{s'}\big)}{1-G\big(\varphi_x^{s'}\big)}\left[\frac{1}{\sigma}R(P\rho\varphi_c)^{\sigma-1} - f\right] + \left[\frac{1}{\sigma}R\big(P\rho\tilde{\varphi}(\varphi_x^{s'})\big)^{\sigma-1} - f\right]$$

$$= \frac{G\big(\varphi_x^{s'}\big)}{1-G\big(\varphi_x^{s'}\big)}\pi_d(\varphi_c) + \pi_d\big(\tilde{\varphi}(\varphi_x^{s'})\big) = \pi_d\big(\tilde{\varphi}(\varphi_x^{s'})\big), \tag{A3.2}$$

since $\pi_d(\varphi_c) = 0$ at $\varphi^* = \varphi_c$.

As a result,

$$\frac{\bar{\pi}_2}{1-G(\varphi_x^{s'})} = \pi_d\big(\tilde{\varphi}(\varphi_x^{s'})\big) + \frac{\big[G(\varphi_x^*)-G\big(\varphi_x^{s'}\big)\big]}{1-G\big(\varphi_x^{s'}\big)}\pi_{x1}(\tilde{\varphi}_{x1}\big(\varphi_x^{s'}\big) + \frac{1-G(\varphi_x^*)}{1-G\big(\varphi_x^{s'}\big)}n\pi_{xn}\big(\tilde{\varphi}_{xn}(\varphi_x^*)\big) = \bar{\pi}_{1,3} \text{ at } \varphi^* = \varphi_c. \tag{A3.3}$$

It is obvious that if $\bar{\pi}_2 = \delta f_e$ at $\varphi^* = \varphi_c$, then $\bar{\pi}_{1,3} = \frac{\delta f_e}{1-G(\varphi_x^{s'})}$ at $\varphi^* = \varphi_c$, as shown in Figure 3-7. The result implies that if the slope of $\bar{\pi}_2$ is negative and the slope of $\bar{\pi}_{1,3}$ is nonpositive, then $\bar{\pi}_2$ intersects the FE curve in the horizontal part if $\bar{\pi}_2 \leq \delta f_e$ at $\varphi^* = \varphi_c$; on the contrary, if $\bar{\pi}_2 > \delta f_e$ at $\varphi^* = \varphi_c$, $\bar{\pi}_{1,3}$ must cut the FE curve where FE has positive slope.

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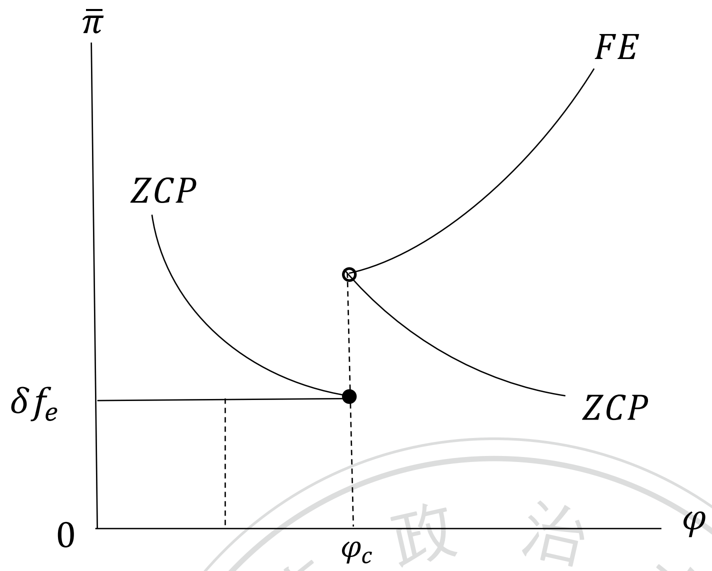

Figure 3-7. ZCP curve with $\bar{\pi}_2 = \delta f_e$ at $\varphi^* = \varphi_c$

Appendix 4. Aggregation in the open economy

All notations we used are explained in the aggregation part in section 9.

1. Aggregate price index:

From (3.2), the aggregate price index with a mass of firms M over a region $\varphi \in (\varphi^*, \infty)$ in a productivity distribution, which has a probability density function $g(\varphi)$ and a cumulative distribution function $G(\varphi)$, is given by:

$$P = \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} M g(\varphi) d\varphi\right]^{\frac{1}{1-\sigma}}. \tag{A4.1}$$

a. Aggregate price index in case 1 and case 3 $(\varphi^* > \varphi_c)$.

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$$P^{1-\sigma} = M\left[\frac{1}{1-G\left(\varphi_x^{s'}\right)}\int_{\varphi_x^{s'}}^{\infty}(\rho\varphi)^{\sigma-1}g(\varphi)d\varphi\right] +$$

$$M_{x1}\left[\frac{1}{G(\varphi_x^*)-G\left(\varphi_x^{s'}\right)}\int_{\varphi_x^{s'}}^{\varphi_x^*}\tau^{1-\sigma}(\rho\varphi)^{\sigma-1}g(\varphi)d\varphi\right] +$$

$$nM_{xn}\left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty}\tau^{1-\sigma}(\rho\varphi)^{\sigma-1}g(\varphi)d\varphi\right] = M(\rho\tilde{\varphi})^{\sigma-1} + M_{x1}(\tau^{-1}\rho\tilde{\varphi}_{x1})^{\sigma-1} +$$

$$nM_{x1}(\tau^{-1}\rho\tilde{\varphi}_x)^{\sigma-1} = M_t p(\tilde{\varphi}_t)^{1-\sigma}. \qquad \text{(A4.2)}$$

Thus, $P = M_t^{\frac{1}{1-\sigma}}p(\tilde{\varphi}_t)$.

b. Aggregate price index in case 2 $(\varphi^* \leq \varphi_c)$.

$$P^{1-\sigma} = G(\varphi_x^s)Mp(\varphi_c)^{1-\sigma} + [1-G(\varphi_x^s)]M\left[\frac{1}{1-G(\varphi_x^s)}\int_{\varphi_x^s}^{\infty}(\rho\varphi)^{\sigma-1}g(\varphi)d\varphi\right] +$$

$$M_{x1}\left[\frac{1}{G(\varphi_x^*)-G(\varphi_x^s)}\int_{\varphi_x^s}^{\varphi_x^*}\tau^{1-\sigma}(\rho\varphi)^{\sigma-1}g(\varphi)d\varphi\right] +$$

$$nM_{xn}\left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty}\tau^{1-\sigma}(\rho\varphi)^{\sigma-1}g(\varphi)d\varphi\right] = G(\varphi_x^s)M(\rho\varphi_c)^{\sigma-1} + [1-$$

$$G(\varphi_x^s)]M(\rho\tilde{\varphi}_x)^{\sigma-1} + M_{x1}(\tau^{-1}\rho\tilde{\varphi}_{x1})^{\sigma-1} + nM_{x1}(\tau^{-1}\rho\tilde{\varphi}_x)^{\sigma-1} = M_t p(\tilde{\varphi}_t)^{1-\sigma}.$$

(A4.3)

2. Aggregate expenditure

From (3.9), the aggregate expenditure with a mass of firms M and a productivity distribution over a subset $\varphi \in (\varphi^*, \infty)$ is given by:

$$R = \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty}r(\varphi)Mg(\varphi)d\varphi\right]. \qquad \text{(A4.4)}$$

DOI:10.6814/NCCU202100569

a. Aggregate expenditure in case 1 and case 3 $(\varphi^* > \varphi_c)$.

$$R = M\left[\frac{1}{1-G(\varphi_x^{s\prime})}\int_{\varphi_x^{s\prime}}^{\infty} r_d(\varphi)g(\varphi)d\varphi\right] + M_{x1}\left[\frac{1}{G(\varphi_x^*)-G(\varphi_x^{s\prime})}\int_{\varphi_x^{s\prime}}^{\varphi_x^*}\tau^{1-\sigma} r_d(\varphi)g(\varphi)d\varphi\right] + nM_{xn}\left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty}\tau^{1-\sigma} r_d(\varphi)g(\varphi)d\varphi\right] = M\left[\frac{1}{1-G(\varphi_x^{s\prime})}\int_{\varphi_x^{s\prime}}^{\infty}\left(\frac{\varphi}{\tilde{\varphi}}\right)^{\sigma-1} r_d(\tilde{\varphi})g(\varphi)d\varphi\right] + M_{x1}\left[\frac{1}{G(\varphi_x^*)-G(\varphi_x^{s\prime})}\int_{\varphi_x^{s\prime}}^{\varphi_x^*}\tau^{1-\sigma}\left(\frac{\varphi}{\tilde{\varphi}_{x1}}\right)^{\sigma-1} r_d(\tilde{\varphi}_{x1})g(\varphi)d\varphi\right] + nM_{xn}\left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty}\tau^{1-\sigma}\left(\frac{\varphi}{\tilde{\varphi}_{xn}}\right)^{\sigma-1} r_d(\tilde{\varphi}_{xn})g(\varphi)d\varphi\right] = Mr_d(\tilde{\varphi}) + M_{x1}r_x(\tilde{\varphi}_{x1}) + nM_{xn}r_x(\tilde{\varphi}_{xn}) = M_tR(P\rho\tilde{\varphi}_t)^{\sigma-1} = M_tr_d(\tilde{\varphi}_t). \quad \text{(A4.5)}$$

b. Aggregate expenditure in case 2 $(\varphi^* \le \varphi_c)$.

$$R = G(\varphi_x^s)Mr_d(\varphi_c) + [1-G(\varphi_x^s)]M\left[\frac{1}{1-G(\varphi_x^s)}\int_{\varphi_x^s}^{\infty} r_d(\varphi)g(\varphi)d\varphi\right] + M_{x1}\left[\frac{1}{G(\varphi_x^*)-G(\varphi_x^s)}\int_{\varphi_x^s}^{\varphi_x^*}\tau^{1-\sigma} r_d(\varphi)g(\varphi)d\varphi\right] + nM_{xn}\left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty}\tau^{1-\sigma} r_d(\varphi)g(\varphi)d\varphi\right] = G(\varphi_x^s)Mr_d(\varphi_c) + [1-G(\varphi_x^s)]Mr_d(\tilde{\varphi}_x) + M_{x1}r_x(\tilde{\varphi}_{x1}) + nM_{xn}r_x(\tilde{\varphi}_{xn}) = M_tR(P\rho\tilde{\varphi}_t)^{\sigma-1} = M_tr_d(\tilde{\varphi}_t). \quad \text{(A4.6)}$$

References

Arnold, J. M. and K. Hussinger, (2005a). "Export Behavior and Firm Productivity in German Manufacturing. A Firm-level Analysis." *Review of World Economics*, 141, 2, 219–43

Bai, X., K. Krishna, and H. Ma, (2017). "How You Export Matters: Export Mode, Learning and Productivity in China." *Journal of International Economics*, 104, 122-137.

