Int. J. Production Economics 223 (2020) 107514

Contents lists available at ScienceDirect

International Journal of Production Economics

journal homepage: http://www.elsevier.com/locate/ijpe

Optimal replenishment decisions for perishable products under cash, advance, and credit payments considering carbon tax regulations

Yan Shi [a,b], Zhiyong Zhang [a], Sheng-Chih Chen [c,d], Leopoldo Eduardo Cárdenas-Barrón [e,*], Konstantina Skouri [f]

[a] *School of Economics and Commerce, South China University of Technology, Guangzhou, PR China*
[b] *School of Mathematics and Computer Science, Hezhou University, Hezhou, PR China*
[c] *Master's Program of Digital Content and Technologies, College of Communication, National ChengChi University, Taipei, 11605, Taiwan, ROC*
[d] *Graduate Institute of Development Studies, National ChengChi University, Taipei, 11605, Taiwan, ROC*
[e] *Department of Industrial and Systems Engineering, School of Engineering and Sciences, Tecnológico de Monterrey, E. Garza Sada 2501 Sur, Monterrey, Nuevo León, C. P. 64849, Mexico*
[f] *Department of Mathematics, University of Ioannina, Greece*

ARTICLE INFO

Keywords:
Environmental regulations
Inventory
Payment types
Perishable goods
Expiration dates

ABSTRACT

This study integrates payment schemes and inventory decisions under sustainability issues. In view of legislation to reduce carbon emissions, the aim of the paper is to gain insights into how payment options affect inventory decisions for perishable products under the widely-used carbon tax regulation. Precisely, this paper establishes an inventory replenishment model from the buyer's perspective in which: (a) the buyer is charged for each unit carbon emissions with a constant tax rate (i.e., carbon-tax regulation) (b) the seller offers one of three commonly-used payment schemes (i.e., advance payment, cash payment, and credit payment), and (c) the product gradually degrades with time and cannot be sold after its expiration date (or sell-by date). Then, the existence and uniqueness of the optimal solution under each payment term is proven, which simplifies the search for the global solution to a local minimum. Finally, numerical experiments are conducted and the results among those three payment schemes are compared in order to answer the following two major questions: How does the payment scheme affect carbon emissions? Which payment scheme is the least expensive for the buyer, when carbon taxes are charged? Computational results provide evidences that the advance payment in companion with price discount is the least costly for the buyer but leads to highest carbon emissions per unit time among all three payment schemes. On the other hand, the credit payment is the best of all three payment schemes to curb carbon emissions and thus protect the climate and environment.

1. Introduction

Both sellers and buyers use a variety of payment terms to settle their business transactions of goods and services. In general, there are three commonly-used payment terms: (1) cash in advance in which the buyer pays the seller the total purchase cost prior to delivery (i.e., advance payment), (2) cash on delivery in which the buyer pays off the total purchase cost upon receiving goods or services (i.e., cash payment), and (3) permissible delay in payment in which the seller grants a short-term interest-free loan to the buyer (i.e., credit payment).

In the Inventory literature, Harris (1913) adopted a cash payment to develop the traditional economic order quantity (EOQ) model. Feng et al. (2017) then explored this cash-on-delivery (COD) model for perishable goods when demand rate depends on selling price, displayed stocks, and expiration date. Currently, Chen (2018) discussed production and inventory decisions with a cash payment for a single-manufacturer multi-retailer system of perishable products. Goyal (1985) established an EOQ model where the seller grants a credit payment. Wu et al. (2014b) further investigated the optimal ordering policy with price and stock sensitive demand under an upstream credit payment. Zhang (1996) built an optimal payment scheme to pay for small-amount bills in advance in order to save time and money. Teng

* Corresponding author.
E-mail addresses: shiyanhzxy@163.com (Y. Shi), zhyzhang@scut.edu.cn (Z. Zhang), scchen@nccu.edu.tw (S.-C. Chen), lecarden@tec.mx (L.E. Cárdenas-Barrón), kskouri@uoi.gr (K. Skouri).

https://doi.org/10.1016/j.ijpe.2019.09.035
Received 29 January 2019; Received in revised form 2 September 2019; Accepted 30 September 2019
Available online 4 October 2019

et al. (2016) expanded this advance payment to perishable products with expiration dates. Teng (2009) used a cash-credit payment (i.e., some in cash and the remainder in credit) to reduce default risks with credit-risk customers. Wu et al. (2016) further expanded this downstream partial trade credit to perishable items with maximum life time. Taleizadeh (2014) proposed an EOQ model with an advance-cash payment (i.e., some in advance and the remainder in cash) for evaporating items. Concurrently, Zhang et al. (2014) set up an EOQ model where the seller offers a price discount with an advance-credit payment (i.e., some in advance and the remainder in credit). Recently, Li et al. (2017), Wu et al. (2018a), as well as Chang et al. (2019) generalized all of the models mentioned above to explore an advance-cash-credit (ACC) payment (i.e., some in advance, some as cash-on-delivery, and the remainder in credit).

