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碩士論文
Master Thesis

探討牛熊市之市場狀態下波動度風險溢酬與預期報酬

Variance Risk Premium and Expected Returns in Bull and Bear Markets

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摘要

在金融市場中最主要和關鍵的問題是如何預測市場的預期報酬，許多研究顯示預期報酬在很大程度上取決於經濟狀態。波動度風險溢籌已被證實對預期收益的可預測性，這是有個問題浮現在腦中，我們如何知道哪種市場狀態主導了波動度風險溢籌對預期報酬的預測能力？爲了研究不同市場狀態下市場預期收益的可預測性差異，我們利用S&P500 期貨的高頻數據，區分了 20 年來熊市或牛市市場狀態下波動度風險溢籌的可預測範圍。我們發現在不同的市場狀態下，市場的型態是截然不同的，它極大地影響了波動度風險溢籌對預期報酬的可預測性。在我們的實證結果中，熊市中的可預測回報時間長度要比牛市中的短。

關鍵詞: 波動度風險溢酬、報酬可預測性、市場狀態依賴性、高頻資料

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Abstract

The principal and critical issue in the financial market is how to predict the market' s expected return and many studies show expected returns depend strongly on the economic times. The variance risk premium has been proved its predictability of expected returns. However, a problem occurs, how do we know which market state dominates the predictability?

In order to investigate the difference in the predictability of expected market returns under different market states, we use high-frequency data of S&P500 futures to differentiate the forecast horizons of variance risk premium in bullish and bearish for over two decades. We realize that market situations vary in different market states, which tremendously affects the predictability of variance premium. In our empirical investigation, the predictable return horizons in bear markets are shorter than in bull markets.

Keywords: Variance risk premium, Return predictability, State dependence, High-frequency data

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1 Introduction

Throughout the ages, the principal and critical issue in the financial market is how to predict the market's expected return; in the meanwhile of earning profits, investors also care about the risk behind returns. Traditionally, studies like French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Whitelaw (1994) measure the relational between financial market risk-return by volatility or equity. However, their connection between expected returns and risk is not convincing and has a long-term reality, Lewellen (2004) and Cochrane (2008) also propose with evidences that the predictability of returns has become weakness. Recently, Bali and Zhou (2016); Bollerslev, Marrone, Xu, and Zhou (2014); Bollerslev, Tauchen, and Zhou (2009); Choi, Mueller, and Vedolin (2017); Han and Zhou (2011); Prokopczuk and Simen (2013) have shown that the difference between the expectation of risk-neutral returns variation and risk-neutral variance named as variance risk premium has been proved its predictability of market expected returns by multiple empirical results and robustness checks in the international equity markets, including stock, treasury, and commodity markets.

While we can use the model-free approach, proposed by Bollerslev et al. (2009), to calculate the variance risk premium, market returns cannot be predicted well in out-of-sample tests (Welch and Goyal (2007)). However, many papers (e.g., Cheema, Nartea, and Man (2018); Dangl and Halling (2008); Hammerschmid and Lohre (2017); Henkel, Martin, and Nardari (2008)) show that the returns could not forecast well but differentiate

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market state with different traditional predictors; in other words, expected returns' predictability depends on the economic times. As an extra inspiration by the study Li and Zakamulin (2020) on the predictability of stock volatility varies in bull and bear markets, in this paper, we want to gauge that variance risk premium, which is a volatility-related approach, is whether to perform dissimilar predictability under different markets states.

In our work, we first calculate implied and realized variation by CBOE VIX index and S&P 500 futures to compute the variance risk premium. In the case of our model-free approach of variance risk premium, we also calculate implied and realized premium in the model-free methods. Much of the literature relies on information in option prices to measure compensation for risks that change over time, and options also provide an ideal tool to examine how different types of risk are known in advance by investors and applied to pricing. In the past, we used the Black-Scholes model, a mathematical model that projects the pricing variation of stocks, futures, or options developed by Black and Scholes (1973); Merton (1976), to get the implied volatility by giving the European options' market price. Nevertheless, because the CBOE VIX index represents the annual implied volatility of the S&P 500 index with thirty days to expiration in percentage, Bollerslev et al. (2009) and Kilic and Shaliastovich (2019) have experimented with transforming the 'new' VIX index to the time we use as the implied variance is a more effortless and robust way to calculate implied variance than backstepping from the Black-Scholes model. Realized variance is an exact measure of volatility, which estimates return variations of prices continuous stochastic process (Barndorff-Nielsen and Shephard (2002)). For that reason, we use the intraday high-frequency price data to compute the return variation, and in case we need a monthly realized variance, we also aggregate the overnight returns and intraday return to represent the accurate return variance of the given month.