DOI:10.6814/NCCU202100569

Baldwin, J. R. and W. Gu, (2003). "Export-market Participation and Productivity Performance in Canadian Manufacturing." *Canadian Journal of Economics*, 36, 634-657.

Bernard, A. B. and J. B. Jensen, (1995). "Exporters, Jobs, and Wages in U.S. Manufacturing: 1975-1987." *Brookings Papers on Economic Activity: Microeconomics*. 67-119.

Bernard, A. B. and J. B. Jensen, (1999). "Exceptional exporter performance: cause, effect, or both?" *Journal of International Economics*, 47(1), 1-25.

Crespi, G., Criscuolo., C, and Haskel, J., (2008). "Productivity, exporting and the learning-by-exporting hypothesis: Direct evidence from UK firms." *Canadian Journal of Economics*, 41(2), 619–638.

De Loecker, J., (2007). "Do Exports Generate Higher Productivity? Evidence from Slovenia." *Journal of International Economics*, 73, 69-98.

De Loecker, J., (2013). "Detecting Learning by Exporting." *American Economic Journal: Microeconomics*, 5 (3), 1–21

Krugman, P. R., (1979). "Increasing Returns, Monopolistic Competition, and International Trade." *Journal of International Economics*, 9, 469-479.

Melitz, M. J., (2003). "The Impact of Trade on Intra-Industry Reallocations and

DOI:10.6814/NCCU202100569

Aggregate Industry Productivity." *Econometrica*, 71, 1695-1725.

Temouri, Y., Vogel., A, and Wagner, J., (2013). "Self-selection into export markets by business services firms—Evidence from France, Germany and the United Kingdom." *Structural Change and Economic Dynamics*, Volume 25, June 2013, 146-158

DOI:10.6814/NCCU202100569

Chapter 4 The Relationship between Export and Foreign Direct Investment with Technology Upgrade

1. Introduction

When a firm decides to serve customers in foreign countries, it can do it through several channels. Which method to use depends on many factors such as the distance of the destination country, the variable cost for shipping a product abroad, the fixed cost of building a facility that can produce in the local country. This paper discusses the relationship (complement or substitute) between two methods, export and foreign direct investment (FDI). In theory, they are alternative strategies if the product is narrowly defined. However, many researches find evidence of complementary relationship between export and FDI, including Lipsey and Weiss (1981), Lipsey and Weiss (1984), Clausing (2000) and Head and Ries (2001). One of the reasons that leads to the complementarity between export and FDI might be the data used in the research is not sufficiently narrowly defined. For example, a vertical specialization multinational exports an intermediate good to its affiliate, then one observes the firm is engaged in FDI and exporting simultaneously. However, this does not contradict the theory's prediction since the intermediate good and the final product are different goods when the data is disaggregated enough.

Given the strong empirical evidence of complementary relationship, researchers also consider the possibility of a firm engaging in both activities even with the product narrowly defined. Rob and Vettas (2003) construct a model with a single seller selling a single product, but facing an uncertain growing demand in the destination country. They find that a multinational will serve the foreign customers by both export and FDI

DOI:10.6814/NCCU202100569

under certain conditions. Since the firm produces a single product, the paper explains a complementary relationship between export and FDI even with product that is sufficiently narrowly defined. Conconi *et al*. (2016) provide an empirical result that supports the prediction of Rob and Vettas (2003). Head and Ries (2004) provide several possibilities to explain the inconsistence between the theory and observed data of export and FDI, such as unobserved variables that stimulate export and FDI simultaneously, a vertical specialization firm exporting intermediate goods to its affiliate. They also apply the market-crowding effect to illustrate that firms may use different methods to serve a given foreign market even all of them have common initial conditions. As such, a pool of identical firms is divided into exporters and multinationals in the equilibrium. Helpman *et al.* (2004) discuss a firm's decision between export and FDI in a monopolistic competition market with firm heterogeneity. They arrive at the result that the most productive firms serve the foreign market through FDI, the less productive firms export, and the least productive firms serve the domestic market only. This finding explains the coexistence of export and FDI in a country (the high productive firms serve the foreign market by FDI while the less productive firms sell their product through export). It also predicts that a firm might use different ways to serve different foreign countries due to difference in trade cost. However, for the special case that all countries are identical in trade cost and the countries' size do not differ too much, all countries share the same cutoffs of export and FDI. Consequently, from the point of view of a single firm, export and FDI are alternative tools to serve all foreign countries. Following this line of research, we are interested in the question: for a single firm, can a complementary relationship arise between export and FDI given that all foreign countries are identical? We construct a model that allows a productivity upgrade due to export or FDI to answer this question. The model is described in the following section.

The model is based on Melitz (2003) and our learning-by-exporting paper, but with

DOI:10.6814/NCCU202100569

some modifications in the open economy case. First, we assume there are two ways for a firm to serve a foreign market, export or FDI. All the n foreign countries are identical. The advantage of export is a lower fixed cost while that of FDI has zero transport cost. Second, we assume there is a learning effect on both export and outward FDI. That is, both activities can induce a productivity growth, but the magnitude of the growth can be symmetric or asymmetric. For the asymmetric growth case, a firm will acquire a higher productivity growth of the two tools if it uses both export and FDI. The setting of the fixed cost and constant variable cost of export, and the fixed cost of FDI leads to the result that the most productive firms are more profitable to serve the foreign market by opening an affiliate, while the less productive firms are better to sell its product through export. As a result, in a symmetric productivity growth case, it is apparent that FDI and export play a substitute role for any single firm since it will choose either export or FDI to serve its foreign customers. However, the situation changes under the assumption that the two instruments have asymmetric impact on a firm's productivity. A firm might choose a tool that earns relative lower profit to serve one of the markets in order to acquire a higher productivity growth, and use another tool which make higher profit to serve the remaining markets with the higher productivity it has obtained. The case that a firm uses both FDI and export to serve different (but identical) foreign markets reveals a weakly complementary relationship between the two instruments. The result indicates that, from a firm's point of view, to export or to make FDI for a single product need not be perfect substitutes even the destination countries are identical, if there are some factors (ex: productivity) arising from export or FDI which affect the firm's profit differently.

The remainder of this paper is composed as follows. Section 2 sets up the model, describing consumer preferences, firms' technology, and the assumption of the

DOI:10.6814/NCCU202100569

productivity growth. Section 3 discusses the case that export and FDI have a symmetric impact on a firm's productivity. Section 4 presents the case that export induces lager growth on productivity compared with FDI. Section 5 shows the case that FDI has a lager impact on productivity upgrade. Section 6 discusses the effect of change in transport cost. Section 7 concludes the paper.

2. The model

We construct a model to investigate the relationship between export and FDI in a monopolistic competition market, based on our learning-by-exporting structure. To focus on the relationship between a firm's FDI and export decision, we will not discuss the firms which serve the domestic market only in this paper.[1]

Consider the case that there are one home country and n foreign countries. All foreign countries are identical and the consumers in each country have the same CES form preference $\mathrm{U} = \left[\int_{w\in\Omega} q(w)^{\rho} dw\right]^{\frac{1}{\rho}}$ ($\rho \in (0,1)$), where w represents the varieties available to consumers. The elasticity of substitution is constant and equal to $\sigma = \frac{1}{1-\rho}$. The cost function for a firm is $l = f + \frac{q}{\varphi}$ (wage is normalized to one). We allow each firm to serve a foreign market through two channels, export or FDI. Furthermore, we assume FDI, like export, has a learning effect. Namely, a productivity upgrade can be induced by exporting as well as FDI.

2.1. Productivity growth and cost assumptions

[1] The part of the non-exporters is discussed in our learning-by-exporting paper.

DOI:10.6814/NCCU202100569

Like our learning-by-exporting model, firms draw a productivity from a probability distribution with probability density function $g(\varphi)$ and cumulative distribution function $G(\varphi)$ in the open economy. After realized the productivity level it has drawn, a firm decides its action, to sell domestically only, export or FDI (or both). As mentioned above, we will not discuss the non-trade firms here, since the aim of this paper is to explore the relationship between export and FDI.

Denote φ_g as the productivity level that a firm has drawn from a productivity distribution. The productivity of the firm will grow to $\alpha\varphi_g$ $(\beta\varphi_g)$ if the firm exports (makes FDI) to $n \geq 1$ countries, where $\alpha, \beta \in [1,2]$. Moreover, if the firm has exported (made FDI) to some countries and made FDI (exported) to other countries, it will reach the productivity level equals $Max\{\alpha, \beta\}\varphi_g$. Consider the case $\beta > \alpha$, which means FDI has higher productivity growth than export, and a firm has drawn a productivity level φ_g. After realized the productivity, if the firm decides to make FDI to one county and to export to the remaining $n-1$ countries, then the firm will reach the productivity level $\beta\varphi_g$ even it has made FDI to only one country.