It is evident that the degrading or deterioration rate of a perishable product gradually increases with time and reaches 100% at its expiration date (or sell-by date). Conversely, the demand rate for a perishable item is gradually decreasing over time and near zero when the product reaches its expiration date because the product is no longer fresh and cannot be sold after its expiration date. Furthermore, health-conscious customers can easily identify the freshness of a perishable product by checking its expiration date. As a result, the expiration date of a perishable product is an important factor in a customer's purchasing decision. However, relatively little attention has been paid to the importance of the expiration date until recently. Sarkar (2012) explored an EOQ model by incorporating the fact that the deterioration rate of a perishable product increases over time and reaches 100% at its sell-by date. Chen and Teng (2014) adopted the deterioration rate linked to expiration date to develop an EOQ model where the seller offers buyers an upstream credit payment to attract more sales. Wu et al. (2014a) further expanded the model into a supplier-retailer-buyer supply chain in which the supplier offers the retailer an upstream credit payment, while the retailer in turn provides a downstream credit payment to buyers. Teng et al. (2016) studied the case in which the supplier asks the retailer to pay the acquisition cost in an advance-cash payment. Wu et al. (2018a) expanded the case to which the supplier asks the retailer an ACC payment, while the retailer offered a credit payment to customers. Li et al. (2017) further explored the problem by adding a pricing strategy and using a discounted cash-flow analysis. Currently, Tiwari et al. (2018) developed joint pricing and lot-sizing model for deteriorating items with upstream and downstream cash-credit payments as well as allowable shortages. Li and Teng (2018) studied pricing and lot-sizing decisions for perishable goods when demand rate depends on selling price, reference price, product freshness, and displayed stocks. Feng and Chan (2019) derived optimal pricing and cycle time for new products under upstream and downstream credit payments. Recently, Li et al. (2019a) explored pricing, lot-sizing, and backordering decisions when a seller demands an advance-cash-credit payment scheme.

There is growing consensus that carbon emissions generated from firms' business activities can lead to major global climate change. In addition, the economic benefits from reducing carbon emissions, by far the largest of which, is the mitigation of air pollution which is now a major challenge for cities across the world. In China, for example, the economic cost of the 1.23 million air pollution-related deaths in 2010 amounted to 9.7–13.2% of China's GDP. In the US the cost of 103,027 air pollution-related deaths was equivalent to 3.2–4.6% US GDP. In the UK 23,036 air pollution-related deaths cost the equivalent of 4.6–7.1% of GDP. As a result, to a society or country, the economic benefits of reducing carbon emissions outweigh the cost of mitigation carbon emissions. As a result, governments and factories are under growing pressure to curb the amount of carbon emissions. Factories can reduce their carbon footprint by replacing energy-inefficient equipment and facilities, redesigning products and packaging, modifying batch sizes, etc. However, most of the previous studies on inventory models focus on maximizing profit or minimizing cost. Only a few of them take environmental issues, such as the reduction in carbon emissions, into consideration.

Benjaafar et al. (2013) analyzed the effect of different emissions regulations and concluded that firms could effectively reduce their carbon emissions by making operational adjustments and by collaborating with other members of their supply chain. He et al. (2015) derived optimal lot-size and emissions under the two most widely-used carbon regulations to curb the carbon emissions generated from firms: carbon-tax (i.e., firms are charged a dollar amount for every ton of emissions they produce) and cap-and-trade (i.e., governments issue a set number of emissions "allowances" to firms each year. These allowances can be auctioned to the highest bidder as well as traded on secondary markets to create a carbon price). Dye and Yang (2015) quantified the impact of credit period (i.e., the length of a credit payment) and the impact of environmental regulations on inventory management when demand rate depends on credit period. Xu et al. (2016) explored the joint production and pricing decisions for multiple products under both carbon-tax and cap-and-trade regulations. Tsao et al. (2017) further extended Dye and Yang (2015) by considering default risks (i.e., the risk that buyers will be unable to make the required credit payments on their debt obligations) from granting credit payments in newsvendor models. Recently, Aljazzar et al. (2018) studied the use of credit payments in a two-echelon coordinated supply chain, and found that using credit payments will reduce carbon emissions generated within the supply chain.

