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## An integrated forecasting model for the coffee bean supply chain

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### ABSTRACT

Coffee is the most traded commodity after petroleum. The Vietnamese coffee bean industry has raised concerns lately over an inefficient coffee value chain; bets on coffee price uncertainty are increasing worldwide in the current. Accurate optimization of coffee bean prices helps manufacturers to control an unpredictable market and upgrade cooperativeness in sustainable agriculture. The authors proposed a forecasting method to deal with demand volatility and uncertainty in volumes and coffee bean prices. In this paper, we applied the forecasting nonlinear grey Bernoulli model (NGBM) (1,1). NGBM (1,1), which is based on the parameter optimization algorithm, can increase the precision of predictions. NGBM (1,1) was integrated with Fourier residual modification model to forecast coffee bean price, which was a crucial factor in the Vietnamese coffee bean supply chain. The price of coffee beans was calculated using a differential equation in an uncertain system, along with actual data collected over the past six years. The results of this study demonstrate that an integrated forecasting model is an effective forecasting method. This research can help companies to control risks that come with uncertain coffee prices and reduce risks in the sustainable agriculture supply chain.

### KEYWORDS

Optimization; forecasting Model; supply Chain Management; coffee Bean

### 1. Introduction

As with supply chain management for manufacturers, agri-food supply chain management involves integration and collaboration among farmers, agricultural agents, manufacturers, distributors, and retailers to meet consumer needs and expectations for quantity, quality, and price. Agri-food supply chains are economic systems that distribute benefits and apportion risks among participants. One of the food industry's core competencies is selecting appropriate suppliers, which provide quality agricultural products for food-processing activities.

Mussatto et al. (2011) and Feria-Morales (2002) revealed that coffee is one of most popular beverages in the world; and is also the second-most traded commercial goods behind petroleum. The coffee industry is expected to face severe challenges in the decades to come because of uncertainty in commodity prices. In the agricultural supply chain, management always wants to reduce risks, as there is uncertainty in the fluctuation of prices in the market. The extreme susceptibility to weather

variability in the agricultural production process is stochastic and not under farmers' control. In the meantime, improvements in modelling price uncertainty in agricultural supply chains have been emphasized, and most of the effort invested consisted of predicting future trends through effective grey forecasting based on past and present data. One model has observed an extremely unfavourable trend in the future distribution of indigenous Arabica coffee due to accelerated global climate change.

In the 1990s, the French brought coffee seeds to Vietnam, which eventually became the world's second-largest exporter of coffee, supplying approximately one-quarter of the United Kingdom's coffee. Coffee contributes to Vietnam's gross domestic product (GDP), creates jobs, and stimulates socio-economic development. There was a disadvantage arrangement that uncertainty fluctuations in the market price of coffee beans are negative to the farmers' benefit of growing coffee beans. Thus, focusing on arrangements to promote international trade in Vietnamese coffee beans

through price forecasting is important in helping Vietnam maintain a good strategy in the global market for coffee.

### **The coffee industry in Vietnam**

Feria-Morales (2002) claimed that Vietnam, one of the top-10 coffee-producing countries, provides high-quality green beans used as seeds for propagation. According to Franca et al. (2005), the climate, soil, cultivation practices, and irrigation systems of specific locations play essential roles in developing good quality coffee.

Agricultural commodity booms have challenged the coffee economies in Vietnam and other developed countries. Moving from a planned economy to a market-oriented one, Vietnam has become one of the fastest-growing economies in Asia from exporting coffee bean, just like Brazil in South America. Vietnamese coffee has risen to the top of the world's coffee manufacturers in world-record time, which has been affirmed by equally fast switches policies to create a suitable market network. However, the Vietnamese coffee industry faces problems of productivity, quality, and price, thus affecting the sustainable development of the industry. Vietnam coffee is the main crop that brings the export value of about 3 USD billion dollar per year. However, Vietnam only gets a low profit percentage because Vietnam always exports raw coffee bean. Following the Vneconomy website by coffee exports, in August 2018 Vietnamese coffee bean was estimated to reach 143 thousand tons with a value of 260 USD million dollar. Accumulated coffee export in the first eight months of 2018 is estimated at 2.5 USD billion dollar, up by 15.5% in volume. However, this rate decrease by 2.5% in value over the same period in 2017. The Vietnamese coffee industry still lacks reputable brands and has a few advertising marketing activities. Therefore, Vietnam still has to export through the third market, which means export prices face uncertainty.

The export price of Vietnamese coffee is determined based on the international coffee prices; thus, the domestic coffee industry is fundamentally insecure and unstable. Furthermore, quality improvement in Vietnam's coffee beans was slow due to the lack of standardized procedures. As

a result of these two weaknesses, Vietnamese coffee's export price has always been lower than the same product from other countries, and the price has remained uncertain for a long time. According to Asche, Jaffry, and Hartmann (2007) and Feng and Tan (2019), price always an important criterion when selecting a vendor in a supply chain. García-Martos, Caro, and Jesús Sánchez (2015) claimed that price forecasting was an interesting problem in profit maximization strategy. Li et al. (2018) indicated that the optimization of price uncertainty problems with small samples and insufficient information significantly improved the performance models' forecasting. In this research, to forecast coffee beans' future performance, we developed the NGBM (1,1) and adopted the Fourier series model to forecast coffee bean prices and make coffee bean selection less risk-averse.