The costs for a firm to export include iceberg transport cost τ $(\tau > 1)$ and a per period per country fixed cost f_x; the cost for a firm to make FDI is a per period per country fixed cost f_I, but no transport cost for selling through FDI. Furthermore, we assume the productivity level that makes zero FDI profit is higher than the productivity that makes zero export profit, implying that FDI requires higher productivity than export. To satisfied this condition, we need to assume $f_I > \tau^{\sigma-1} f_x$. To see this, defined φ_I^* (φ_x^*) as a productivity level that makes zero FDI (export) profit, then we have:

DOI:10.6814/NCCU202100569

$$\pi_I(\varphi_I^*) = \frac{r_I(\varphi_I^*)}{\sigma} - f_I = 0 \text{ and } \pi_x(\varphi_x^*) = \frac{r_x(\varphi_x^*)}{\sigma} - f_x = 0,$$

where π_I (r_I), π_x (r_x) is FDI and export profit (revenue), respectively. Divide r_I by r_x to get:

$$\frac{r_I(\varphi_I^*)}{r_x(\varphi_x^*)} = \tau^{\sigma-1}\left(\frac{\varphi_I^*}{\varphi_x^*}\right)^{\sigma-1} = \frac{\sigma f_I}{\sigma f_x} \Rightarrow \varphi_I^* = \frac{1}{\tau}\varphi_x^*\left(\frac{f_I}{f_x}\right)^{\frac{1}{\sigma-1}}.$$

Therefore, $\varphi_I^* > \varphi_x^*$ if and only if $\frac{1}{\tau}\left(\frac{f_I}{f_x}\right)^{\frac{1}{\sigma-1}} > 1$, that is $f_I > \tau^{\sigma-1} f_x$.

2.2. The cutoff productivity between export and FDI

We now find a cutoff productivity that makes firm indifference between export and FDI to one country. Denote $\pi^x(\varphi)$ and $\pi^I(\varphi)$ as a combined profit of export and FDI (to one country) respectively, that is:

$$\pi^x(\varphi_g) = \pi_d(\alpha\varphi_g) + \pi_x(\alpha\varphi_g),$$
$$\pi^I(\varphi_g) = \pi_d(\beta\varphi_g) + \pi_I(\beta\varphi_g),$$

where φ_g is the productivity drawn from the productivity distribution, while $\pi_d(\varphi)$, $\pi_x(\varphi)$ and $\pi_I(\varphi)$ represent domestic sales profit, export sales profit, and FDI profit.

Defined a productivity level φ_{XI} that makes the firm's combined profit from export and from FDI equal, that is, φ_{XI} satisfied:

DOI:10.6814/NCCU202100569

$\pi^I(\varphi_{\mathrm{XI}}) = \pi^x(\varphi_{\mathrm{XI}})$, or

$$\pi_d(\beta\varphi_{\mathrm{XI}}) + \pi_I(\beta\varphi_{\mathrm{XI}}) = \pi_d(\alpha\varphi_{\mathrm{XI}}) + \pi_x(\alpha\varphi_{\mathrm{XI}}) \tag{4.1}$$

We can solve φ_{XI} as an explicit function of the open economy equilibrium cutoff φ^* and other exogenous variables as follows[2]:

$$\pi_d(\beta\varphi_{\mathrm{XI}}) + \pi_I(\beta\varphi_{\mathrm{XI}}) = \pi_d(\alpha\varphi_{\mathrm{XI}}) + \pi_x(\alpha\varphi_{\mathrm{XI}})$$

$\Rightarrow \left[\frac{r_d(\beta\varphi_{XI})}{\sigma} - f\right] + \left[\frac{r_I(\beta\varphi_{XI})}{\sigma} - f_I\right] = \left[\frac{r_d(\alpha\varphi_{XI})}{\sigma} - f\right] + \left[\frac{r_x(\alpha\varphi_{XI})}{\sigma} - f_x\right]$ (f is the fixed cost of production).

$$\Rightarrow r_d(\beta\varphi_{XI}) + r_I(\beta\varphi_{XI}) = r_d(\alpha\varphi_{XI})(1+\tau^{1-\sigma}) + \sigma(f_I - f_x).$$

Using $\frac{r_d(\alpha\varphi_{XI})}{r_d(\varphi^*)} = \left(\frac{\alpha\varphi_{XI}}{\varphi^*}\right)^{\sigma-1} \Rightarrow r_d(\alpha\varphi_{XI}) = \left(\frac{\alpha\varphi_{XI}}{\varphi^*}\right)^{\sigma-1}\sigma f$, then

$$\frac{r_d(\beta\varphi_{XI})+r_I(\beta\varphi_{XI})}{r_d(\varphi^*)} = 2\left(\frac{\beta\varphi_{XI}}{\varphi^*}\right)^{\sigma-1} = \frac{\sigma\left[(1+\tau^{1-\sigma})\left(\frac{\alpha\varphi_{XI}}{\varphi^*}\right)^{\sigma-1} f + (f_I - f_x)\right]}{\sigma f}.$$

From the second equal sign of the equation above we arrived at:

$$\varphi_{\mathrm{XI}} = \varphi^*[2\beta^{\sigma\text{-}1} - (1+\tau^{1\text{-}\sigma})\alpha^{\sigma\text{-}1}]^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}}.$$

Clearly, if α is not greater than β too much, an increase in productivity increases more profit in $\pi^I(\varphi_{\mathrm{XI}})$ than in $\pi^x(\varphi_{\mathrm{XI}})$ since FDI has no transport cost. We assume

[2] φ^* is defined as $\pi_d(\varphi^*) = 0$ under open economy equilibrium. The determination of φ^* is relatively complicated, we discuss one of the cases in appendix A6.

DOI:10.6814/NCCU202100569

the condition $\frac{\beta}{\alpha} > \left[\frac{1+\tau^{1-\sigma}}{2}\right]^{\frac{1}{\sigma-1}}$ is satisfied, then it can be shown that when a firm has drawn $\varphi_g > \varphi_{XI}$, the combined profit of FDI to one country is larger than the combined profit of export. On the contrary, if $\varphi_g < \varphi_{XI}$, export to one country is more profitable.[3]The condition also guarantees the existence of the φ_{XI}.

3. The symmetric productivity growth *($\alpha = \beta$)*

We start with the symmetric growth case where export and FDI affect the productivity growth in the same magnitude, namely $\alpha = \beta$.

Let $\alpha = \beta = \gamma$, then under our assumption of the cost structure we can derive the result[4]:

$$\gamma\varphi_{\mathrm{XI}} > \varphi_I^* > \varphi_x^*.$$

Since the productivity grows to $\gamma\varphi_g$ if the firm export or FDI to foreign countries, we can divide the range of $\gamma\varphi_g$ into four regions (figure 4-1). We can now analyze what a firm will do if it has $\gamma\varphi_g$ is in each region.

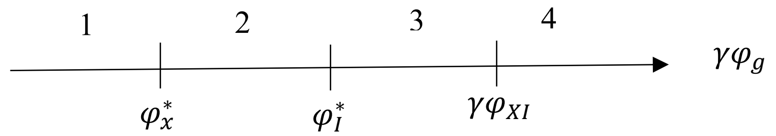

Figure 4-1 Four regions in the symmetric productivity growth case.

[3] Proof in appendix 1.
[4] See appendix 2.

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Region 1

In this region, firms will export to one country for the purpose of the productivity growth. Since a firm's profit from foreign sales is negative either through export or through FDI ($\because \gamma\varphi_g < \varphi_x^* < \varphi_I^*$), we do not need have to consider export or FDI to $n > 1$ countries (Export or FDI to more countries doesn't make the productivity grow further). The combined profit of export to one country is greater than the combined profit of FDI to one country. (Since $\gamma\varphi_g < \gamma\varphi_{XI}$, we have $\varphi_g < \varphi_{XI}$ in region 1)

Region 2

Firms with $\gamma\varphi_g$ in region 2 have a positive export profit. However, the profit for a firm's FDI to any additional country is negative. As a result, to maximize profit, a firm will choose to export to n countries and does not make any FDI abroad.

Region 3

If $\gamma\varphi_g$ is in region 3, both export and FDI make a positive profit, but, export earns more than FDI. To see this, notice that $\gamma\varphi_g < \gamma\varphi_{XI}$ in this region, thus $\varphi_g < \varphi_{XI}$. By the definition of φ_{XI} we have:

$$\pi^I(\varphi_{\mathrm{XI}}) = \pi^x(\varphi_{\mathrm{XI}}), \text{ or}$$

$$\pi_d(\beta\varphi_{\mathrm{XI}}) + \pi_I(\beta\varphi_{\mathrm{XI}}) = \pi_d(\alpha\varphi_{\mathrm{XI}}) + \pi_x(\alpha\varphi_{\mathrm{XI}}).$$

As $\alpha = \beta = \gamma$ in the symmetric productivity growth case, the second equation above becomes:

DOI:10.6814/NCCU202100569

$$\pi_d(\gamma\varphi_{\mathrm{XI}}) + \pi_I(\gamma\varphi_{\mathrm{XI}}) = \pi_d(\gamma\varphi_{\mathrm{XI}}) + \pi_x(\gamma\varphi_{\mathrm{XI}})$$

Under the condition $\frac{\beta}{\alpha} > \left[\frac{1+\tau^{1\text{-}\sigma}}{2}\right]^{\frac{1}{\sigma\text{-}1}}$, $\pi^I(\varphi) < \pi^x(\varphi)$ if $\varphi < \varphi_{XI}$. Using $\varphi_g < \varphi_{XI}$ in region 3, we have $\pi^I(\varphi_g) < \pi^x(\varphi_g)$, that is:

$$\pi_d(\gamma\varphi_g) + \pi_I(\gamma\varphi_g) < \pi_d(\gamma\varphi_g) + \pi_x(\gamma\varphi_g), \text{ and so}$$

$$\pi_x(\gamma\varphi_g) > \pi_I(\gamma\varphi_g).$$

Since export is more profitable than FDI, the best strategy for a firm with $\gamma\varphi_g$ in this region is to export to n countries.