No studies in the existing literature have considered the impacts of three commonly-used payment types (i.e. advance, cash, and credit payments) on both the total amount of carbon emissions per unit time and the total relevant cost per unit time. To fill the gap in the literature, we develop an EOQ model in which demand rate of a perishable product is linked to the expiration date, buyers are charged with a fixed carbon-tax, and the seller asks buyers to pay the purchase cost by one of the following three payment schemes: advance payment, cash payment, or credit payment. We then mathematically demonstrate that the optimal solution uniquely exists to the proposed problem under each of the three payment schemes, which simplifies the search for the global solution to a local minimum. Furthermore, we run several numerical examples, and try to answer the following two major questions: What is the impact of different payment schemes on the optimal total relevant cost per unit time? Which payment scheme produces the least total carbon emissions? Next, we perform a sensitivity analysis on the buyer's total relevant cost, and then explore managerial insights. Finally, we believe that the findings in this study will be of interest to readers because they bring new and important light to the field of inventory control and carbon reduction with various payment types.

The remainder of the paper is organized as follows. Section 2 presents notation used and assumptions made to build the model. The mathematical models for different payment schemes are formulated in Section 3. Theoretical results are established in Section 4. Section 5 presents numerical examples to provide management insights. Concluding remarks and future research are presented in Section 6. To make the paper easy to understand, all proofs are presented in appendixes.

2. Notation and assumptions

The following notation and assumptions are introduced to construct an EOQ inventory model for perishable products under cash, advance, and credit payments in which firms are charged carbon-tax.

2.1. Notation

The following parameters and variables are used in developing the problem.

c_h	The holding cost (excluding interest) per unit per unit time in dollars
$\widehat{c}_h$	The amount of carbon emissions per unit per unit time in inventory
c_p	The purchase cost per unit in dollars
$\widehat{c}_p$	The amount of carbon emissions associated per unit purchased
CE	The total amount of carbon emissions per replenishment cycle
D	The market annual demand rate in units
I_c	The interest charged per dollar per unit time
I_e	The interest earned per dollar per unit time
$I(t)$	The inventory level at time t in units
K	The ordering cost per order in dollars
$\widehat{K}$	The amount of carbon emissions per order
L	The advance payment period in units of time
m	The time to expiration date or the maximum shelf life in units of time, $m > 0$
M	The credit period in units of time granted by the seller to the buyer, $M > 0$
n	The number of equal installments in an advance payment
Q	The buyer's order quantity in units
r_2	The rate of price discount for an advance payment
s	The buyer's selling price per unit in dollars, $s > c_p > 0$
$\theta(t)$	The degrading rate at time t, $0 \le \theta(t) \le 1$
τ	The tax paid on each carbon unit emitted (the carbon unit price)
Decision variable	
T	The buyer's replenishment cycle length in units of time, $0 \le T \le m$

For convenience, the asterisk symbol on a variable denotes the optimal solution of the variable. For example, T^* is the optimal solution of T.

2.2. Assumptions

All perishable products continuously degrade over time and cannot be sold when time exceeds the expiration date m. Following Sarkar (2012), Wu et al. (2014a), Wang et al. (2014), Chen and Teng (2014), Teng et al. (2016), Wu et al. (2017, 2018b), and Li et al. (2019b), we assume the following degrading rate:

$$\theta(t) = \frac{1}{1+m-t}, 0 \le t \le m. \tag{1}$$

There is no replacement, repair, financing, or salvage value of perished items during the replenishment cycle $[0, T]$.

In general, the seller asks the buyer to pay the total purchase cost in one of the following three most commonly-used payment schemes: advance payment, cash payment, or credit payment. To avoid default risks or to forecast demand more accurately, the seller may ask the buyer to prepay the total purchase cost with n equal installments in L units of time prior to the time of delivery (i.e., an advance payment). On the other hand, to increase demand, the seller may grant the buyer an interest-free credit period M to settle the total purchase cost (i.e., a credit payment).

Requesting the buyer to make an advance payment not only gives the seller interest earned, but also has no default risk (to the seller). However, it is evident that the longer the prepayment period, the lower the demand rate because the buyer prefers "buy now and pay later" to "pay now and get later". As a result, the seller usually offers a price discount as an incentive to encourage the buyer to prepay the total purchase cost.

In contrast to an advance payment, the longer the credit period, the higher the sales volume and the higher the default risk, as described in Li et al. (2017, 2018). Hence, granting buyers a credit payment has a positive impact on sales while having negative impacts on both interest loss and default risk.

To curb the amount of carbon emissions, following Benjaafar et al. (2013) and Xiang and Lawley (2018), we assume that the buyer is charged a fixed dollar amount τ for every ton of emissions produced (i.e., a carbon tax regulation) by the local government.