## **II. Literature review**

In agri-food supply chains, an organization's behaviour involves complex decision-making from the customer in complex networks consisting of small- and medium-sized enterprises. The agri-food supply chain became part of interactions with multinational companies providing input to the supply chain and engaging in its retail services. Leroy et al. (2006) showed that high consumer demand requires speciality coffee markets. Consumers paid the premium price required for special coffee that retains the original quality. Therefore, forecasting further system improvement from previously observed data seems to be a practical alternative. With worldwide overproduction, this study considers coffee bean prices in the future. Specific system development laws based on real data must be discovered to obtain a reliable forecast. These laws would reduce the risk of market price volatility in the future.

### **Grey theory**

The grey forecasting model (GM) originated by Deng (1989) focuses on solving problems using small sample sizes and insufficient information. The conventional GM is based on knowledge of the least square method and first-order linear ordinary differential equations. The GM is

principally suited for handling situations in which only limited data are convenient for forecasting, and the system follows the time series data. In recent years, grey systems theory has been successfully employed in the fields of agriculture, industrial textiles, and industrial transportation. The GM (1,N) is the main model in grey forecasting, Kayacan, Ulutas, and Kaynak (2010) in grey systems theory, GM (n,m) denotes a grey model, where n is the order of the ordinary differential equations and m is the number of Grey variables. Tien (2012) suggested that the various predictions of the existing model GM (1,n) can be an unexpected and large sample, which is also a strength of the basic grey methodology. The prediction accuracy can be handled in situations, which include uncertain and insufficient information. The differential equations of the GM (1, 1) model have time-varying coefficients.

Shrivastava and Panigrahi (2014) showed that forecasting models are evaluated based on multiple accuracy measures to reduce price risk. Coffee price forecasting is one of the overwhelming threats to market participants, managers, and farmers due to the high volatility of coffee bean prices. Hovelaque, Duvaleix-Tréguer, and Cordier (2009) argued that the sustained competitiveness of Vietnam's coffee bean industry depends on its ability to find stable and sustainable solutions to the imminent crisis in Vietnam's coffee sector. This research aimed to integrate the combined non-linear grey Bernoulli model – (NGBM) (1,1) and Fourier series to forecast coffee bean prices in the future.

#### ***Nonlinear grey Bernoulli model (1,1)***

In 2008, Chen (2008) re-examined an example in Deng's edition, and after applying the NGBM to the analytical results, demonstrated that it effectively enhanced the precision of the methodology. The nonlinearity of natural phenomena increases NGBM's precision. The traditional Grey Model was easy to understand and simple to calculate, with satisfactory accuracy. In case, if the data to adjust the model have large variability, that acquire will not get higher forecasting precision. This investigation examines the improved grey model and the knowledge of the elementary ordinary nonlinear differential equations. Although those

studies applied NGBM (1, 1), which has provided promising results, NGBM (1, 1) was sometimes unsuitable in some extraordinary situation example: trending of business or the result in the future. The NGBM was a novel grey forecasting model that integrates Bernoulli differential equations suitable for forecasting various developing trend curves in coffee prices. According to Hsu (2010), and Zhou et al. (2009), in the forecasting model, the short-term forecasting is within 1–2 years, medium-term is from 3 to 5 years, and the period of more than 5 years is considered to be long time. In this paper, the NGBM is applied to medium- and long-term coffee bean forecasting and an algorithm to optimize the parameters of the NGBM is proposed.

Shaik and Walid (2011) showed that by optimizing techniques effectively, the estimation of NGBM, could enhance the accuracy of the prediction model. Although those studies applied NGBM (1, 1), which has provided promising results, NGBM (1, 1) was sometimes unsuitable in some particular situations. The authors theorize that if the data fluctuated wildly or included lots of noise, the data forecasted by NGBM (1,1) would also fluctuate wildly. To deal with this problem, this research adopts the improvements of NGBM (1, 1) and integrated Fourier series to build an effective model with higher predictive accuracy. This methodology, Providing strategy value to boost the distribution and consumption for optimization coffee bean value chain system and contributing more sustainability.

#### ***Fourier series***

In the model, non-linear relations González, Jaramillo, and Carmona. (2008) adopted the Fourier series aiming to be used to forecast volatility values for which the series presents an apparent annual behaviour of the spectrum plot of the fluctuation series with high-precision. The Fourier series was developed by Brigham and Brigham (1988) to enhance forecasting accuracy when using highly fluctuating data sets. According to Huang and Lee (2011) a Fourier series where the fundamental frequency was assumed to be a modification model can refine the forecasting effectiveness of stochastic volatility data and time series. Fourier series are also adaptive models to refine the forecasting effectiveness of random and uncontrolled data, which