Secondly, label market states as bullish and bearish to reflect the real-time economic situation. There are two different methods to state the market condition, forecasting or

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labeling; forecasting means using what has already happened to predict the future; labeling means dating the past periods depending on the present situation. In our paper, we like to evidence study on S&P 500 futures from 2000 to 2019; as a result, we desire to operate the labeling method because of the accuracy and stability.

Consequently, we accomplish empirical investigations on variance risk premium and expected returns in bull and bear markets. Our main results can conclude into two parts. First and foremost, the horizon of expected returns predicted by variance risk premium increased in recent years. That is, the expected returns of S&P 500 futures are statistically significant at one-, three-, and six-month return horizons in Kilic and Shaliastovich (2019) from January 1996 to August 2014, yet in our work from January 2000 to December 2019 are statistically significant at one-, three-, six-, nine-, twelve-, and fifteen-month return horizons. Furthermore, the expected returns horizon is considerably elongated when the market state is bullish than in bearish; in preciseness, the horizon can be up to twenty-four months in the bull markets but only six months in the bear market.

1.1 Related literatures

Our paper focuses on evidence studying the variance risk premium and expected returns in bull and bear markets. Bollerslev et al. (2009) shows that the stock market returns are predictable within quarterly return horizons by the difference between "model-free" implied and realized variances, providing a robust approach to calculate the variance risk premium. Moreover, in the following work, Bollerslev et al. (2014) demonstrates that variance risk premium is also a predictor in global stock markets; Choi et al. (2017) stands for variance risk premium to remain statistically significant on the prediction of bond returns; Prokopczuk and Simen (2013) also shows evidence for the predictability on commodity markets. Hence, we firmly believe that the variance risk premium has forecasting power

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on expected returns, but whether it works only in good economic or bad economic time, alternatively in both market conditions.

Specifically, there is a problem that why we need to research the predictability of variance risk premium under different market states. Li and Zakamulin (2020) offers evidence that the predictability of realized volatility is closely linked to market conditions. In addition, Cheema et al. (2018) also presents that either time-series strategy or cross-sectional strategy can only forecast momentum returns well in the same market state, UP(bull) or DOWN(bear). Accordingly, we expect that variance risk premium's predictability shows various efforts under different market states.

Especially, classifying the bull and bear market becomes one of the main issues in our paper; to encounter this problem, we focus on labeling the market states bullish and bearish despite distinguishing between economic boom or recession. Moreover, how to date the market states has separated into two different ways in past research; one stands for a significant period, and another denotes substantial price fluctuations. For instance, about a significant period means that two market states combined to an economic cycle will rise(fall) over a considerable period in the bull(bear) market. When it comes to this opinion, the algorithm provided by Sossounov and Pagan (2003) offers a set of rules for multiple censor periods length to find the turn points of regime switches.

On the other hand, if we focus on substantial price fluctuations, we need to check the difference between market price and the previous peak or trough price to identify the transition point between bull and bear markets. Based on this idea, a famous work, Lunde and Timmermann (2004), provided a breakthrough to filter market to bull or bear market. In summary, two different algorithms supply us with to locate the turn points of market states changing. Fortunately, Kole and van Dijk (2010) compares four different methods of identifying bullish and bearish markets, including Sossounov and Pagan (2003) and Lunde and Timmermann (2004); Lunde and Timmermann (2004) outperforms Sossounov

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and Pagan (2003) under several experiments. As a result, we implement Lunde and Timmermann (2004) as the label methodology to determine the bull and the bear market states in this paper.

Based on the above empirical studies, we can theoretically research the variance risk premium and expected returns in bull and bear markets and show the results of variance risk premium is state-dependent.

The remaining paper is coordinated in the following manners. Section 2, as the methodology section, clarifies the empirical methodology of bull and bear market labeling algorithm, computation of realized variance via high-frequency data, transformation CBOE VIX index to implied variance, measurement of variance risk premium, and our integral regression equation to evaluate predictability. Section 3 illustrates the empirical results. In the end, Section 4 makes conclusions of the paper and points out some future works.