The perishable product cannot be sold after the expiration date m. Hence, we assume $T \le m$. In addition, the longer the credit period, the higher the default risk. Therefore, there are no good reasons for the seller to grant a credit period M which is longer than m, so we assume that $M \le m$.

Furthermore, most perishable goods such as meat and seafood, fruits and vegetables, etc. have very short shelf life. Health conscious consumers like to examine them personally prior to purchase. Hence, it is reasonable to assume that shortages are not allowed.

Finally, it assumes that the replenishment rate is infinite, and lead time is negligible.

3. Mathematical model

Given the above notation and assumptions, $I(t)$, the inventory level at time t during the replenishment cycle $[0, T]$ is depleted by demand and deterioration, and hence governed by the following differential equation:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D, 0 \le t \le T \le m, \tag{2}$$

Solving (2) with boundary condition $I(T) = 0$ yields

$$I(t) = e^{-\int_0^t \theta(v)dv} \int_t^T De^{\int_0^x \theta(v)dv} dx = \int_t^T De^{\int_t^x \theta(v)dv} dx. \tag{3}$$

Therefore, the order quantity per replenishment cycle time T is as follows:

$$Q = I(0) = \int_0^T De^{\int_0^x \theta(v)dv} dx. \tag{4}$$

The ordering cost per cycle is $OC = K$. The holding cost per cycle time T is given by:

$$HC = c_h \int_0^T I(t)dt = c_h \int_0^T \int_t^T De^{\int_t^x \theta(v)dv} dxdt. \tag{5}$$

Following Benjaafar et al. (2013) and Xiang and Lawley (2018), the total amount of carbon emissions per replenishment cycle includes the fixed carbon emissions associated with placing an order (e.g., carbon emissions due to transportation and production) $\widehat{K}$, the variable amount of carbon emissions associated with each unit (e.g., carbon emissions due to the handling of each unit) $\widehat{c}_p$ multiplied by the order quantity Q, and the integration of the amount of carbon emissions associated with the storage of each unit held per unit of time $\widehat{c}_h$ (e.g., carbon emissions involved such as refrigeration in the storage of each unit) multiplied by inventory level $I(t)$ throughout the replenishment cycle. Therefore, the total amount of carbon emissions per replenishment cycle is given by:

$$CE = \widehat{K} + \widehat{c}_p Q + \widehat{c}_h \int_0^T I(t)dt. \tag{6}$$

Next, the case of cash payment is discussed, then advance payment is studied, and finally credit payment is analyzed.

3.1. The case of cash payment

In this case, the buyer pays for the items upon receipt of them. Hence, the total acquisition (or purchase) cost AC_1 and the interest charged IC_1 per cycle respectively are as follows:

$$AC_1 = c_p Q = c_p \int_0^T De^{\int_0^x \theta(v)dv} dx, \tag{7}$$

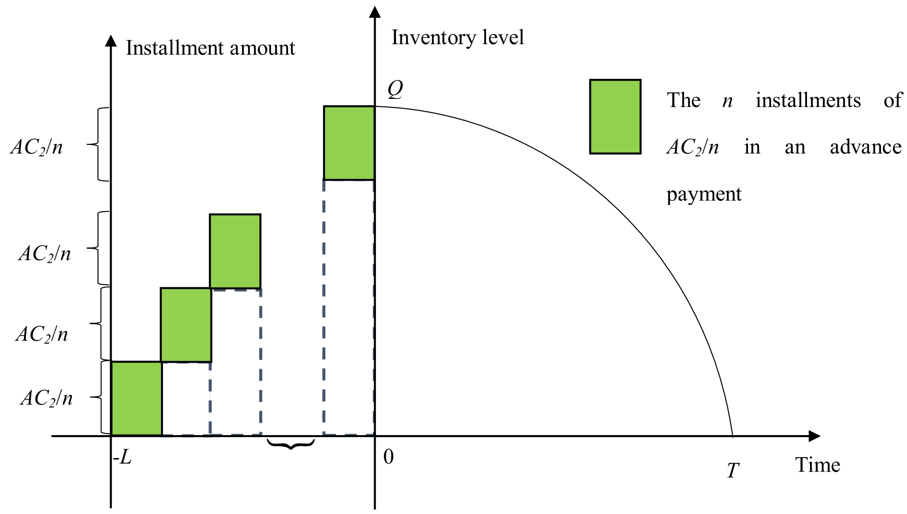

Fig. 1. Graphical representation for the inventory system with an advance payment.

and

$$IC_1 = I_c c_p \int_0^T I(t)dt = I_c c_p \int_0^T \int_t^T De^{\int_t^x \theta(v)dv} dxdt. \tag{8}$$