describes the trends and fluctuations. The parameter estimation of a Fourier series is formulated as one of the combinatorial optimization problems. A Fourier series was the model parameters by optimization techniques that can effectively intensify the model's precision level. González, Jaramillo, and Carmona. (2008) suggested that the trend of a Fourier series, predicted with maintenance planning and market research in the optimized model, could enhance the prediction's accuracy. A Fourier series was also more accurate at demand forecasting than using maintenance planning and market research to predict fluctuations. Fourier Series showed the effects of data eventuality, or centre-symmetry situation could be adjusted to fit the result of one-time accumulated generating operation (1-AGO) of raw data. However, mathematics in Fourier Series precision can be increased owing to the nonlinearity of natural phenomena the authors adapted. The non-linear Grey Bernoulli Model is used to adjust the model to acquire higher forecasting precision. Abate and Whitt (1992) found that many methods have been proposed to increase accuracy by modelling the residuals of the original grey prediction. In fact, the residual diagnosis was a commonly used correction approach for time series prediction. One of those residual correction approaches was to use the Fourier series. Besides, Odan and Reis (2012), the linear forecasting methods were widely used because they were easy to develop and implement, in addition to being simple to understand and interpret. However, in the real world, data have varied degrees of nonlinearity. The theory of combination forecasting and the skill of modelling are practicable in a complex economic system with uncompleted information. Because energy consumption system was complexity and non-linear. Thus, forecasting the uncertainty and stochastic, the authors adopted three models to determine which was the best methodology in this research to predict fluctuations in coffee bean price.

### III. Methodology

Forecasting techniques have developed rapidly, thus becoming superior to conventional statistical models. González (2008) determined, after

researching the Fourier series, that the analytical results demonstrated that applying a Fourier series effectively enhanced the forecast's accuracy. As previously mentioned, the mathematics in a Fourier series becomes increasingly sophisticated, making the series difficult to understand and apply. The forecasting results in a wide amplitude fit with a Fourier series. Thus, this study demonstrated how to improve the Fourier series model by showing the error indices of fluctuation forecasting obtained by prior validation. In this paper, studies followed the isomorphism from the original equation and were then combined with the methodology of elementary ordinary nonlinear differential equations. To prove this novel approach works and is effective, this research has utilized the genetic algorithm for the optimization problem and then constructed a Fourier-series-based NGBM. To improve the Fourier series model, we verified the effectiveness of the proposed model. We proposed a research framework Figure 1 consisting of six steps, as follows:

*Step 1:* Collect coffee bean price data. Historical coffee bean price data were collected from the website Giacaphe (2019) from 2014 to 2019. The forecasting techniques available have increased both in number and complexity. The purchasing

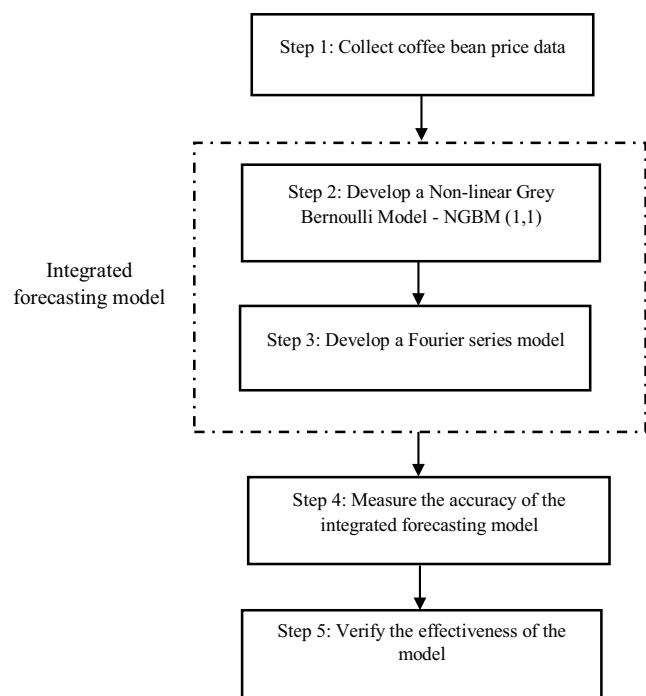


Figure 1. Research framework.

department in manufacturing paid attention to forecasting future tendencies of some events, such as investment in the market, which must be forecasted to obtain higher profits and reduce the investment risk for Vietnam’s coffee prices.

*Step 2:* Develop an NGBM (1,1) model that improved the forecasting effectiveness of the stochastic fluctuation data, boosted an effective methodology for forecasting Vietnam coffee beans price – designing the fluctuation of six-year forecasting in cases involving samples with a large change in data forecasting.

*Step 3:* Develop a Fourier series forecasting model for coffee bean prices. Every simulation result determined that the suggested model offered a more accurate forecast than several other types of forecasting models. Hence, this research improved the Fourier series based on NGBM (1,1) models to refine the forecasting effectiveness of stochastic volatility data, thus providing an effective method for forecasting the price of Vietnam coffee beans and further improving the accuracy of forecasting that can be applied for cases involving sample data with large fluctuations.

*Step 4:* Measure the integrated forecasting model’s accuracy to judge its effectiveness at integrating the NGBM and Fourier series models. In this step, we hypothesize that integrating the Fourier series builds an effective model that increases its predictive accuracy.

*Step 5:* Concerning the evaluation performance of the forecasting volatility model, based on verifying the results of predicted values that match the actual values, in this step, this study adopted mean absolute percentage error (MAPE) approaches to evaluate the forecasting performance model GM (1,1), NGBM (1, 1), and Fourier series models. Model characteristics included periodicity, randomness, and tendency.