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2 Methodology

2.1 Bull and bear market labeling algorithm

As mentioned above, Lunde and Timmermann (2004) provides a method to detect turn points by setting a minimum price change threshold since the last peak(trough) value. First, we declare P_t as market price at time t. Then, assume variable $\lambda_{Bull}(\lambda_{Bear})$ to be the threshold of a price changing ratio that activates the switching from bear(bull) market state to bull(bear) market state and the original work investigation on symmetrical (λ_{Bull} = 20%, λ_{Bear} = 20%) and asymmetrical (λ_{Bull} = 20%, λ_{Bull} = 15%) threshold. Based on the Li and Zakamulin (2020), they report that using asymmetrical threshold gets better filtering market states. As follows, we will separate into two situations, bull market state switches to bear market state, and bear market state switches to bull market state.

2.1.1 Bear market state switches to bull market state

In this situation, we know that the time at t_0 starts in a bear market state; we need to find the turning point during t_0 to t. First of all, find the trough price $P^{min}_{t_0,t}$ in this period

$$P^{min}_{t_0,t} = min\{P_{t_0}, P_{t_0+1}, \ldots, P_t\}, \tag{2.1}$$

and in order to compare with the threshold λ_{Bull}, we define the relative change δ_t

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$$\delta_t = \frac{P_t - P_{t_0,t}^{min}}{P_{t_0,t}^{min}}. \tag{2.2}$$

Switch the market state when $\delta_t > \lambda_{Bull}$, it represents that we found a trough price at t_{trough} from t_0 to t and the price at time t is up λ_{Bull}, then the algorithm identifies this as a turning point at t_{trough} by label t_0 to t_{trough} as bear market and start bull market from $t_{trough+1}$.

2.1.2 Bull market state switches to bear market state

Contrary to above, we start in a bull market state; Foremost, we find the peak price $P_{t_0,t}^{max}$ from t_0 to t

$$P_{t_0,t}^{max} = max\{P_{t_0}, P_{t_0+1}, \ldots, P_t\}, \tag{2.3}$$

and then compute δ_t

$$\delta_t = \frac{P_{t_0,t}^{max} - P_t}{P_{t_0,t}^{max}}. \tag{2.4}$$

As the same consideration with the opposite approach, swap the market state through label t_0 to t_{peak} as bull market and start bear market from t_{peak+1} if $\delta_t > \lambda_{Bear}$.

2.2 Variance risk premium

Based on the model-free fashion the paper provides us with the variance risk premium calculation, we must first compute the market's implied variance and realized variance.

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2.2.1 Implied variance

Traditionally, we use European call options to construct a portfolio to implement a model-free procedure to calculate the risk-neutral expectation of returns variation between time t and $t+1$ as the implied variance (iv_t).

As for the 'new' VIX index is the implied volatility of S&P 500 contracts, Bollerslev et al. (2009) and Bekaert and Hoerova (2014) use the squared VIX index to express implied variance except using call options, which aligns with current industry standards and also replicates the effectiveness of a traditional method that has been recognized as accurate. In order to quantify the monthly implied variance iv_t

$$iv_t = \frac{VIX^2}{12}, \tag{2.5}$$

we divided it by twelve to transfer it from annually to monthly.

2.2.2 Realized variance

In order to calculate the realized variances (rv_t) in a model-free approach, Andersen, Bollerslev, Diebold, and Ebens (2001); Andersen, Bollerslev, Diebold, and Labys (2001); Barndorff-Nielsen and Shephard (2002) propose and demonstrate a robust way to measure the return variation by utilizing five-minute intraday data, defined as the summation of high-frequency and overnight squared returns of each period. Fisrt, we calculate the h_t intraday returns

$$r_{i,h_t} = p_{\frac{it}{h_t}} - p_{\frac{(i-1)t}{h_t}}, i = 1, \ldots, h_t, \tag{2.6}$$

where p_t denotes the logarithmic price of the equity price; h_t is the number of five-minute intervals in period t. (e.g., 86 within-day five-minute squared returns and 22 trading days

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per month concludes to 1892 five-minute returns in a period t.)

And then, based on

$$rv_t = \sum_{i=1}^{h_t} {r_{i,h_t}}^2, \tag{2.7}$$

represent as the true risk-neutral variance of $t-1$ to t in percentage form.

2.2.3 Variance risk premium

Once we have derived implied variance and realized variance at time t, we use the difference between the expectation of risk-neutral returns variation and risk-neutral variance of $t-1$ to t to derive variance risk premium (vrp_t) based on the Bollerslev et al. (2009).

$$vrp_t \equiv iv_t - rv_t \tag{2.8}$$

Finally, we classify the market states and compute variance risk premium from the given time and market, and now we need to define the evaluation method to estimate the predictability of variance risk premium under bull and bear markets.