Combining the results from (5)–(8), the total relevant cost per unit time including ordering cost, purchase cost, holding cost excluding interest charged, interest charged for on-hand inventory, and the cost of carbon emissions is obtained as follows:

$$TC_1(T) = \frac{1}{T}(OC + AC_1 + HC + IC_1 + \tau \cdot CE)$$

$$= \frac{1}{T}\left[(K + \tau\widehat{K}) + (c_p + \tau\widehat{c}_p)\int_0^T De^{\int_0^x \theta(v)dv}dx + (c_h + \tau\widehat{c}_h + I_c c_p)\int_0^T \times \int_t^T De^{\int_t^x \theta(v)dv} dxdt\right]. \tag{9}$$

3.2. The case of advance payment

In this case, the seller asks the buyer to prepay the acquisition cost AC_2 with n equal installments during L units of time prior to the time of delivery. In general, the seller usually offers the buyer a price discount r_2 as an incentive to induce the buyer's willing to pay in advance.

Consequently, the buyer's acquisition cost per cycle time T is

$$AC_2 = (1 - r_2)c_p Q = (1 - r_2)c_p \int_0^T De^{\int_0^x \theta(v)dv} dx. \tag{10}$$

Similar to Teng et al. (2016), the capital cost per cycle prior to delivery for the advance payment as depicted in Fig. 1 is given by:

$$CC_2 = I_c \cdot \frac{AC_2}{n} \cdot \frac{L}{n} \cdot (1 + 2 + \ldots + n) = \frac{1+n}{2n} I_c L(1 - r_2)c_p \int_0^T De^{\int_0^x \theta(v)dv} dx. \tag{11}$$

Furthermore, the interest charged for on-hand inventory is as follows:

$$IC_2 = I_c(1 - r_2)c_p \int_0^T I(t)dt = I_c(1 - r_2)c_p \int_0^T \int_t^T De^{\int_t^x \theta(v)dv} dxdt. \tag{12}$$

Combining the results in (5), (6), and (11)–(13), the total relevant cost per unit time in advance payment is:

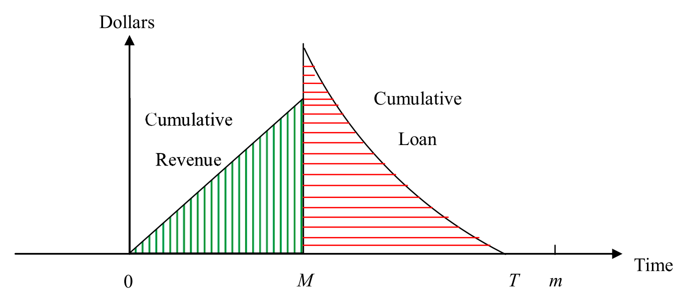

Fig. 2. Graphical representation of the case of $M \le T$.

$$TC_2(T) = \frac{1}{T}(OC + AC_2 + HC + CC_2 + IC_2 + \tau \cdot CE) = \frac{1}{T}\Big\{(K + \tau\widehat{K}) + \left[(1 - r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n} I_c L(1 - r_2)c_p\right]\int_0^T De^{\int_0^x \theta(v)dv} dx + \left[c_h + \tau\widehat{c}_h + I_c(1 - r_2)c_p\right]\int_0^T \int_t^T De^{\int_t^x \theta(v)dv} dxdt\Big\} \tag{13}$$

3.3. The case of credit payment

In this case, the buyer receives a fixed credit period M from the seller. It is clear that the buyer's acquisition cost per replenishment cycle time T is

$$AC_3 = c_p Q = c_p \int_0^T De^{\int_0^x \theta(v)dv} dx. \tag{14}$$

As to the interest charged and interest earned in this case, based on the values of M and T, there are two potential cases: (1) $M \le T$. and (2) $M \ge T$. The sub-case of $M \le T$ is discussed first, and then the other sub-case of $M \ge T$.

3.3.1. Sub-case 1 of $M \le T$

In this sub-case, since $M \le T$ the buyer needs to pay the interest for the items in stock after time M as depicted in Fig. 2. Hence, the interest charged per cycle is as follows:

$$IC_{31} = I_c c_p \int_M^T I(t)dx = I_c c_p \int_M^T \int_t^T De^{\int_t^x \theta(v)dv} dxdt. \tag{15}$$

On the other hand, the interest earned from time 0 to M is the interest rate I_e multiplied by the area of the green triangle as shown in Fig. 2.