Region 4

By similar analysis of region 3, we can show $\pi_I(\gamma\varphi_g) > \pi_x(\gamma\varphi_g)$ in region 4. A firm with $\gamma\varphi_g$ in this region earns highest profit if it chooses to make FDI to n countries. Figure 4-2 presents a firm's best strategy in each region.

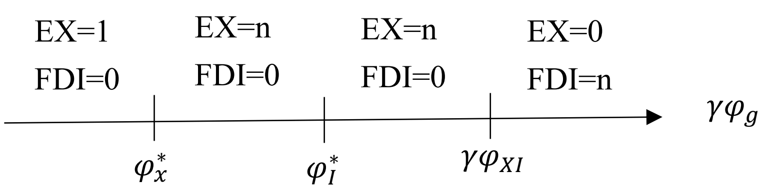

Figure 4-2 A firm's best strategy in the symmetric productivity growth case

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We can see when export and FDI have a symmetric impact on productivity growth, a firm will choose either export or FDI but not both, implying a perfect substitute relation between export and FDI. The result is similar to that obtained in Helpman et. (2004)[5].

4. The asymmetric productivity growth with $\boldsymbol{\alpha > \beta}$

The case $\alpha > \beta$ indicates that export has a larger productivity growth effect than FDI. To analyze this case, we need to define a cutoff which generates equal export and FDI profit. Defined a cutoff productivity φ' such that φ' satisfies:

$$\pi_x(\varphi') = \pi_I(\varphi').$$

Then $\pi_I(\varphi) > \pi_x(\varphi)$ if $\varphi > \varphi'$; $\pi_I(\varphi) < \pi_x(\varphi)$ if $\varphi < \varphi'$. Since

$\pi_I(\varphi) - \pi_x(\varphi) = \frac{1}{\sigma}[r_d(\varphi)(1-\tau^{1-\sigma})] - (f_I - f_x)$ is increasing in φ, it can be shown that $\varphi' = \varphi^*(1\text{-}\tau^{1\text{-}\sigma})^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}}$.[6]

The relation of each cutoff in the case of $\alpha > \beta$ are[7]:

$$\alpha\varphi_{\mathrm{XI}} > \beta\varphi_{\mathrm{XI}} > \varphi' > \varphi_I^* > \varphi_x^*.$$

To simplify the analysis, we make the following assumption: The number of the foreign countries n is large enough such that if a firm has drawn a productivity level

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[5] A special case of Helpman et. (2004) with all countries are symmetric in trade cost.
[6] See appendix 3.
[7] See appendix 3.

DOI:10.6814/NCCU202100569

which is high enough to earn a positive export profit, it is always more profitable for the firm to export (FDI) to at least one country if $\alpha > \beta$ ($\beta > \alpha$) compared with using the tool that induce lower productivity growth to serve all foreign countries[8].

For example, if export leads to a higher productivity growth than FDI ($\alpha > \beta$) and a firm has drawn a productivity level φ_g that satisfied $\alpha\varphi_g > \beta\varphi_g > \varphi^{'} > \varphi_I^*$, then the firm will serve one country through export in order to obtain the productivity $\alpha\varphi_g$ (Although the profit of making FDI is greater than export), and makes FDI to the remaining $n-1$ countries, with the productivity level $\alpha\varphi_g$. In other words, we block the case such that:

$$\pi_d(\beta\varphi_g) + n\pi_I(\beta\varphi_g) > \pi_d(\alpha\varphi_g) + \pi_x(\alpha\varphi_g) + (n-1)\pi_I(\alpha\varphi_g) \text{ if } \alpha > \beta.$$

Now let us turn to a firm's best strategy in the case $\alpha > \beta$. Five cutoffs divide the range of productivity into six regions, as shown in figure 4-3

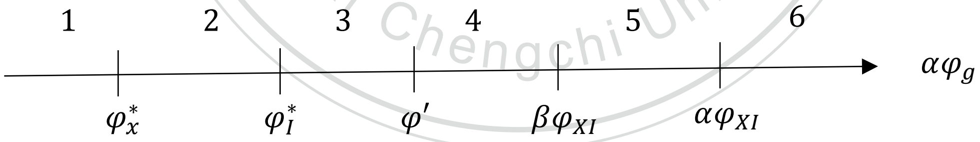

Figure 4-3 Six regions in the case with $\alpha > \beta$.

First, notice that in the present case, a firm will always export to at least one country and its productivity will grow to $\alpha\varphi_g$. This is obvious, since it is less costly for the firm to export than making FDI and export leads to a higher productivity growth. We

[8] The assumption simplifies our analysis and does not change the main results. See appendix 4 for details.

DOI:10.6814/NCCU202100569

now discuss what a firm will do if $\alpha\varphi_g$ is located in each region.

Region 1

If $\alpha\varphi_g$ is in region 1, then both export and FDI have negative profit and we have $\varphi_g < \varphi_{XI}$. The firm will export to one country to increase its combined profit. Recall that we only discuss the case with the firm having export. This means the combined profit of export to one country is greater than to sell domestically only. The detailed discussion of this point is in our learning by exporting paper.

Region 2 and Region 3

The export profit is positive and is greater than FDI profit, that is, $\pi_x(\alpha\varphi_g) > 0$, and $\pi_x(\alpha\varphi_g) > \pi_I(\alpha\varphi_g)$ in region 2 and region 3. Notice that FDI profit is positive in region 3, however it is smaller than export profit since $\alpha\varphi_g < \varphi'$. As a result, firms with $\alpha\varphi_g$ in region 2 and region 3 will export to n countries.

Region 4 ~ Region 6

If $\alpha\varphi_g$ is greater than φ', both export and FDI have positive profit and FDI is more profitable. To maximize profit, a firm will use FDI to serve the foreign countries. However, according to our assumption, it is even more profitable if the firm export to at least one country to acquire a higher productivity growth since $\alpha > \beta$. Namely, n is large enough so that

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$\pi_d(\alpha\varphi_g) + \pi_x(\alpha\varphi_g) + (n-1)\pi_I(\alpha\varphi_g) > \pi_d(\beta\varphi_g) + n\pi_I(\beta\varphi_g)$ holds for $\alpha\varphi_g > \varphi'$

As a result, a firm with productivity in these regions will export to one country to obtain the higher productivity growth and make FDI to the remaining $n-1$ countries with the higher productivity $\alpha\varphi_g$ (If a firm chooses to make FDI to n countries, the productivity will grow to $\beta\varphi_g$, which is lower than $\alpha\varphi_g$). We summarize the results in figure 4-4.

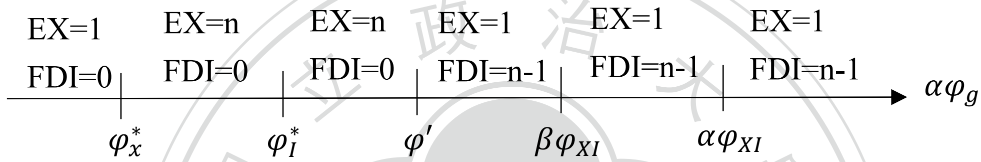


Figure 4-4 A firm's best strategy in the case with $\alpha > \beta$.

As show in Figure 4-4, in some regions, a firm exports to one country and makes FDI to $n-1$ countries even the trade cost are identical for all countries. This gives rise to a weakly complementary relationship between export and FDI. The differ from the perfectly substitute relationship in the symmetric growth case.

5. The asymmetric productivity growth with $\beta > \alpha$

This is the case that FDI leads to a higher growth on the productivity. We can derive that $\varphi' > \varphi_I^*$, $\varphi' > \beta\varphi_{XI} > \alpha\varphi_{XI}$[9]. But $\beta\varphi_{XI} > \varphi_x^*$ is not guaranteed to hold under

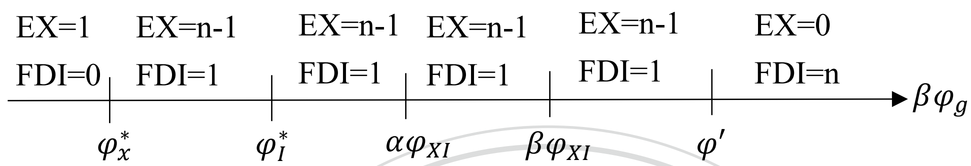
[9] See appendix 3.

DOI:10.6814/NCCU202100569

$\beta > \alpha$[10]. As a result, there are several possible cases if $\beta > \alpha$.

We report the results in figure 4-5, which shows a firm's best strategy if it has drawn the productivity φ_g such that $\beta\varphi_g$ is in each region. The analysis method is similar to the previous cases:

1.



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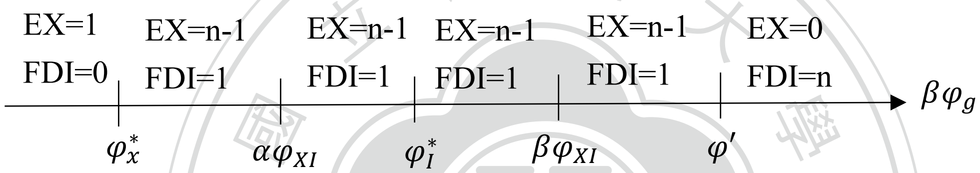

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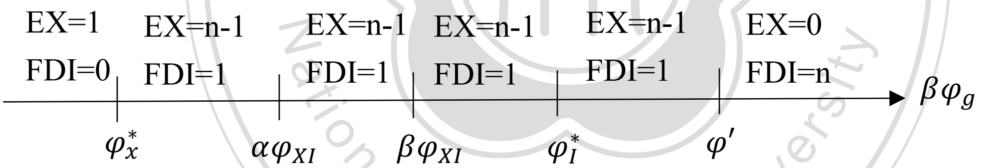

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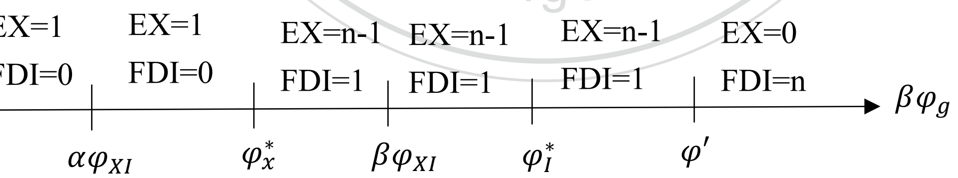

5.