2.3 Predictability return regression

By the model-free approach of variance risk premium, regarded as a predictor of expected returns refer to the unit time of a month. As to define the "monthly" return horizon, our regression shows in the following format:

$$\frac{1}{h}\sum_{j=1}^{h} r_{t,j} = b_0(h) + b_1(h)(vrp_j) + u_{t+h,t}, \tag{2.9}$$

where h is the horizon of the regression, r denotes a monthly log return.

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Furthermore, to evaluate the statistical significance, we use the Newey—West standard errors with optimal number of lags (Newey and West (1994)) to implement robust t-statistic to test the estimated slope coefficients.

Our study will use the above methods to differentiate market states and use model-free methods to calculate variance risk premium. In our subsequent empirical study on S&P 500 futures, we will first verify whether our calculation is consistent and correct with previous studies and expectations. In the end, we will empirically demonstrate whether variance risk premium's ability to predict market returns differs under different market states.

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3 Empirical Results

In our work, we use the five-minute S&P 500 futures data from TICKDATA and CBOE VIX index from Yahoo Finance as our experiment dataset from 2000 to 2019 to investigate variance risk premium predictability in bull and bear markets.

3.1 Label bull and bear market states

As mentioned above, we first need to label the market states of each period in our empirical study time. Figure 3.1 on page 13 shows the market states filtered by the algorithm of Lunde and Timmermann (2004) with the symmetrical and asymmetrical parameters. The upper panel set symmetrical thresholds of $\lambda_{bull} = \lambda_{bear} = 20\%$; and in the downer panel, formed asymmetrical thresholds of $\lambda_{bull} = 15\%, \lambda_{bear} = 20\%$.

Both panels of this figure demonstrate the two well-known economic recessions after Two thousand. The first one is the dot-com bubble in the United States from the beginning of 2000, and another one is the global financial crisis of 2007—2008. However, after the global financial crisis, there was another notable economic recession about the European debt crisis but only dating by the algorithm with asymmetrical thresholds in the downer panel.

As a result, we use the dating results under asymmetrical thresholds to accomplish the following experimentations. Table 3.1 informs the descriptive statistics of each market states depending on asymmetrical thresholds. In sum, the first observation of this descrip-

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tive statistics is that the duration of the bull market state is significantly longer than the bear market state no matter in minimum, maximum, and average. The second observation is that the bull market represents higher return and lower volatility, and the bear market denotes the lower return but higher volatility.

Table 3.1: Statistics of bull and bear market states

Statistics	Bull	Bear
Number of phases	3	3
Minimum duration	26	5
Average duration	62	18
Maximum duration	99	33
Mean return	1.25	-2.81
Standard deviation	3.43	5.44

3.2 Variance risk premium

We use five-minute intraday data to calculate variance risk premium through the difference between implied and realized variance shown in Figure 3.2. To verify the correctness of the method, we first compare with past studies through explainable statistics and find that the general statistics are close to each other. However, we find that our three factors' mean, standard deviation, skewness, and kurtosis are more extensive because of one more bear market than previous work.

Table 3.2: Statistics of realized variance, implied variance, and variance risk premium under different market states

	rv_t	rv_t^{bull}	rv_t^{bear}	iv_t	iv_t^{bull}	iv_t^{bear}	vrp_t	vrp_t^{bull}	vrp_t^{bear}
Mean	30.52	17.08	76.57	36.82	27.69	68.49	6.38	10.60	-8.08
Std. dev.	52.27	16.51	92.36	36.26	21.24	55.15	40.93	17.24	78.84
Skewness	6.92	2.48	4.05	3.57	2.60	2.43	-7.82	-0.63	-4.39
Kurtosis	67.73	7.47	20.29	17.62	9.66	6.15	89.32	8.41	24.01
AR(1)	0.61	0.45	0.48	0.82	0.68	0.68	0.22	0.16	0.19

In order to test our idea of the influence of an additional bear market state, we will do descriptive statistics on different market states. Table 3.2 shows that indeed, in bear