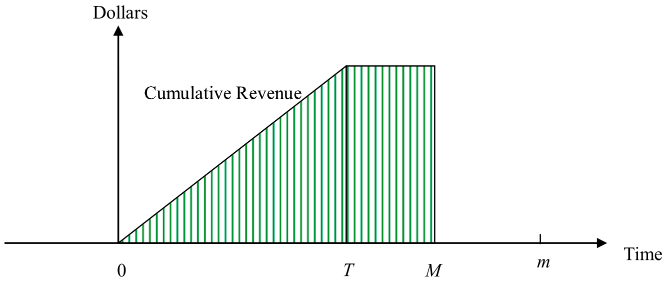

Fig. 3. Graphical representation of the case of $M \geq T$.

Since the cumulative revenue at time M is sDM, the interest earned per replenishment cycle is given by:

$$IE_{31} = \frac{1}{2} I_e (sDM) M = \frac{1}{2} I_e sDM^2. \tag{16}$$

Combining (5), (6) and (15)–(17), the total relevant cost per unit time in the credit payment with $M \leq T$ is:

$$TC_{31}(T) = \frac{1}{T}(OC + AC_3 + HC + IC_{31} - IE_{31} + \tau \cdot CE)$$

$$= \frac{1}{T}\Bigg[(K + \tau \widehat{K}) + (c_p + \tau \widehat{c}_p)\int_0^T De^{\int_0^x \theta(v)dv}dx + (c_h + \tau \widehat{c}_h)\int_0^T \int_t^T De^{\int_t^x \theta(v)dv}dxdt$$

$$+ I_c c_p \int_M^T \int_t^T De^{\int_t^x \theta(v)dv}dxdt - \frac{1}{2} I_e sDM^2\Bigg] \tag{17}$$

3.3.2. Sub-case 2 $M \geq T$

In this sub-case, the replenishment cycle time T is shorter than or equal to the credit period M. Consequently, there is no interest charged, hence, $IC_{32} = 0$. However, the interest earned in this sub-case is the interest rate I_e multiplied by the area of the green trapezoid on the interval [0, M] as shown in Fig. 3. Since the cumulative revenue at time T is sDT, the interest earned per replenishment cycle is given by:

$$IE_{32} = \frac{1}{2} I_e sDT^2 + I_e s(M - T)DT = I_e sDT\left(M - \frac{1}{2}T\right). \tag{18}$$

Similarly, combining (5), (6), and (18), the total relevant cost per unit time in the credit payment with $M \geq T$ is as follows:

$$TC_{32}(T) = \frac{1}{T}(OC + AC_3 + HC + IC_{32} - IE_{32} + \tau \cdot CE)$$

$$= \frac{1}{T}\Bigg[(K + \tau \widehat{K}) + (c_p + \tau \widehat{c}_p)\int_0^T De^{\int_0^x \theta(v)dv}dx + (c_h + \tau \widehat{c}_h)\int_0^T \int_t^T De^{\int_t^x \theta(v)dv}dxdt$$

$$- I_e sDT\left(M - \frac{1}{2}T\right)\Bigg] \tag{19}$$

Consequently, the buyer's total cost per unit time under the case of credit payment is given by

$$TC_3(T) = \begin{cases} TC_{31}(T), & T \geq M, \\ TC_{32}(T), & T \leq M. \end{cases} \tag{20}$$

It is clear that $TC_{31}(M) = TC_{32}(M)$, so $TC_3(T)$ is well defined. In the following section, theoretical results, optimal solutions, and important propositions are derived.

4. Theoretical results

This section studies the property of the total relevant cost per unit time and derives the optimal replenishment cycle time under the three payment options above. Likewise, the case of cash payment is discussed first, then the case of advance payment is explored, and finally the case of credit payment is studied.

4.1. The case of cash payment

To obtain theoretical results, we define the following discrimination term

$$\Delta_1 = -T^2 \frac{dTC_1(T)}{dT}\bigg|_{T=m} = -m^2 \frac{dTC_1(m)}{dT}$$

$$= K + \tau \widehat{K} - (c_p + \tau \widehat{c}_p)\left[mDe^{\int_0^m \theta(v)dv} - \int_0^m De^{\int_0^x \theta(v)dv}dx\right]$$

$$- (c_h + \tau \widehat{c}_h + I_c c_p)\Bigg[m\int_0^m De^{\int_t^m \theta(v)dv}dt$$

$$- \int_0^m \int_t^m De^{\int_t^x \theta(v)dv}dxdt\Bigg]. \tag{21}$$

Theorem 1. .(a) $TC_1(T)$ is a strictly pseudo-convex function of T, and hence there exists a unique minimum solution T_1^*.

(b) If $\Delta_1 \geq 0$, then $TC_1(T)$ is minimized at $T_1^* = m$.
(c) If $\Delta_1 < 0$, then there exists a unique $T_1^* \in (0, m)$ that minimizes $TC_1(T)$.