**Grey forecasting model**

The progress of construction GM (1,1) model is described below:

**Theorem:** Let  $X^{(0)}$ ,  $X^{(1)}$ , and  $Z^{(1)}$  be the original sequence data except that  $X^{(0)}$  is non-negative. If  $\hat{a} = (a, b)^T$  was a parameter sequence and

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad (1)$$

The least squares estimated sequence of the GM (1,1) model in equation. (2) satisfies the condition  $\hat{a} = (B^T B)^{-1} B^T Y$ . With all notations adopted from Theorem (3), if  $[a, b]^T = (B^T B)^{-1} B^T Y$ , then  $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$  was a whitening equation of the GM (1,1) model in equation (4). The time response sequence of the GM (1,1) model in equation. (5) was given below:

$$\hat{x}^{(1)}(k + 1) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}, k = 1, 2, \dots, n \quad (2)$$

The parameters  $(-a)$  and  $b$  of the GM (1,1) model accumulated the development coefficient and calculated by the ordinary least squares method. The former reflected the development states of  $\hat{x}^{(1)}$  and  $\hat{x}^{(0)}$ . According to Cui et al. (2013) the variables denoted by the system of interest external or pre-defined. The GM (1,1) model was the system whitened by applying the inverse accumulated generating operation (IAGO) technique for parameter variance, rather than a first order differential equation.

**Nonlinear grey Bernoulli model – (NGBM (1,1))**

According to Chen, Chen, and Chen (2008) designed the Nonlinear Grey Bernoulli Model – NGBM (1,1) model was as follow:

*Step 1: Theorem:* Let’s matrix  $X^{(0)}$  stand for the nonnegative original history of time series data  $X^{(0)} = \{x^{(0)}(t_i)\}$ ,  $i = 1, 2, \dots, m$ , where  $x^{(0)}(t_i)$  corresponds to the system output at a time  $t_i$  and  $m$  is the total number of the accurate data sequence.

*Step 2:* A new sequence  $X^{(1)}$  could be approximately rewritten as the following one-time accumulated generating operation (1-AGO), which is:

$$X^{(1)} = \{x^{(1)}(t_i)\}, \quad \text{where} \\ x^{(1)}(t_i) = \sum_{i=1}^k x^{(0)}(t_i) \text{ Where } t = 1, 2, 3, \dots, i$$

*Step 3:* In NGBM (1,1) methodology the yields of the Grey differential equation showed:

$$x^{(0)}(t_k) + az^{(z)}(t_k) = b[z^{(1)}(t_k)]^n \tag{3}$$

The first-order accumulate progression satisfies as follow difference equation:

$$\frac{dx^{(1)}(t_k)}{dt_k} + ax^{(1)}(t_k) = b[x^{(1)}(t_k)]^n \tag{4}$$

where

$$z^{(1)}(t_k) = px^{(1)}(t_k) + (1 - p)x^{(1)}(t_{k-1}), k = 2, 3, \dots, m, \tag{5}$$

$p$  was denoted the value of production coefficient with a closed the background value of  $[0, 1]$ , which traditionally set balance as 0.5. To demonstrate the expressions regarding parameters  $a$  and  $b$  in the NGBM (1,1) model. In this case study, the power  $n$  is used to be the adjustable parameter that the real numbers represent and serve as an adjustable parameter in terms of a matrix. Based on the previous research, if the parameter has  $n = 0.0381$ , NGBM (1,1) will get excellent flexibility.

*Step 4:* Chen, Chen, and Chen (2008) believed that the accuracy forecasting model was directly stochastic volatility by the parameter  $p$  in the background value. The accuracy values of this parameter could be calculated by applying a specific simultaneous optimization. However, in this research, equation (4), depending on the value of parameters  $a$  and  $b$  could be calculated by adapting the ordinary least squares method (OLS):

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_m \tag{6}$$

where

$$Y = \begin{bmatrix} x^{(0)}(t_2) \\ x^{(0)}(t_3) \\ \vdots \\ x^{(0)}(t_m) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(t_2) & (z^{(1)}(t_2))^n \\ -z^{(1)}(t_3) & (z^{(1)}(t_3))^n \\ \vdots & \vdots \\ -z^{(1)}(t_m) & (z^{(1)}(t_m))^n \end{bmatrix} \tag{7}$$

In equations (8) and (9), Zheng et al. (2011) believed that model accuracy was directly influenced by the optimal solution of parameters  $p$  in the background power, given that the background value was expressed. Adopting the actual forecasting example of coffee bean price rates to

demonstrate the improvement in the accuracy of NGBM (1,1), and serves as an adjustable parameter detail below:

*Step 5:* From the solution of equation (4) could be obtained after the parameters  $a$  and  $b$  are predicted value. Such as:

When  $n \neq 1, k = 1, 2, 3, \dots$

*Step 6:* The response function of inverse accumulated generating operation (I-AGO) to  $\hat{x}^{(1)}(t_k)$ , the estimated the coffee bean price of  $\hat{x}^{(0)}(t_k)$  could be forecasted as follows:

### Development Fourier series modification by NGBM (1,1)

Adapting González, Jaramillo, and Carmona. (2008) separated the idea of Fourier series systems and those of forecasting boxes as an important test.