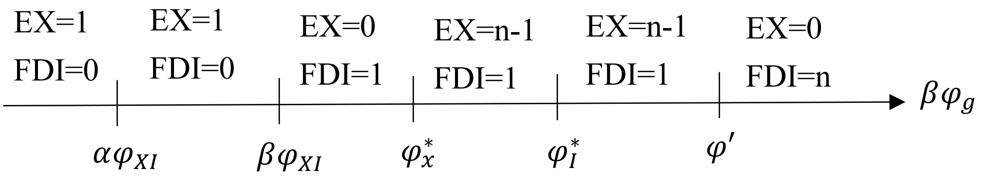

Figure 4-5 A Firm's best strategy in the case with $\beta > \alpha$.

10 See appendix 5.

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It is worth noticing that we don't have the case such that $\varphi' > \beta\varphi_{\mathrm{XI}} > \varphi_I^* > \varphi_x^* > \alpha\varphi_{XI}$, since φ_{XI} is defined as $\pi_d(\beta\varphi_{\mathrm{XI}}) + \pi_I(\beta\varphi_{\mathrm{XI}}) = \pi_d(\alpha\varphi_{\mathrm{XI}}) + \pi_x(\alpha\varphi_{\mathrm{XI}})$, and $\beta > \alpha$ implies $\pi_I(\beta\varphi_{\mathrm{XI}}) < \pi_x(\alpha\varphi_{\mathrm{XI}})$. But $\varphi' > \beta\varphi_{\mathrm{XI}} > \varphi_I^* > \varphi_x^* > \alpha\varphi_{XI}$ means $\pi_x(\alpha\varphi_{XI}) < 0$ and $\pi_I(\beta\varphi_{XI}) > 0$, which is contradicted with $\pi_I(\beta\varphi_{\mathrm{XI}}) < \pi_x(\alpha\varphi_{\mathrm{XI}})$.

We can see that, although the location of the $\alpha\varphi_{\mathrm{XI}}$ and $\beta\varphi_{\mathrm{XI}}$ are different in each case, the baseline results are similar. The most productive firms make FDI to n countries, and firms which make positive, and greater export profit (compared with FDI) export to $n-1$ countries, however, FDI to one country to increase its productivity further. This shows a weakly complementary relationship between export and FDI. The least productive firms export (or make FDI) to one country.

Combining the finding of symmetric and two asymmetric cases, we conclude that FDI and export are perfect substitute only when the learning effects of FDI and export have the same magnitude; otherwise, there may be a weakly complementary relationship between export and FDI. The results are summarizing in figure 4-6.

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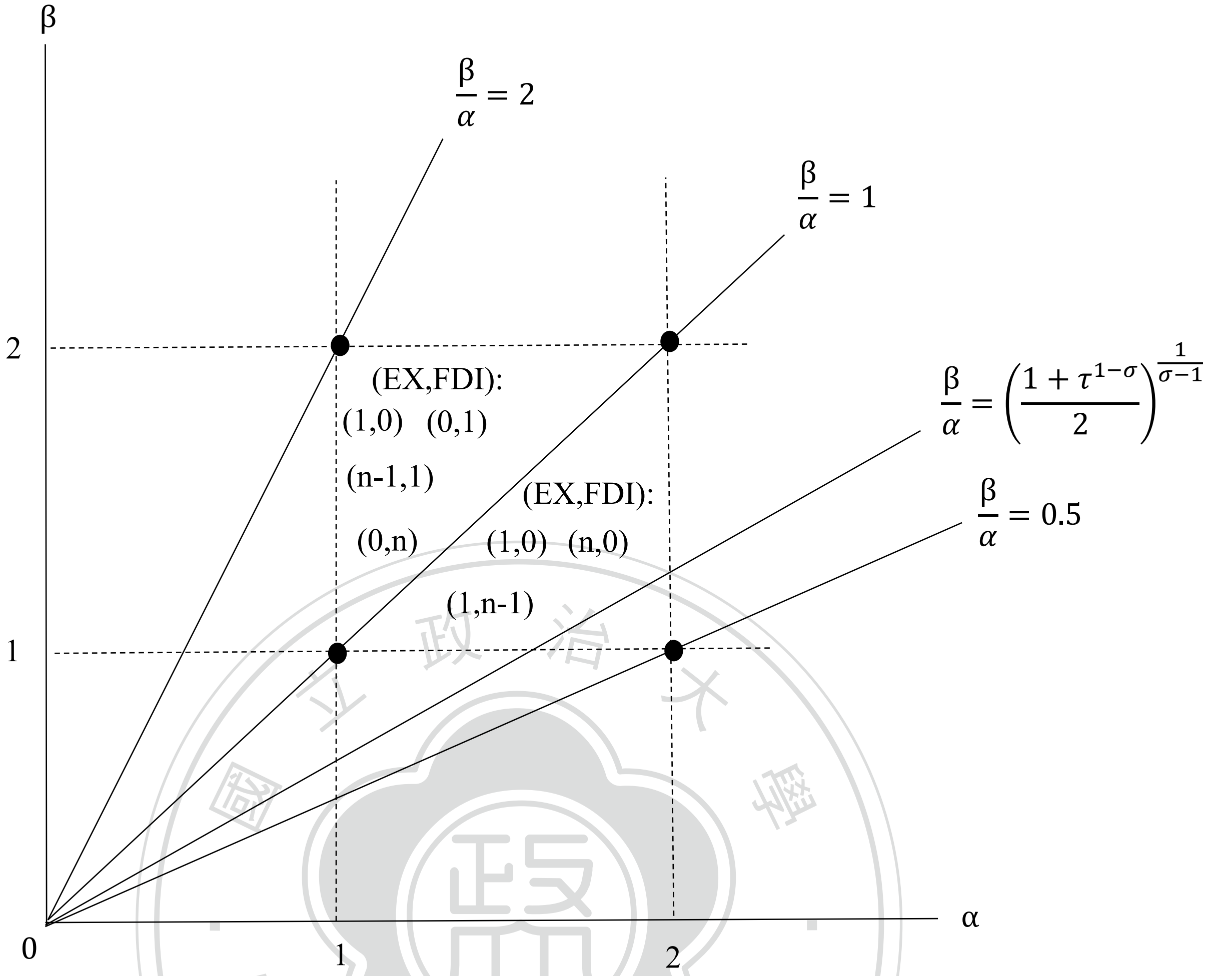

Figure 4-6 All possible results in symmetric and asymmetric productivity growth cases.

6. The effect of an increase in τ

An increase in the variable trade cost shifts the FE and the ZCP curves in this model. The FE curve will shift upward in the portion where $\varphi^* > \varphi_c$ since $\varphi_x^{s\prime}$ is increase in τ[11]. The change of the ZCP curve depends on the parameters and the distribution of $G(\varphi)$. Most likely, it shifts downward due to the reduction in the export profit because of the higher variable cost. The shifts of the two curves change the equilibrium cutoff. For a more possible case, it decreases the equilibrium cutoff φ^*(φ^* increases only in

[11] See appendix 6 for the discuss of FE and ZCP curve in this model.

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the case that ZCP shifts upward enough). We analyze the case that if an increase in the variable trade cost makes the equilibrium cutoff smaller. This makes the region $[\varphi_x^*, \varphi']$ narrower, which depresses export in all three cases. To see this, differentiate $\varphi' - \varphi_x^*$ with respect to τ and we have:

$$\frac{\partial(\varphi' - \varphi_x^*)}{\partial\tau} = \frac{\partial\varphi^*}{\partial\tau}\left[(1-\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}} - \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right] + \varphi^*\left[-\tau^{-\sigma}(1-\tau^{1-\sigma})^{\frac{\sigma}{1-\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}} - \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right] < 0, \text{ since } \frac{\partial\varphi^*}{\partial\tau} < 0.$$

Furthermore, we can ensure that an increase in the variable trade cost decreases the cutoff φ'. Differentiating φ' with respect to τ, we arrive:

$$\frac{\partial\varphi'}{\partial\tau} = \frac{\partial\varphi^*}{\partial\tau}(1-\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}} + \varphi^*(-\tau^{-\sigma})(1-\tau^{1-\sigma})^{\frac{\sigma}{1-\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}} < 0.$$

As a result, the region for a firm to made FDI to n or $n-1$ countries is expanded.

As expected, an increase in the variable trade cost may well lead firms to replace export with FDI. This is because an increase in τ makes the cost of export more expensive, but with no effect on the trade cost of FDI. (The effects are inconclusive if $\frac{\partial\varphi^*}{\partial\tau} > 0$)

7. Conclusion

We have constructed a model to discuss firms' decision of how they serve foreign markets based on the structure of our learning-by-exporting paper. The main

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assumptions of the model are: (1) There are two possible instruments to serve the foreign markets, export or foreign direct investment. (2) A firm's productivity grows if it exports or engages in FDI to serve foreign countries. The magnitude of the growth in productivity due to export can be different from that due to FDI. (3) The cost of export or FDI are identical to all importing countries.

Under the settings, we have found that even all countries have identical export and FDI cost, a firm may export (made FDI) to one country in order to acquire a higher productivity, and made FDI (export) to the remaining countries when export and FDI have an asymmetric effect on the productivity growth. The results indicate that the asymmetric effect of export and FDI on a firm's productivity may be one of the factors that leads to a complementary relationship between export and FDI. That is, a firm may use both export and FDI to serve foreign markets even all of them are identical. Moreover, as expected, an increase in the variable trade cost is most likely to replace export with FDI. The result in the symmetric productivity growth case is basically the same as the special case of Helpman et. (2004) with all countries are identical in trade cost. A perfect substitute result appears, and firms will use either export or FDI to serve all foreign markets.