Proof. . See Appendix A.

Theorem 1 simplifies the search for the global solution to a local minimum. A simple interpretation of Theorem 1 is as follows: It is clear from (21) that Δ_1 has an opposite sign of the first-order derivative of $TC_1(T)$ with respect to T at T = m. We know from Part (a) of Theorem 1 that $TC_1(T)$ is a strictly pseudo-convex function of T. Since $\frac{dTC_1(T)}{dT}\Big|_{T=0} < 0$, $\Delta_1 \geq 0$ implies $\frac{dTC_1(T)}{dT}\Big|_{T=m} \leq 0$. Consequently, $TC_1(T)$ is decreasing in [0, m), so $TC_1(T)$ is minimized at $T_1^* = m$. On the other hand, if $\Delta_1 < 0$, then $\frac{dTC_1(T)}{dT}\Big|_{T=m} > 0$. Therefore, there exists a unique $T_1^* \in (0, m)$ such that $\frac{dTC_1(T)}{dT}\Big|_{T=T_1^*} = 0$, which minimizes $TC_1(T)$.

The impact of carbon unit price on the optimal carbon emissions per unit time is derived by the following proposition.

Proposition 1. (a) An increase in τ, c_h, c_p, I_c, or K increases $TC_1(T)$.
(b) $\frac{d}{d\tau}\left(\frac{CE}{T}\right)\Big|_{T=T_1^*} < 0$.

Proof. . See Appendix B.

Proposition 1 shows that the optimal total relevant cost per unit time increases as the carbon tax τ, the holding cost c_h, the purchasing cost c_p, the interest charged I_c, or the ordering cost K increases, which is trivial. Proposition 1 also demonstrates that the higher the carbon unit price, the lower the total amount of c00arbon emissions per unit time, encouraging firms to reduce carbon emissions.

4.2. The case of advance payment

Similarly, for convenience, we define another discrimination term

$$\Delta_2 = -T^2 \frac{dTC_2(T)}{dT}\bigg|_{T=m} = -m^2 \frac{dTC_2(m)}{dT}$$

$$= K + \tau\widehat{K} - \left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n} I_c L (1-r_2)c_p\right]\left[mDe^{\int_0^m \theta(v)dv} - \int_0^m De^{\int_0^x \theta(v)dv}dx\right]$$

$$-\left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]\left[m\int_0^m De^{\int_t^m \theta(v)dv}dt - \int_0^m\int_t^m De^{\int_t^x \theta(v)dv}dxdt\right]. \tag{22}$$

Then one can obtain Theorem 2 below.

Theorem 2. .(a) $TC_2(T)$ is a strictly pseudo-convex function in T, and hence there exists a unique minimum solution T_2^*.

(b) If $\Delta_2 \geq 0$, then $TC_2(T)$ is minimized at $T_2^* = m$.
(c) If $\Delta_2 < 0$, then there exists a unique $T_2^* \in (0, m)$ that minimizes $TC_2(T)$.

Proof. . See Appendix C.

Similar to the interpretation of Theorem 1, it is obvious from (22) that Δ_2 has an opposite sign of the first-order derivative of $TC_2(T)$ with respect to T at T = m. From Part (a) of Theorem 2, we know that $TC_2(T)$ is a strictly pseudo-convex function in T. Since $\frac{dTC_2(T)}{dT}|_{T=0} < 0$, if $\Delta_2 \geq 0$ then $\frac{dTC_2(T)}{dT}|_{T=m} \leq 0$, which implies that $TC_2(T)$ is decreasing in [0, m). Hence, $TC_2(T)$ is minimized at $T_2^* = m$.Otherwise, $\Delta_2 < 0$ implies $\frac{dTC_2(T)}{dT}|_{T=m} > 0$. As a result, there exists a unique $T_2^* \in (0, m)$ such that $\frac{dTC_2(T)}{dT}|_{T=T_2^*} = 0$, which minimizes $TC_2(T)$.

The effect of each important parameter on the objective function is analytically explored as described below.

Proposition 2. (a) An increase in τ, c_h, c_p, I_c, K, or L increases $TC_2(T)$, while an increase in r_2, or n decreases $TC_2(T)$.

(b) For the advance payment, an increase in τ decreases the optimal carbon emissions per unit time, i.e., $\frac{d}{d\tau}\left(\frac{CE}{T}\right)|_{T=T_2^*} < 0$.

Proof. . See Appendix D.