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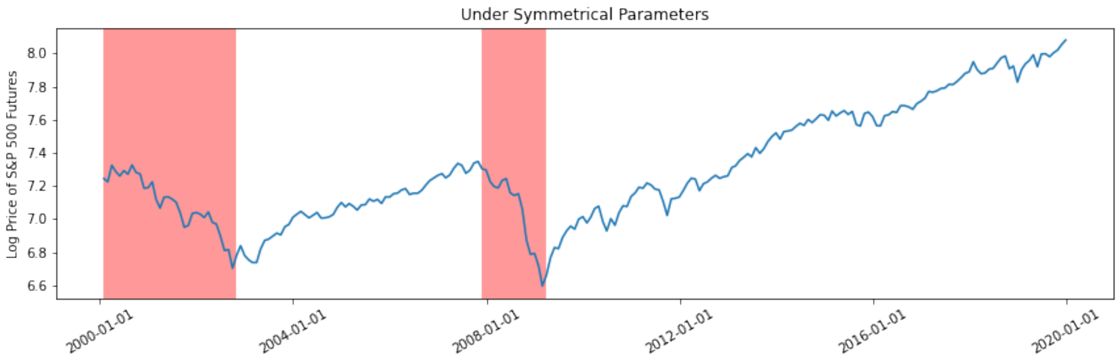
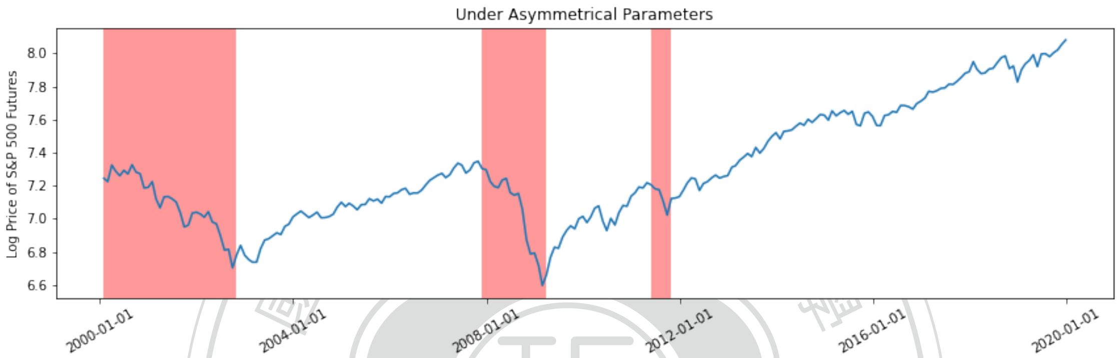


(a) Symmetrical parameters of $\lambda_{bull} = \lambda_{bear} = 20\%$



(b) Asymmetrical parameters of $\lambda_{bull} = 15\%, \lambda_{bear} = 20\%$

Figure 3.1: Market states

Notes. This figure shows the market states of S&P500 futures from 2000 to 2019. The line represents the log price of the S&P500 futures. The areas with background color indicate the bear market state. The upper panel (a) illustrates the labeled algorithm with symmetrical parameters $\lambda_{bull} = \lambda_{bear} = 20\%$. The downer panel (b) presents the market states under asymmetrical parameters of $\lambda_{bull} = 15\%, \lambda_{bear} = 20\%$.

markets, the implied variance, realized variance, and variance risk premium are more volatile than in bull markets, which means that the market is experiencing massive declines much more rapidly than when it is rising, or as the saying goes, "slow rises and sharp falls."

After confirming the accuracy of the data, we need to do further testing of the predictive power to ensure that the variance risk premium still has predictive power. In order to experiment with the predictability, we use the traditional predictor variables utilized in Bollerslev et al. (2009), $log(P_t/E_t)$, $log(P_t/D_t)$, $DFSP_t$ (Th spread between Moody's monthly seasoned Baa and AAA corporate bond yield), and $TMSP_t$ (The spread between 10-Year Treasury Constant Maturity and 3-Month Treasury Constant Maturity).