*Theorem: Step 1:* Assumed that  $x$  be the initial values of  $m$  entries, and  $v$  is the predicted value obtaining from NGBM (1,1). The residual series named  $\varepsilon$  which could be written as:

$$\varepsilon = \{e(k)\}, k = 2, 3, \dots, m$$

where

$$e(k) = x(k) - v(k) k = 2, 3, \dots, m \tag{12}$$

*Step 2:* With the same definitions of a Fourier series development by the residual sequence of NGBM (1,1) expressed as follow the equation  $\hat{\varepsilon}(k)$  approximately:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_{(0)} + \sum_{i=1}^z \left[ \left[ a_i \cos\left(\frac{2\pi i}{m-1}(k)\right) \right] + b_i \sin\left(\frac{2\pi i}{m-1}(k)\right) \right] \tag{13}$$

$$k = 1, 2, 3, \dots, m$$

Where

$$Z = \frac{(m-1)}{2} - 1 \tag{14}$$

*Step 3:* In this step, the minimum deployment frequency of the Fourier series, and  $Z$  only take an integer value. Therefore, according to González, Jaramillo, and Carmona. (2008), the residual series was rewritten as

$$\varepsilon = PC \tag{15}$$

where

$$C = [a_0, a_1, b_1, a_2, b_2, \dots, a_z, b_z]^T \tag{17}$$

The parameters  $a_0, a_1, b_1, a_2, b_2, \dots, a_z, b_z$  were obtained by using the OLS method from equation (8) and (9), the results of which followed equation:

$$C = (P^T P)^{-1} P^T \varepsilon^T \tag{18}$$

Combining the expressions of parameters  $a$  and  $b$  to improve the Fourier series model. After that, the optimal solution of parameters  $\hat{\varepsilon}(k)$  could be obtained by minimizing the in-sample model of forecasting error using a Fourier series. This step combined the expressions of parameters  $a$  and  $b$  of Zheng (2012) developed the Fourier series model that corresponds to the smallest in-sample forecasting error.

*Step 4:* Once the parameters were estimated, the development of Fourier series mathematics then calculated as the following expression:

$$\hat{\varepsilon}(k) = \frac{1}{2} a_0 + \sum_{i=1}^z \left[ a_i \cos\left(\frac{2\pi i}{m-1}(k)\right) + b_i \sin\left(\frac{2\pi i}{m-1}(k)\right) \right] \tag{19}$$

*Step 5:* From the predicted mathematics series  $v$ , and  $\hat{\varepsilon}$ , the Fourier series developed  $\hat{v}$  of series  $v$  was measured the price forecasting by

$$\hat{v} = \{\hat{v}_1, \hat{v}_2, \hat{v}_3, \dots, \hat{v}_k, \dots, \hat{v}_m\}$$

Where:

$$\hat{v} = \begin{cases} \hat{v}_1 = v_1 \\ \hat{v}_k = v_k + \hat{\varepsilon}_k \end{cases} \quad k = 2, 3, \dots, m \tag{20}$$

On the basis of the relation, this research establishes the optimized model and solve the optimization forecasting value, then given practical application.

### Evaluative precision of forecasting models

The mean absolute percentage error (MAPEs) was calculated to find the forecasting error. There were some common approaches to evaluate the price uncertainty for the forecasting model. The model characteristics include periodicity, randomness, and tendency. In addition to improving the background value using integration, this model has improved its accuracy by correcting the model's periodical errors. According to González, Jaramillo, and Carmona. (2008) the forecasting MAPEs of the fluctuation series were obtained with the validation data for different sizes of the data set used to fit the Forecasting model. According to Lewis (1982), forecasting accuracy could be measured by MAPE, defined as follows:

$$MAPE = \frac{1}{n} \sum_{k=2}^n \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\%$$

where  $x^{(0)}(k)$  was the actual value, and  $\hat{x}^{(0)}(k)$  the forecasted value, in time period  $k$ . In addition, Lewis (1982) divided the MAPE index into four grades in terms of the evaluative precision of the forecasting model (see Table 1), and added a source line below the table MAPE grade levels, as follows:

To ensure that no errors occurred during the calculation, it was important to avoid mathematical concerns over future values due to incomplete information when performing accurate forecasts. If the MAPE values were small or the predicted values closest to the actual values, the forecasting model would have the optimal result equation.

**Table 1.** Coffee bean prices (2014–2019).

Years	Lam Dong Price (USD/Ton)
2014	1725
2015	1540
2016	1637
2017	1637
2018	1651
2019	1683



#### IV. An illustrative example of the Vietnamese coffee supply chain

##### *Vietnamese coffee bean supply chain*

In 2017, Vietnam was selected to become one of the suppliers of Arabica coffee for the Starbucks coffee company worldwide. Starbucks's choice of Vietnam as a supplier proves that Vietnamese coffee will retain a high price in the future, which is both a challenge and a chance for Vietnam's coffee bean in the agribusiness supply chain. However, the food industry's sustained competitiveness will depend on the ability to estimate price uncertainties in the future, demonstrating permanent and sustainable solutions to address the imminent crisis in Vietnam's coffee industry. This paper primarily focuses on estimating unpredictable prices to help wholesale companies or agencies to control the input material's stability and risk allocation. Vietnam grows Arabica coffee in the Lam Dong, Dien Bien, Son La, Quang Tri, and Dak Nong provinces. Each region has its own unique coffee flavour. In this study, we created a model and applied it to a price-forecasting case study in the most famous coffee province in Vietnam: Lam Dong.

##### *Collect data of the coffee bean prices*

Based on the data collected from the website Giacaphe (2019) in Table 1- uncertain coffee prices obtained for the six years from 2014 to 2019 in Lam Dong, the research represents Vietnamese coffee beans. Procedures of the optimized NGBM (1,1) and Fourier series models were established by minimizing an objective function, with the constraints being  $0 \leq p \leq 1$  and  $n \neq 1$ , to obtain the global optimization of parameters  $p$  and  $n$  (just for NGBM (1,1) and Fourier Series). In the coffee industry, forecasting the future tendency of some events, such as investment in the market, must be forecasted to obtain higher profits and reduce the investment risk for Vietnam's coffee prices.