Although we have obtained some interesting results for a firm using both export and FDI to serve foreign markets, the model exhibits some limitations due to assumptions made in it. Relaxing some of the assumptions can be a direction of future research. For example, the complementary relationship of the two instruments is weak in the sense that a firm will export (make FDI) to one country and make FDI (export) to the remaining $(n-1)$ countries. If the productivity growth from export and FDI are affected by the number of export or FDI countries, we may study the problem of

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optimal export and FDI countries. In that case a stronger complementary relationship between export and FDI may emerge.

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Appendices.

Appendix 1.

We prove this by showing $\frac{\partial(\pi^I-\pi^x)}{\partial\varphi}$ is increasing in φ. The combined profit of FDI to one country is:

$$\pi^I(\varphi) = \pi_d(\beta\varphi) + \pi_I(\beta\varphi) = \left[\frac{1}{\sigma}R(P\rho\beta\varphi)^{\sigma-1} - f\right] + \left[\frac{1}{\sigma}R(P\rho\beta\varphi)^{\sigma-1} - f_I\right].$$

Thus,

$$\frac{\partial\pi^I}{\partial\varphi} = \frac{2}{\sigma}(\sigma-1)R(P\rho\beta\varphi)^{\sigma-2}P\rho\beta.$$

The combined profit of export to one country is:

$$\pi^x(\varphi) = \pi_d(\alpha\varphi) + \pi_x(\alpha\varphi) = \left[\frac{1}{\sigma}R(P\rho\alpha\varphi)^{\sigma-1} - f\right] + \left[\frac{1}{\sigma}\tau^{1-\sigma}R(P\rho\alpha\varphi)^{\sigma-1} - f_x\right].$$

Thus,

$$\frac{\partial\pi^x}{\partial\varphi} = \frac{1}{\sigma}(\sigma-1)R(P\rho\alpha\varphi)^{\sigma-2}(1+\tau^{1-\sigma})P\rho\alpha.$$

As a result,

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$$\frac{\partial(\pi^I-\pi^x)}{\partial\varphi}=\frac{\partial\pi^I}{\partial\varphi}-\frac{\partial\pi^x}{\partial\varphi}=\frac{1}{\sigma}(\sigma-1)R(P\rho)^{\sigma-1}\varphi^{\sigma-2}[2\beta^{\sigma-1}-\alpha^{\sigma-1}(1+\tau^{1-\sigma})].$$

$\frac{\partial(\pi^I-\pi^x)}{\partial\varphi}>0$ if $[2\beta^{\sigma-1}-\alpha^{\sigma-1}(1+\tau^{1-\sigma})]>0$, or equivalently $\frac{\beta}{\alpha}>\left[\frac{1+\tau^{1\text{-}\sigma}}{2}\right]^{\frac{1}{\sigma\text{-}1}}$.

Appendix 2.

Since $\varphi_I^*>\varphi_x^*$ is ensured by our assumption on cost ($f_I>\tau^{\sigma-1}f_x$), we only need to show $\gamma\varphi_{\mathrm{XI}}>\varphi_I^*$ if $\alpha=\beta=\gamma$. We know

$$\gamma\varphi_{\mathrm{XI}}=\gamma\varphi^*[2\beta^{\sigma\text{-}1}-(1+\tau^{1\text{-}\sigma})\alpha^{\sigma\text{-}1}]^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}},$$

$$\varphi_I^*=\varphi^*\left(\frac{f_I}{f}\right)^{\frac{1}{\sigma-1}}.$$

Substitute $\alpha=\beta=\gamma$ into $\gamma\varphi_{\mathrm{XI}}-\varphi_I^*$ to get:

$$\gamma\varphi_{\mathrm{XI}}-\varphi_I^*=\gamma\varphi^*[2\gamma^{\sigma\text{-}1}-(1+\tau^{1\text{-}\sigma})\gamma^{\sigma\text{-}1}]^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}}-\varphi^*\left(\frac{f_I}{f}\right)^{\frac{1}{\sigma-1}}.$$

Therefore, $\gamma\varphi_{\mathrm{XI}}-\varphi_I^*>0$ if and only if $\gamma\varphi^*[2\gamma^{\sigma\text{-}1}-(1+\tau^{1\text{-}\sigma})\gamma^{\sigma\text{-}1}]^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}}>\varphi^*\left(\frac{f_I}{f}\right)^{\frac{1}{\sigma-1}}$. It can be shown that the condition of $\gamma\varphi_{\mathrm{XI}}-\varphi_I^*>0$ is equivalent to $f_I>\tau^{\sigma-1}f_x$, which is our assumption on cost. In other words, $f_I>\tau^{\sigma-1}f_x$ also guarantees $\gamma\varphi_{\mathrm{XI}}>\varphi_I^*$ in the symmetric productivity growth case.

Appendix 3.

Since $\varphi_I^*>\varphi_x^*$ is true under the assumptions $f_I>\tau^{\sigma-1}f_x$ and $\alpha\varphi_{\mathrm{XI}}>\beta\varphi_{\mathrm{XI}}$ when

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$\alpha > \beta$, we only need to prove the inequality $\beta\varphi_{\mathrm{XI}} > \varphi' > \varphi_I^*$.

1. Proof of $\varphi' > \varphi_I^*$.

By the definition of φ' we have $\pi_I(\varphi') - \pi_x(\varphi') = \frac{1}{\sigma}[r_d(\varphi')(1-\tau^{1-\sigma})] - (f_I - f_x) = 0$. Use $r_d(\varphi') = \left(\frac{\varphi'}{\varphi^*}\right)^{\sigma-1}\sigma f$ to get $\frac{1}{\sigma}\left[\left(\frac{\varphi'}{\varphi^*}\right)^{\sigma-1}\sigma f(1-\tau^{1-\sigma})\right] - (f_I - f_x) = 0$. Then, we have:

$\varphi' = \varphi^*(1\text{-}\tau^{1\text{-}\sigma})^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}}$.

Since $\varphi_I^* = \varphi^*\left(\frac{f_I}{f}\right)^{\frac{1}{\sigma-1}}$, $\varphi' > \varphi_I^*$ if and only if: $\varphi^*(1\text{-}\tau^{1\text{-}\sigma})^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}} > \varphi^*\left(\frac{f_I}{f}\right)^{\frac{1}{\sigma-1}}$, which is same as the assumption $f_I > \tau^{\sigma-1}f_x$.

2. Proof of $\beta\varphi_{\mathrm{XI}} > \varphi'$.

We know $\beta\varphi_{\mathrm{XI}} = \beta\varphi^*[2\beta^{\sigma-1} - (1+\tau^{1-\sigma})\alpha^{\sigma-1}]^{\frac{1}{1-\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}}$, and $\varphi' = \varphi^*(1\text{-}\tau^{1\text{-}\sigma})^{\frac{1}{1\text{-}\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}}$. Therefore, $\beta\varphi_{\mathrm{XI}} > \varphi'$ if and only if:

$$\beta\varphi^*[2\beta^{\sigma-1} - (1+\tau^{1-\sigma})\alpha^{\sigma-1}]^{\frac{1}{1-\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}} > \varphi^*(1-\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left(\frac{f_I-f_x}{f}\right)^{\frac{1}{\sigma-1}}.$$

This can be simplified to $\left(\frac{\alpha}{\beta}\right)^{\sigma-1} > 1$, and the condition always holds if $\alpha > \beta$. Since $\sigma > 1$.

Thus, the inequality $\alpha\varphi_{\mathrm{XI}} > \beta\varphi_{\mathrm{XI}} > \varphi' > \varphi_I^* > \varphi_x^*$ holds if $f_I > \tau^{\sigma-1}f_x$ and

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$\alpha > \beta$ are satisfied.

Appendix 4.

The words "*does not change the main results"* suggest that the weakly complementary relationship between export and FDI is not due to the large n assumption. The number of n will affect the region of the weak complement, but not affect the existence of the region. We explain this by showing that the weakly complementary relationship region exists in both cases of $\alpha > \beta$ and $\beta > \alpha$ for $n > 1$.

a. $\alpha > \beta$

Under the large n assumption, a firm's decision could be:

1. $\alpha\varphi_g < \varphi_x^* \Rightarrow \begin{cases} EX = 1, FDI = 0 \ if \ \varphi_g < \varphi_{XI} \\ EX = 0, FDI = 1 \ if \ \varphi_g > \varphi_{XI} \end{cases}$,

2. $\varphi_x^* < \alpha\varphi_g < \varphi' \Rightarrow EX = n, FDI = 0$,

3. $\varphi' < \alpha\varphi_g \Rightarrow EX = 1, FDI = n - 1$.