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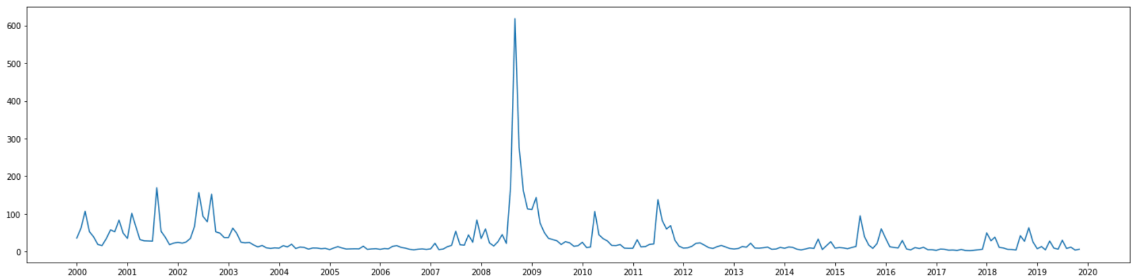
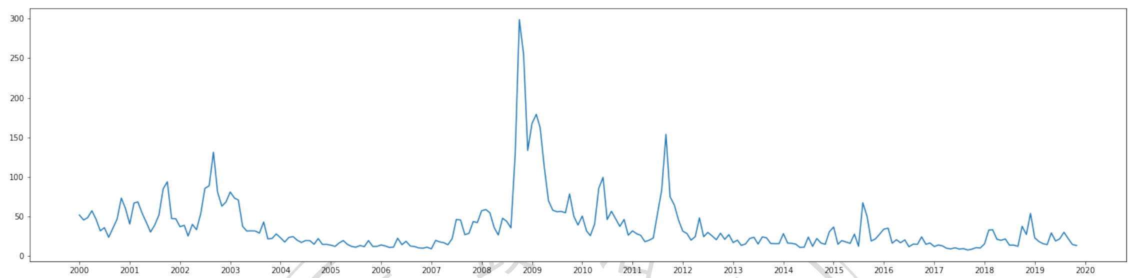
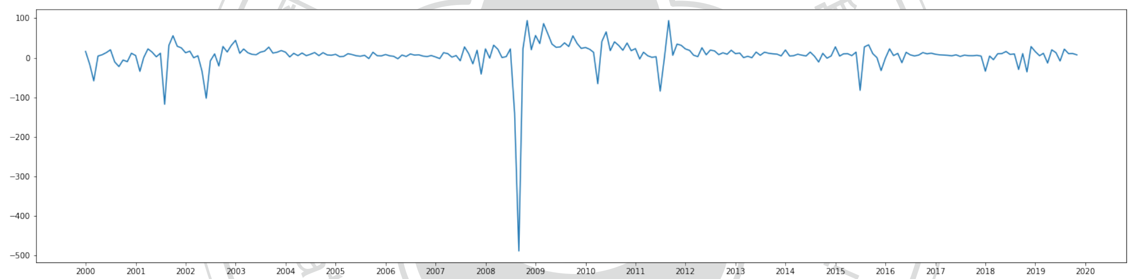


(a) Realized variance



(b) Implied variance



(c) Variance risk premium

Figure 3.2: Realized variance, implied variance, and variance risk premium

Notes. This figure shows three variables from 2000 to 2019. The line represents the value of each variable.

Table 3.3 shows the summary statistics of our chosen predictors and their relationship with implied variance, realized variance, and variance risk premium. We discover that the predictors differ somewhat from the previous studies regarding narrative statistics, empirically concluding that the fluctuations are not as large as our sample because of the low occurrence of extreme events in the past study period. However, we can see that their first-order autocorrelations are pretty high, and the correlation between one-month forward return is not much significant than variance risk premium as before works.

Next, we also did regression results for monthly, quarterly, and annual returns con-

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Table 3.3: Summary statistic

	r_{t+1}	rv_t	iv_t	vrp_t	$log(P_t/E_t)$	$log(P_t/D_t)$	$DFSP_t$	$TMSP_t$
Summary statistics								
Mean	0.35	30.52	36.82	6.38	3.14	3.99	1.04	1.69
Std. dev.	17.24	52.27	36.26	40.93	0.39	0.19	0.43	1.18
Skewness	-0.78	6.92	3.57	-7.82	2.17	0.14	3.03	-0.28
Kurtosis	1.44	67.73	17.62	89.32	5.92	2.07	11.49	-0.97
AR(1)	0.08	0.61	0.82	0.22	0.98	0.97	0.96	0.86
Correlation matrix								
r_{t+1}	1.00	-0.46	-0.06	0.54	-0.05	-0.13	-0.07	-0.01
rv_t		1.00	0.63	-0.72	0.25	-0.12	0.40	0.14
iv_t			1.00	0.09	0.44	-0.33	0.72	0.27
vrp_t				1.00	0.07	-0.14	0.13	0.06
$log(P_t/E_t)$					1.00	-0.13	0.55	0.25
$log(P_t/D_t)$						1.00	-0.63	-0.31
$DFSP_t$							1.00	0.30
$TMSP_t$								1.00

taining the aforementioned traditional predictor variables in Tables 3.4.