##### *Validation of the forecasting model*

In this section, three forecasting models were used to compare the proposed model with several

different kinds of NGBM (1,1) models, along with the optimized Fourier series, showed the effectiveness of the proposed model in high fluctuation data sets. This paper demonstrated the forecasting framework for coffee bean prices applied to the real case of Vietnam. The procedure for the terms of the proposed prediction model has two stages. The first stage established the NGBM (1,1) to roughly predict future data using a set of the most recent data. The second stage used a Fourier series to refine the residual error in the NGBM (1,1) more details in the next part.

##### *Grey forecasting model*

The sequence of raw data  $X^{(0)} = (1725.00, 1540.00, 1637.00, 1637.00, 1651.00, 1683.00)$  simulates this sequence  $X^{(0)}$  using the following three GM (1,1) models and comparing the accuracy of the simulation:

From Eq. (2)  $x^{(0)}(k) + az^{(1)}(k) = b$ , compute the accumulation generation of  $X^{(0)}$  as follows:

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), x^{(1)}(4), x^{(1)}(5)) = (3265.00, 4902.00, 6539.00, 8190.00, 9873.00).$$

We check the quasi-smoothness from  $\sigma^{(1)}(k) = \frac{x^{(0)}(k)}{x^{(1)}(k-1)}$ . It follows that  $\sigma^{(1)}(2) = 0.69$ ,  $\sigma^{(1)}(3) = 0.37$ ,  $\sigma^{(1)}(4) = 0.28$ , and  $\sigma^{(1)}(5) = 0.22$ ,  $\sigma^{(1)}(6) = 0.18$ . Therefore,  $k > 5$ ,  $\sigma^{(1)}(k) \in [0.1, 1]$  with  $\sigma = 0.90$ , that is, the law of quasi-exponentially and the condition of quasi-smoothness is stratified. Thus, we can establish a GM (1,1) model for  $X^{(1)}$ . Using the adjacent neighbours of sequence, let  $Z^{(1)} = (z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n))$  be the sequence generated from  $X^{(1)}$  by the adjacent neighbour means sequence  $Z^{(1)} = (x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), x^{(1)}(4), x^{(1)}(5)) = (2495, 4083.5, 5720.5, 7365, 9031.5)$ .

In addition, matrix  $B$  and constant vector  $Y_N$  are accumulated as follows:

$$B = \begin{bmatrix} -2595 & 1 \\ -4083.5 & 1 \\ -5720.5 & 1 \\ -7364.5 & 1 \\ -9031.5 & 1 \end{bmatrix} \quad Y_N = \begin{bmatrix} 1540 \\ 1637 \\ 1637 \\ 1651 \\ 1683 \end{bmatrix}$$

Using the least squares estimation, we obtain the

sequence of parameters  $\hat{a} = [a, b]^T$  as follows  $\hat{a} = (B^T B)^{-1} B^T Y = \begin{bmatrix} -0.0182 \\ 1524.7043 \end{bmatrix}$ . We establish the following model  $\frac{dx^{(1)}}{dt} - 0.0182x^{(1)} = 1524.7043$  and its time response from  $\hat{x}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-a(k)} + \frac{b}{a} = -81693.8e^{-0.0102} - 83418.88333$  Substituting different values of  $k$  into the equation:

- $k = 1X^{(1)}(1) = 1725.00$
- $k = 2X^{(1)}(2) = 3446.54$
- $k = 3X^{(1)}(3) = 5226.05$
- $k = 4X^{(1)}(4) = 7011.07$
- $k = 4X^{(1)}(4) = 8774.14$
- $k = 5X^{(1)}(5) = 10523.81$
- $k = 6X^{(1)}(6) = 11593.12$
- $k = 7X^{(1)}(7) = 13346.22$
- $k = 8X^{(1)}(8) = 16948.95$
- $k = 9X^{(1)}(9) = 18800.31$
- $k = 10X^{(1)}(10) = 22606.11$

Compute the simulated values of  $X^{(0)}$  using the original series according to the accumulated generating operation by using  $\hat{x}^{(0)}(k+1) = \alpha^{(1)}\hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$ . Using the same calculation and procedures, all forecasted values of all companies from 2020 to 2025 are found for the Vietnamese coffee evaluation shown in Table 3.