The decision 3 implies that for $\varphi' < \alpha\varphi_g$, we have:

$$\pi_d(\alpha\varphi_g) + \pi_x(\alpha\varphi_g) + (n-1)\pi_I(\alpha\varphi_g) > \pi_d(\beta\varphi_g) + n\pi_I(\beta\varphi_g) \quad \text{(a1)}$$

The LHS of the inequality (a1) is the combined profit of a firm which exports to

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one country and makes FDI to the remaining $n-1$ countries. Consequently, its productivity grows to the higher level $\alpha\varphi_g$ due to export. The RHS is a combined profit of the firm which makes FDI to n countries. Its productivity grows to a lower level $\beta\varphi_g$ since it doesn't export to any country. Obviously, inequality (a1) tends to hold when n is getting larger. However, it holds for some $\varphi' < \alpha\varphi_g$ even if $n = 1$. Substituting $n = 1$ into (a1), we have:

$$\pi_d(\alpha\varphi_g) + \pi_x(\alpha\varphi_g) > \pi_d(\beta\varphi_g) + \pi_I(\beta\varphi_g).$$

By the definition of φ_{XI}, the inequality above holds when $\varphi_g < \varphi_{XI}$. That is, inequality (a1) holds in the region $\varphi' < \alpha\varphi_g < \alpha\varphi_{XI}$ even $n = 1$. Therefore, (a1) must hold for some region in $\varphi' < \alpha\varphi_g$ when $n > 1$.

b. $\beta > \alpha$

Under the large n assumption, a firm's decisions could be:

1. $\beta\varphi_g < \varphi_x^* \begin{cases} EX = 1, FDI = 0 \ \ if \ \varphi_g < \varphi_{XI} \\ EX = 0, FDI = 1 \ \ if \ \varphi_g > \varphi_{XI} \end{cases}$,

2. $\varphi_x^* < \beta\varphi_g < \varphi' \ \Rightarrow \ EX = n-1, FDI = 1,$

3. $\varphi' < \beta\varphi_g \ \Rightarrow \ EX = 0, FDI = n.$

The decision 2 implies that for $\varphi_x^* < \beta\varphi_g < \varphi'$, we have:

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$$\pi_d(\beta\varphi_g) + \pi_I(\beta\varphi_g) + (n-1)\pi_X(\beta\varphi_g) > \pi_d(\alpha\varphi_g) + n\pi_X(\alpha\varphi_g) \qquad \text{(a2)}$$

The analysis is similar to the case $\alpha > \beta$; (a2) tends to hold as n is large. When $n = 1$, (a2) becomes

$\pi_d(\beta\varphi_g) + \pi_I(\beta\varphi_g) > \pi_d(\alpha\varphi_g) + \pi_X(\alpha\varphi_g)$, which holds if $\varphi_g > \varphi_{XI}$.

This implies that (a2) holds for $\beta\varphi_{XI} < \beta\varphi_g < \varphi'$ even if $n = 1$. As a result, (a2) must hold for some region in $\varphi_x^* < \beta\varphi_g < \varphi'$ when $n > 1$.

Appendix 5.

Now we want to show the conditions $f_I > \tau^{\sigma-1} f_x$ and $\beta > \alpha$ do not guarantee the inequality $\beta\varphi_{XI} > \varphi_x^*$. Since

$$\beta\varphi_{\mathrm{XI}} = \beta\varphi^*[2\beta^{\sigma-1} - (1+\tau^{1-\sigma})\alpha^{\sigma-1}]^{\frac{1}{1-\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}},$$

$$\varphi_x^* = \varphi^*\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}.$$

Therefore, $\beta\varphi_{XI} > \varphi_x^*$ if and only if $\beta\varphi^*[2\beta^{\sigma-1} - (1+\tau^{1-\sigma})\alpha^{\sigma-1}]^{\frac{1}{1-\sigma}}\left(\frac{f_I - f_x}{f}\right)^{\frac{1}{\sigma-1}} > \varphi^*\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$, which is the same as $f_I > f_x\left[1 + 2\tau^{\sigma-1} - (1+\tau^{\sigma-1})\left(\frac{\alpha}{\beta}\right)^{\sigma-1}\right]$.

Notice that $\left[1 + 2\tau^{\sigma-1} - (1+\tau^{\sigma-1})\left(\frac{\alpha}{\beta}\right)^{\sigma-1}\right]$ is decreasing is $\left(\frac{\alpha}{\beta}\right)$ and equals to $\tau^{\sigma\text{-}1}$ if $\left(\frac{\alpha}{\beta}\right) = 1$. As a result, under the conditions $f_I > \tau^{\sigma-1} f_x$ and $\beta > \alpha$, the

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inequality $\beta\varphi_{XI} > \varphi_x^*$ does not necessarily hold.

Appendix 6.

The equilibrium cutoff φ^* is determined by the corresponding free entry condition (FE curve) and the zero cutoff point condition (ZCP curve) in each case. To determine the cutoff we need to consider all firms, including the non-exporters, as we did in our learning-by-exporting paper. As such, we can obtain the FE and the ZCP curves by modifying the two curves in our learning-by-exporting model. We shall use the symbols in our learning-by-exporting model and discuss the determination of φ^* in the case $\alpha > \beta$.

First, if $\varphi^* > \varphi_c$ (φ_c is the original productivity for all firms when they enter the domestic market), a firm can survive only if it exports or makes FDI to at least one country to acquire a productivity growth. Let the least productivity level for a firm to make zero combined profit of domestic sales and export (FDI) sales be φ_Z (φ_Z'), then the least productivity for a firm to survive would be $\min\{\varphi_Z, \varphi_Z'\}$. Thus we have:

$$\pi^x(\varphi_Z) = \pi_d(\alpha\varphi_Z) + \pi_x(\alpha\varphi_Z) = 0,$$
$$\pi^I(\varphi_Z') = \pi_d(\beta\varphi_Z') + \pi_I(\beta\varphi_Z') = 0.$$

Since $\pi_d(\alpha\varphi_Z) > \pi_x(\alpha\varphi_Z)$ and $\pi_d(\beta\varphi_Z') > \pi_I(\beta\varphi_Z')$, we have $\pi_x(\alpha\varphi_Z) < 0$ and $\pi_I(\beta\varphi_Z') < 0$. The results imply $\alpha\varphi_Z < \varphi_x^*$ and $\beta\varphi_Z' < \varphi_I^*$.

Combining the results with the cutoffs in the case $\alpha > \beta$, we have:

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$\alpha\varphi_Z < \varphi_x^* < \alpha\varphi_{XI}$,

$\beta\varphi_Z' < \varphi_I^* < \beta\varphi_{XI}$.

Therefore, both φ_Z and φ_Z' are smaller than φ_{XI}. According to the definition of φ_{XI} and the assumption $\frac{\beta}{\alpha} > \left[\frac{1+\tau^{1\text{-}\sigma}}{2}\right]^{\frac{1}{\sigma\text{-}1}}$, we have $\pi^x(\varphi_Z) > \pi^I(\varphi_Z)$ as $\varphi_Z < \varphi_{XI}$. Since $\pi^x(\varphi_Z) = 0$, we have $\pi^I(\varphi_Z) < 0$, which implies $\varphi_Z' > \varphi_Z$ ($\because \pi^I(\varphi_Z') = 0$).

The above results also imply that the least productivity for a firm to successfully survive if $\varphi^* > \varphi_c$ and $\alpha > \beta$ is the productivity level φ_Z such that $\pi^x(\varphi_Z) = \pi_d(\alpha\varphi_Z) + \pi_x(\alpha\varphi_Z) = 0$. Notice that $\alpha\varphi_Z$ is identical to the cutoff $\varphi_x^{s\prime}$ in our learning-by-exporting model, which gives us $\varphi_Z = \frac{\varphi_x^{s\prime}}{\alpha}$. The probability for a firm to survive if $\varphi^* > \varphi_c$ is $1 - \mathrm{G}(\frac{\varphi_x^{s\prime}}{\alpha})$, which is higher than what we have in our learning-by-exporting model. This is due to a larger productivity growth on export ($\alpha > 1$) compared with the learning-by-exporting model ($\alpha = 1$).

We now modify the FE and ZCP curves in our learning-by-exporting model to get FE and ZCP we wanted here. There are two major things we need to modify: one is the probability for a firm to acquire a productivity that is higher than a specific level; the other is the measurement of the average productivity $\tilde{\varphi}(\varphi)$.

Since export induces a productivity into the level $\alpha\varphi$ if a firm has drawn a productivity equal to φ, the probability that a firm who acquiring a productivity that is higher than a specific level φ through export is $1 - \mathrm{G}(\frac{\varphi}{\alpha})$. The average productivity for firms that have a productivity greater than a specific level φ also changes. To be

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sure, the least productivity drawn that makes the productivity level φ is $\frac{\varphi}{\alpha}$, since, after export, the productivity of a firm become α times the productivity it has drawn. As a result, the average productivity for all firms that have a productivity greater than a specific level φ^* is modified to be:

$$\tilde{\varphi}\left(\frac{\varphi^*}{\alpha}\right) = \left[\frac{1}{1-G\left(\frac{\varphi^*}{\alpha}\right)}\int_{\frac{\varphi^*}{\alpha}}^{\infty}(\alpha\varphi)^{\sigma-1}\, g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}.$$

With the two modifications and $\alpha > \beta$, we can write the FE and ZCP curves that characterize the equilibrium. Using the symbols in our learning-by-exporting paper, we have:

The FE curve:

$$\overline{\pi} = \frac{\delta f_e}{1-G(\frac{\varphi_x^{s\prime}}{\alpha})} \text{ if } \varphi^* > \varphi_c,$$

$$\overline{\pi} = \delta f_e \text{ if } \varphi^* \leq \varphi_c.$$

The ZCP curve:

$$\overline{\pi} = \bar{\pi}_{1,3} \text{ if } \varphi^* > \varphi_c,$$

$$\overline{\pi} = \bar{\pi}_2 \text{ if } \varphi^* \leq \varphi_c,$$

where:

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$$\bar{\pi}_{1,3} = \pi_d\left(\tilde{\varphi}\left(\frac{\varphi_x^{S\prime}}{\alpha}\right)\right) + \frac{G\left(\frac{\varphi_x^*}{\alpha}\right) - G\left(\frac{\varphi_x^{S\prime}}{\alpha}\right)}{1 - G\left(\frac{\varphi_x^{S\prime}}{\alpha}\right)}\pi_{x1}\left(\tilde{\varphi}\left(\frac{\varphi_x^{S\prime}}{\alpha}\right)\right) + \frac{G\left(\frac{\varphi'}{\alpha}\right) - G\left(\frac{\varphi_x^*}{\alpha}\right)}{1 - G\left(\frac{\varphi_x^{S\prime}}{\alpha}\right)} n\pi_{xn}\left(\tilde{\varphi}\left(\frac{\varphi_x^*}{\alpha}\right)\right) +$$

$$\frac{1 - G\left(\frac{\varphi'}{\alpha}\right)}{1 - G\left(\frac{\varphi_x^{S\prime}}{\alpha}\right)}\left[\pi_x\left(\tilde{\varphi}\left(\frac{\varphi'}{\alpha}\right)\right) + (n-1)\pi_I\left(\tilde{\varphi}\left(\frac{\varphi'}{\alpha}\right)\right)\right] \text{ , and}$$

$$\bar{\pi}_2 = \pi_d(\tilde{\varphi}(\varphi_c)) + \left[G\left(\frac{\varphi_x^*}{\alpha}\right) - G\left(\frac{\varphi_x^S}{\alpha}\right)\right]\pi_{x1}\left(\tilde{\varphi}\left(\frac{\varphi_x^S}{\alpha}\right)\right) + \left[G\left(\frac{\varphi'}{\alpha}\right) - \right.$$

$$\left. G\left(\frac{\varphi_x^*}{\alpha}\right)\right] n\pi_{xn}\left(\tilde{\varphi}\left(\frac{\varphi_x^*}{\alpha}\right)\right) + \left[1 - G\left(\frac{\varphi'}{\alpha}\right)\right]\left[\pi_x\left(\tilde{\varphi}\left(\frac{\varphi'}{\alpha}\right)\right) + (n-1)\pi_I\left(\tilde{\varphi}\left(\frac{\varphi'}{\alpha}\right)\right)\right] .$$

$\tilde{\varphi}(\varphi_c)$ in $\bar{\pi}_2$ is the average productivity for all surviving firms in the case $\varphi^* \leq \varphi_c$, including non-exporters with the original productivity level φ_c and a group of firms that serve the foreign markets. $\tilde{\varphi}(\varphi_c)$ is equal to:

$$\tilde{\varphi}(\varphi_c) = \left\{G\left(\frac{\varphi_x^S}{\alpha}\right)\varphi_c^{\sigma-1} + \left[1 - G\left(\frac{\varphi_x^S}{\alpha}\right)\right]\frac{1}{1 - G\left(\frac{\varphi_x^S}{\alpha}\right)}\left[\int_{\frac{\varphi_x^S}{\alpha}}^{\infty}(\alpha\varphi)^{\sigma-1} g(\varphi)d\varphi\right]\right\}^{\frac{1}{\sigma-1}} =$$

$$\left\{G\left(\frac{\varphi_x^S}{\alpha}\right)\varphi_c^{\sigma-1} + \left[1 - G\left(\frac{\varphi_x^S}{\alpha}\right)\right]\tilde{\varphi}^{\sigma-1}\left(\frac{\varphi_x^S}{\alpha}\right)\right\}^{\frac{1}{\sigma-1}}.$$

$\pi_{x1}(\tilde{\varphi})$ is the average profit earning from export sales for all export firms that export to only one country, and $\pi_{xn}(\tilde{\varphi})$ is the average profit earning from export sales for all firms whose best strategy is to export to n countries. $\pi_x(\tilde{\varphi}) + (n-1)\pi_I(\tilde{\varphi})$ represents the average profit earning from export sales to one country and FDI to $(n-1)$ country. These firms have the highest productivity, which is greater than φ'.

If we compare the FE and the ZCP curves with our learning-by-exporting model, it is obvious the FE curve shifts downward in the portion where $\varphi^* > \varphi_c$. The change in the ZCP is influenced by the productivity distribution. If the distribution follows

DOI:10.6814/NCCU202100569

Pareto distribution, $G(\varphi) = 1 - \left(\frac{b}{\varphi}\right)^k$, with the shape parameter $k > \sigma - 1$, then the average productivity measure $\tilde{\varphi}\left(\frac{\varphi^*}{\alpha}\right)$ for all firms that have productivity greater or equal to a specific productivity level φ^* is same as the average productivity measure $\tilde{\varphi}(\varphi^*)$ in the learning-by-exporting model. That is:

$$\tilde{\varphi}\left(\frac{\varphi^*}{\alpha}\right) = \left[\frac{1}{1-G\left(\frac{\varphi^*}{\alpha}\right)}\int_{\frac{\varphi^*}{\alpha}}^{\infty}(\alpha\varphi)^{\sigma-1}\, g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} = \left(\frac{k}{1+k-\sigma}\right)^{\frac{1}{\sigma-1}}\varphi^*,$$

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty}(\varphi)^{\sigma-1}\, g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} = \left(\frac{k}{1+k-\sigma}\right)^{\frac{1}{\sigma-1}}\varphi^*.$$

Consequently, compared with the ZCP curve in the learning-by-exporting model, both $\bar{\pi}_{1,3}$ and $\bar{\pi}_2$ shift upward.

References

Clausing, K. A., (2000). "Does Multinational Activity Displace Trade?" *Economy Inquiry*, 38(2), 190-205.

Conconi, P., Sapir A, and Zanardi M., (2016). "The internationalization process of firms: from exports to FDI." *Journal of International Economics*, 99, 16-30.

Head, K. and J. Ries, (2001). "Overseas Investment and Firms Exports." *Review of International Economics*, 9(1), 108-122.

Head, K. and J. Ries, (2004). "Exporting and FDI as alternative strategies." *Oxford*

DOI:10.6814/NCCU202100569

Review of Economic Policy, 20, 409-423

Helpman, E., Melitz, M. J., and Yeaple, S. R., (2004). "Export Versus FDI with Heterogeneous Firms." *American Economic Review*, 94(1), 300-316.

Lipsey, R. E. and M. Y. Weiss, (1981). "Foreign Production and Exports in manufacturing industries." *The Review of Economics and Statistics*, 63(4), 488-494.

Lipsey, R. E. and M. Y. Weiss, (1984). "Foreign Production and Exports of Individual Firms." *The Review of Economics and Statistics*, 66(2), 304-308.

Melitz, M. J., (2003). "The Impact of Trade on Intra-Industry and Aggregate Industry Productivity." *Econometrica*, 71, 1695-1725.

Rob, R. and N. Vettas, (2003). "Foreign direct investment and exports with growing demand." *Review of Economics Studies*, 70, 629–648.

DOI:10.6814/NCCU202100569

Chapter 5 Conclusions

This thesis investigates three topics in international trade from chapter 2 to chapter 4. In chapter 2, we endogenize the competition mode in the Brander-Spencer strategic trade policy model, attempting to find the optimal policy of a government when firms can react to its action by choosing their strategic variables. We conclude that both firms will choose to compete in quantities, the optimal policy of the government is to subsidize its export firm. The policy implication of the model greatly strengthens the result obtained in Brander and Spencer (1985), since the firms will engage in Cournot competition.

We construct a model to study the phenomenon of learning-by-exporting in chapter 3, based on Melitz (2003) framework. The model gives us two salient features that cannot be observed in the self-selection structure. First, compared with no learning effect, a firm is more willing to export if export can increase its productivity. This is due to the fact that a productivity improvement also affects the firm's domestic sales profit. As a result, different from the self-selection structure, the decision of export is not to consider whether the export profit is positive or not, but to compare the combined profit of domestic and export sales after export with the profit when sells domestically only. In our model, some firms export even they have negative profit from export. This implies that some firms might choose to serve the foreign market even if it seems not profitable under existing condition. They expect that there might be a productivity improvement due to the learning effect from exporting, and thus increase profit in the future. Second, there is a striking productivity gap between the exporters and the non-exporters. That is, the least productive exporter is still sufficiently better than the best

DOI:10.6814/NCCU202100569

non-exporters. In other words, there is a productivity region between the least productive exporter and the non-exporters with no firms in it. This result supports the empirical findings that export firms are better than their non-export counterpart. With the learning effect, the productivity difference between the exporters and the non-exporters is widened further.

Chapter 4 explores the relationship between the decision making of export and foreign direct investment (FDI). We extend the model in chapter 3, allowing firms to serve foreign customers through export or making FDI. All countries are identical in countries size and trade cost. The most critical feature is that we assume both export and FDI can upgrade a firm's productivity. The magnitude of the productivity growth induced by the two activities can be symmetric or asymmetric. We obtain the result that a weakly complementary relationship between export and FDI may appear in the asymmetric growth cases. The reason is similar to the case that a firm may decide to export with negative export profit in chapter 3. In the asymmetric growth cases, a firm is likely to export (make FDI) to one country even though making FDI (export) generates a higher profit to an additional country. That is because a firm can sacrifice some of the profit in an attempt to acquire higher productivity, which then influences the firm's combined domestic and foreign sales profit. The result provides one possible explanation for the strong empirical evidence that the two activities play a complementary role.

Although we have obtained various interesting results in the three chapters, the models still exist some limitations due to the simplified assumptions. Relaxing these assumptions could be the natural direction for future research. For instance, following Brander and Spencer (1985) the model in chapter 2 assumes there are only two firms

DOI:10.6814/NCCU202100569

(one domestic and one foreign firm) competing in the third market. Furthermore, only the home government takes action to affect its export firm's profit. As such, the direction of the future research could be to relax these assumptions by allowing multiple domestic and foreign firms to compete in the third market, or allowing the foreign government to take action in the first stage. In chapter 3, we assume the productivity for an export firm changes immediately after the firm exports. This ignores the time dynamics and the probability of failure for a productivity growth. This in turn prevents us from discussing how the risk of failure in productivity growth affects a firm's export willingness. In chapter 4, we assume that the magnitude of productivity upgrade induced by export or FDI does not depend on the number of countries. In other words, export (or making FDI) to one country gives the same level of productivity improvement as export (or making FDI) to numerous countries. This assumption prevents us from discussing the optimal export (FDI) countries. A possible way to relax this assumption is to allow the foreign countries to be different (for example, south and north countries) and let the magnitude of the productivity growth depend on the destination country. This seems to be a promising direction to pursue in the future research.

References

Brander, J.A. and B.J. Spencer, (1985). "Export Subsidies and International Market Share Rivalry", *Journal of International Economics* 18: 83-100.

Melitz, M. J., (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 71, 1695-1725.

DOI:10.6814/NCCU202100569