Table 3.4: Monthly return regression

	Simple					Multiple				
Constant	-0.0110 (-0.054)	2.1956 (0.553)	11.9056 (1.220)	1.0354 (0.901)	0.4088 (1.037)	3.1660 (1.133)	5.0623 (0.694)	1.4003 (1.635)	1.7561 (0.448)	2.1510 (0.750)
vrp_t	0.0565 (4.252)					0.0572 (4.202)	0.0557 (4.687)	0.0583 (4.029)		0.0583 (4.044)
$log(P_t/E_t)$		-0.5881 (-0.448)				-1.0136 (-1.112)			-0.2758 (-0.207)	-0.2872 (-0.290)
$log(P_t/D_t)$			-2.8984 (-1.205)				-1.2711 (-0.698)			
$DFSP_t$				-0.6561 (-0.525)				-1.3616 (-1.654)	-0.5175 (-0.403)	-1.2173 (-1.404)
$TMSP_t$					-0.0349 (-0.136)					
$AdjR^2$(%)	28.7	-0.1	1.3	0	-0.4	29.2	28.7	30.2	-0.4	30

Table 3.4 shows the test statistics in the monthly return horizon are lower than expected except for variance risk premium, which have a statistically significant predictive history in the monthly range in traditional studies. Moreover, because all traditional predictors are insignificant on the t-statistics, we add $log(P_t/E_t)$, $log(P_t/D_t)$, and $DFSP_t$ to the multiple regression based on Bollerslev et al. (2009) to check whether the variance risk premium is still statistically significant. After testing, either t-statistics or adjusted R squared still stays statistically significant. Therefore, we conducted the following quarterly and annual returns regressions experiments based on this approach (shown in Table

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3.5-3.6), and the results were similar. We observed that the predictability of variance risk premium deteriorated with a longer horizon as expected.

Table 3.5: Quarterly return regression

	Simple					Multiple				
Constant	0.1597	1.2401	12.9224	0.5684	0.3100	1.7061	9.7909	0.7443	1.1421	1.3315
	(0.719)	(0.361)	(1.612)	(0.570)	(0.856)	(0.616)	(1.465)	(0.902)	(0.339)	(0.468)
vrp_t	0.0271					0.0274	0.0255	0.0278		0.0278
	(4.934)					(4.881)	(6.759)	(5.105)		(5.136)
$log(P_t/E_t)$		-0.2893				-0.4933			-0.2196	-0.2247
		(-0.255)				(-0.542)			(-0.189)	(-0.227)
$log(P_t/D_t)$			-3.1578				-2.4130			
			(-1.602)				(-1.458)			
$DFSP_t$				-0.2260				-0.5633	-0.1155	-0.4503
				(-0.206)				(-0.658)	(-0.100)	(-0.493)
$TMSP_t$					0.0128					
					(0.056)					
$AdjR^2$(%)	18.2	-0.2	5.2	-0.3	-0.4	18.4	21.1	18.7	-0.6	18.4

Table 3.6: Annually return regression

	Simple					Multiple				
Constant	0.2702	1.1573	14.9133	-0.3511	-0.1794	1.2910	14.2795	-0.3075	2.1068	2.1546
	(1.295)	(0.581)	(5.744)	(-0.938)	(-0.505)	(0.712)	(5.659)	(-0.892)	(1.362)	(1.481)
vrp_t	0.0075					0.0077	0.0052	0.0067		0.0067
	(2.430)					(2.366)	(5.304)	(3.157)		(3.363)
$log(P_t/E_t)$		-0.2674				-0.3253			-0.9402	-0.9418
		(-0.408)				(-0.548)			(-1.710)	(-1.800)
$log(P_t/D_t)$			-3.6590				-3.5083			
			(-5.802)				(-5.707)			
$DFSP_t$				0.6380				-0.5561	1.1124	1.0311
				(1.871)				(1.867)	(2.457)	(2.509)
$TMSP_t$					0.2798					
					(1.975)					
$AdjR^2$(%)	4.2	0.1	24.6	3.3	4.5	4.6	26.5	6.6	7.5	10.9

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3.3 Predictability in bull and bear markets

On the basis of the above results, we can notice a wide range of statistics for variance risk premium in different market states. Besides, variance risk premium has the predictive ability for expected returns but will weaken with the degree of the horizon. Then we come to the most crucial issue of this paper: whether variance risk premium predictability differs in bull and bear markets. According to Table 3.1, we discover that the duration and the market return are polarized in different market states. In addition to Table 3.2, we can realize that variance risk premium is very unlike in bull and bear markets. Therefore, in Table 3.7, we distinguish between the bear and bull states in the two panels to explore the difference in variance risk premium forecasting power in the two different states.