**Nonlinear grey Bernoulli model – (NGBM (1,1))**

The NGBM (1,1) model of the empirical process is described as follows

(1) Denoted the original history of time series data by

(2) The (1-AGO) series  $X^{(1)}$  is defined as

$$X^{(1)} = \{1725.00, 3265.00, 4902.00, 6539.00, 9873.00\}$$

(3) According to the equation  $x^{(0)}(t_k) + az^{(z)}(t_k) = b[z^{(1)}(t_k)]^n$  the equation showed:

$$x^{(0)}(t_k) + -0.0172z^{(z)}(t_k) = -3001961.3[z^{(1)}(t_k)]^n$$

The first-order accumulate progression satisfies as follows difference equation:

$$\frac{dx^{(1)}(t_k)}{dt_k} - 0.0172(t_k) = -3001961.3[x^{(1)}(t_k)]^n(4)$$

Where

$$z^{(1)}(t_k) = 0.5(t_k) + 0.5x^{(1)}(t_{k-1}),$$

$$k = 2, 3, \dots, m, (5)$$

By taking the above the formula settings the optimal parameter value  $p = 0.5$  and  $n = 0.0381$ , which were calculated by adapting the ordinary least squares method (OLS):

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_m$$

where  $Y = \begin{bmatrix} 1540 \\ 1637 \\ 1637 \\ 1651 \\ 1683 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1725 & 4360 \\ -3265 & 6923 \\ -4902 & 9541 \\ -6539 & 12173 \\ -8190 & 14846 \end{bmatrix}$

Building the GNBM (1,1) form equations (8) and (9), estimates of the parameters  $a$  and  $b$  were as follows:

$$\begin{cases} a = -0.0172 \\ b = -3001961.3 \end{cases}$$

(5) From the solution of equation (4) could be obtained after the parameters  $a$  and  $b$  are predicted values. Such as:  $\hat{x}^{(1)}(t_i) =$

(10) Where  $n \neq 1, k = 1, 2, 3, \dots$

(6) The estimated result  $\hat{x}^{(0)}(t_i)$  from Fourier Series modified using equation (10) as follows:

$$\hat{x}^{(0)}(t_i) = \begin{cases} \hat{x}^{(0)}(t_k) = 1725.00 \\ \hat{x}^{(0)}(t_k) = 3439.64 - \hat{x}^{(1)}(t_{k-1}) \end{cases}$$

( $k = 2, 3, \dots, m$ )

Using the same calculation and procedures, all forecasted values by the NGBM (1,1) of the coffee bean price from 2020 to 2025 were calculated for the Vietnamese coffee evaluation shown in Table 3.

**Development Fourier series modification by NGBM (1,1)**

Adapting González, Jaramillo, and Carmona. (2008) separated the idea of Fourier series systems and those of forecasting boxes as an important test.

(1) For the fluctuation data sequence  $X^{(0)} = \{1725.00, 1540.00, 1637.00, 1637.00, 1651.00, 1683.00\}$ . The residual series named  $\varepsilon$  was a difference between the reality which could be written as:

$$,k = 2, 3, \dots, m$$

where (2) The residual series can be estimated as  $\hat{\varepsilon}(k)$  approximately:

(3) With the residual series can be estimated in matrix  $\varepsilon = PC$  form as

The Fourier series mathematics be calculated:

$$\hat{\varepsilon}(k) = 859.1093 + \sum_{i=1}^z \left[ 27.1971 \cos\left(\frac{2\pi i}{5}(k)\right) + 31.6859 \sin\left(\frac{2\pi i}{5}(k)\right) \right]$$

(5) Then the estimated  $\hat{v}$  values to calculate coffee bean price forecasting:

$$\hat{v} = \{1725, 1690.54, 1685.51, 1675.02, 1669.07, 1683.67\}$$

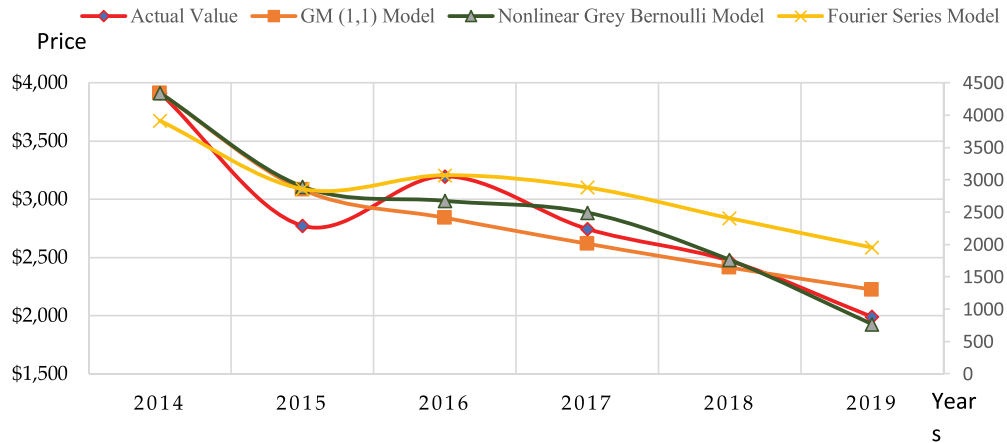


Figure 2. Curves of actual versus simulated values using traditional and optimized GM (1,1), NGBM (1,1), and Fourier series.

Table 3. The MAPE for each model.

Year	Actual Price	GM (1,1) Model		Nonlinear Grey Bernoulli Model		Fourier Series Model	
	Lam Dong	Forecasted value	MAPE (%)	Forecasted value	MAPE (%)	Forecasted value	MAPE (%)
2014	1725	1725.00	0.000	1725.00	0.000	1725.00	0.000
2015	1540	1721.54	11.788	1714.64	11.340	1690.54	9.775
2016	1637	1779.51	8.706	1708.51	4.368	1685.51	2.963
2017	1637	1785.02	9.042	1700.78	3.896	1675.02	2.322
2018	1651	1763.07	6.788	1682.43	2.509	1669.07	1.094
2019	1683	1789.67	6.338	1684.85	0.05	1683.67	0.398
<b>MAPE</b>			<b>7.050</b>		<b>3.926</b>		<b>2.077</b>

Table 2. MAPE grade levels Lewis (1982).