Table 3.7: Regression under different return horizons

Monthly return horizon	1	3	6	9	12	15	18	24
All periods								
Constant	-0.0110	0.1597	0.1928	0.2456	0.2702	0.2898	0.3035	0.3308
	(-0.054)	(0.719)	(0.862)	(1.118)	(1.295)	(1.482)	(1.645)	(2.022)
vrp_t	0.0565	0.0271	0.0202	0.0108	0.0075	0.0053	0.0050	0.0046
	(4.252)	(4.934)	(6.676)	(3.762)	(2.430)	(2.156)	(1.805)	(1.999)
$Adj\,R^2$(%)	28.7	18.2	17.7	7	4.2	2.5	2.7	3.1
Bull market								
Constant	-0.2101	0.5062	0.5948	0.6385	0.6258	0.5717	0.4958	0.4545
	(-1.227)	(3.761)	(4.640)	(4.888)	(4.339)	(3.552)	(2.673)	(2.344)
vrp_t^{bull}	0.1230	0.0467	0.0297	0.0203	0.0165	0.0137	0.0154	0.0134
	(9.122)	(5.723)	(5.069)	(3.636)	(2.929)	(3.321)	(3.315)	(2.385)
$Adj\,R^2$(%)	39.8	21.2	20.3	14.4	11.3	6.5	6.9	6.2
Bear market								
Constant	-1.8751	-1.7947	-1.5411	-1.4286	-1.2366	-0.9340	-0.6932	-0.3737
	(-3.380)	(-3.301)	(-2.675)	(-2.547)	(-2.651)	(-2.444)	(-2.174)	(-1.186)
vrp_t^{bear}	0.0402	0.0180	0.0134	0.0043	0.0015	0.0003	0.0004	0.0012
	(5.786)	(5.220)	(5.618)	(1.653)	(0.607)	(0.158)	(0.261)	(0.783)
$Adj\,R^2$(%)	26.9	14.6	13.5	0.5	-1.5	-1.9	-1.8	-1.3

Table 3.7 shows the performance of all regression results at different time durations. We can first see that the first panel is the situation in the full-time period regardless of the market state. Variance risk premium maintains significant statistics at one-, three-, six-, nine-, twelve-, and fifteen-month return horizons; it peaks at one month and decreases

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after that.

In the bull market state panel, The result exhibits that the strength of the forecast ($Adj\,R^2(\%)$) or statistical significance (t-statistic) is more muscular than the original total samples. The adjusted R squared in the bull market is in the range of 6.2% to 39.8%. Furthermore, the t-statistic is above 2 in our total testing return horizons of up to twenty-four-month. On the contrary, the predictability in the bearish market drops sharply when the horizon comes to nine-month and the adjusted R squared is much lower than the value in the bullish market.

The main reason for this conclusion is that the range of variance risk premium values in the bear market is very different from that in bull markets, and the magnitude and speed of price changes in the bull market are more moderate than in the bear market. However, most of the time, the market is labeled as the bull state; in other words, we can say that bull markets mainly contribute to the significance of variance risk premium in the entire sample forecast.

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4 Conclusion and future works

4.1 Conclusion

This paper uses the high-frequency S&P 500 futures data for twenty years to thoroughly discuss how variance risk premium predictability contrasts with bullish and bearish market states. The results reveal that variance risk premium foreseeability counts robustly on the market state.

Under our empirical experiments, we successfully label the three famous financial crises as bear markets by using the algorithm from Lunde and Timmermann (2004). It denotes a prominent statement that the bull market carries positive returns with low volatility; in contrast, the bear market denotes negative returns with high volatility and much bumpy than before.

Due to wider fluctuation, the traditional predictors lose effectiveness, but the variance risk premium still shows predictability and is even better than before. When we separate the market into bull and bear, variance risk premium significantly outperforms the predicted horizons up to twenty-four-month in bull than only six-month in bear market states. Moreover, the forecasting ability diminishes comparably quickly with extending horizons in the bear market state.

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4.2 Future works

Based on our work and previous conclusion, we assume that variance risk premium is a predictor in the global financial markets. As a result, we can extend this work to other equity markets to check whether the results are diverse in bull and bear markets. In another way, we can change the labeling market states into forecasting bullish or bearish markets to construct this state-sensitive predictor as an indicator in a long-short portfolio.

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