MAPE	≤ 1%	1%-4.99%	5%-10%	> 10%
<b>Performance</b>	Excellent	Good	Qualified	Unqualified

$$\text{where } \hat{v} = \begin{cases} \hat{v}_1 = 1725.00 \\ \hat{v}_2 = v_2 + 1718.2187 \end{cases}$$

Finally, the coffee bean price from 2014–2019 calculated with the Fourier Series Model result is shown in Table 3.

Table 4. The parameter evaluation for forecasting the coffee bean price.

Model	Parameter value
GM (1,1)	$n = 0, p = 0.5$
NGBM (1,1)	$n = 0.0381, p = 0.5$
Fourier Series	$n = 0.0381, p = 0.5$

**Check forecasting accuracy**

MAPE used to evaluate the accuracy of the forecasting model and shown as follows:

After finding each model's future price forecasts, [Figure 2](#) displays the validation of coffee bean prices from 2014 to 2019 for each forecasting model, such as GM (1,1), the NGBM (1,1), and Fourier Series Model. We presented the price forecasts for coffee beans grown in Lam Dong below:

[Wang and Phan \(2015\)](#) explained the remarkable higher accuracy of the NGBM (1,1) model compared with other GM (1,1) models, as exemplified in the fluctuating sequence  $X(0) = (1725, 1540, 1637, 1637, 1651, 1683)$ . In this section, we used coffee prices to compare the accuracy of the NGBM (1,1) with the Fourier series model to demonstrate the improvements in the Fourier series model. [Table 2](#) and [Figure 3](#) show the results. According to [Table 4](#), the NGBM (1,1) model and Fourier series model has  $n = 0.0381$ ,  $p = 0.5$ . In this optimization method, the Fourier series model's average error decreased from 3.926% to 2.077%. However, even though the MAPE percentage was small, for the fluctuating curve given in [Figure 2](#), the Fourier series model surpasses the traditional Fourier series model in the simulation results, being nearly equal to the actual coffee bean price line.

Besides, [Table 3](#) shows the MAPEs from 2012 to 2017 for coffee bean growers in Lam Dong, with an average forecasting accuracy of 7.050% for both GMs (1,1), 3.926% for the NGBM (1,1), and 2.077% for the Fourier series. The model precision improved obviously when applied the Fourier series. These results strongly suggest that the forecasted accuracy was better than 90%. The performance of Fourier series forecasting earned the level of 'excellent' in MAPE. Thus, the Fourier series was an effective model for forecasting coffee bean prices in this case study. Using a predictive mathematical model priced using three forecasting models would help economists estimate market pressures and build a more efficient supply chain to control for volatility levels in the world coffee market.

#### **Fluctuating raw data sequence example**

The parameter evaluation of the fitted models was developed by [Wang and Phan \(2015\)](#) and [Chen \(2008\)](#) below:

As shown in [Figure 2](#), for the given fluctuating curve, the NGBM (1,1) surpasses the performance expectations of the Fourier series model in the simulation results. Price forecasting was the best method to generate new ideas for self-innovation in the coffee industry. In this research, we solved two major problems: first, combining the NGBM (1,1) and Fourier residual modification models to improve the effectiveness of forecasting stochastic volatility data. Second, providing an effective method for forecasting coffee bean prices in the Vietnam case study. Besides, improving the accuracy of six-year forecasts in cases involving sample data with large fluctuations. This price forecasting method should help coffee companies ascend to greater heights by strengthening weak links in Vietnam's agricultural supply chain.

#### **V. Conclusions**


This paper developed an NGBM (1,1) and examined its validity in forecasting the price of coffee grown in Lam Dong. Estimating this unpredictable price would help companies manage their production in a timely manner and control input material stability and risk allocation. Research results suggest the combination of the NGBM (1,1) and Fourier series models to forecast the effect of current price movements on coffee prices in the future. Further analysis revealed the forecasting result of the uncertainty price's Vietnamese coffee bean analysis. The MAPE of GM (1,1) and MAPE of NGBM (1,1) having a percentage of MAPE at around 10%–20% shows that the NGBM (1,1) and GM (1,1) have adequate levels. However, with optimistic accuracy in the real work, the price uncertainty estimated by the Fourier series model with the MAPE has an 'excellence' level as well as with the actual value. In this paper, an applied and improved Fourier series model, and the NGBM (1,1) were based on data from 2014 to 2019 to forecast the uncertainty of coffee prices. A mathematical model for integer forecasting model solved prices of Lam Dong growers. Moreover, this model could be applied to forecast different unit priced coffee in Vietnam or in the world. Coffee bean price forecasts validating the model could potentially improve forecast accuracy for a wide variety of macroeconomic outcomes, such as risk reduction in the market.

Forecasting accuracy was determined using MAPE calculations, and the results showed that the proposed Fourier series model was more effective after integration with NGBM (1,1). This research proposes a new and useful decision-making model that uses a real case study to show the proposed model's applicability and effectiveness. The integrated approach provides empirically meaningful and helpful results for the sustainable development of the food supply chain industry in the future. In addition, further research could help develop a better perspective and an increasingly integrated approach for even more complex supply chains.

### Disclosure statement

No potential conflict of interest was reported by the authors.

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