Proposition 2 demonstrates that the optimal total relevant cost per unit time increases as the carbon tax τ, the holding cost c_h, the purchasing cost c_p, the interest charged I_c, the ordering cost K, or the prepayment length L increases, which is obvious. By contrast, the optimal total relevant cost per unit time decreases as price discount rate r_2, or number of installments n increases. A simple economic interpretation is as follows: An increase in price discount means a decrease in purchase cost, which results in lowering total relevant cost. Likewise, an increase in the number of installments reduces the interest charged for an advance payment, which in turn decreases total relevant cost.

4.3. The case of credit payment

There are two potential sub-cases: (1) $M \leq T$, and (2)$M \geq T$ for the case of credit payment. We investigate them according to their order.

4.3.1. Sub-case of $M \leq T$

Again, for convenience, we set the following two discrimination terms.

$$\Delta_{311} = -M^2 \frac{dTC_{31}(M)}{dT} = K + \tau\widehat{K} - \left(c_p + \tau\widehat{c}_p\right)\left[MDe^{\int_0^M \theta(v)dv} - \int_0^M De^{\int_0^x \theta(v)dv}dx\right]$$

$$-\left(c_h + \tau\widehat{c}_h\right)\left[M\int_0^M De^{\int_t^M \theta(v)dv}dt - \int_0^M\int_t^M De^{\int_t^x \theta(v)dv}dxdt\right] - \frac{1}{2}I_e sDM^2. \tag{23}$$

and

$$\Delta_{312} = -m^2 \frac{dTC_{31}(m)}{dT} = K + \tau\widehat{K} - \left(c_p + \tau\widehat{c}_p\right)\left[mDe^{\int_0^m \theta(v)dv} - \int_0^m De^{\int_0^x \theta(v)dv}dx\right]$$

$$-\left(c_h + \tau\widehat{c}_h\right)\left[m\int_0^m De^{\int_t^m \theta(v)dv}dt - \int_0^m\int_t^m De^{\int_t^x \theta(v)dv}dxdt\right]$$

$$-I_c c_p\left[m\int_M^m De^{\int_t^m \theta(v)dv}dt - \int_M^m\int_t^m De^{\int_t^x \theta(v)dv}dxdt\right] - \frac{1}{2}I_e sDM^2. \tag{24}$$

Then we have the following results.

Lemma 1. $\Delta_{311} > \Delta_{312}$.

Proof. . See Appendix E.

Theorem 3. (a) $TC_{31}(T)$ is a strictly pseudo-convex function in T, and hence there exists a unique solution T_{31}^* that minimizes $TC_{31}(T)$.
(b) If $\Delta_{312} \geq 0$, then $TC_{31}(T)$ is minimized at $T_{31}^* = m$.
(c) If $\Delta_{311} > 0$, and $\Delta_{312} < 0$, then there exists a unique $T_{31}^* \in (M, m)$ that minimizes $TC_{31}(T)$.
(d) If $\Delta_{311} < 0$, then $TC_{31}(T)$ is minimized at $T_{31}^* = M$.

Proof. . See Appendix F.

A simple interpretation of Theorem 3 is similar to that of Theorem 1.

Proposition 3. (a) An increase in τ, c_p, c_h, I_c, or K increases $TC_3(T)$, while an increase in M, I_e, or s decreases $TC_3(T)$.
(b) For the credit payment, an increase in τ decreases the optimal carbon emissions per unit time, i.e., $\frac{d}{d\tau}\left(\frac{CE}{T}\right)\Big|_{T=T_{31}^*} < 0$.

Proof. See Appendix G.

In the case of credit payment, the buyer gets interest earned from sales revenue during the credit period M and pays interest charged after the credit period ends at M. As a result, the longer the credit period M, the more the interest earned during [0, M] and the less the interest charged during [M, T]. Consequently, the higher the credit period M, the lower the total relevant cost $TC_{31}(T)$.

4.3.2. Sub-case 2 of $M \geq T$

Likewise, for simplicity, we set the following discrimination term.

$$\Delta_{32} = -M^2 \frac{dTC_{32}(M)}{dt} = K + \tau\widehat{K} - \left(c_p + \tau\widehat{c}_p\right)\left[MDe^{\int_0^M \theta(v)dv} - \int_0^M De^{\int_0^x \theta(v)dv}dx\right]$$

$$-\left(c_h + \tau\widehat{c}_h\right)\left[M\int_0^M De^{\int_t^M \theta(v)dv}dt - \int_0^M\int_t^M De^{\int_t^x \theta(v)dv}dxdt\right] - \frac{1}{2}I_e sDM^2. \tag{25}$$

Theorem 4. .(a) $TC_{32}(T)$ is a strictly pseudo-convex function in T, and hence there exists a unique minimum solution T_{32}^*.

(b) If $\Delta_{32} \geq 0$, then $TC_{32}(T)$ is minimized at $T_{32}^* = M$.

Table 1
The optimal solution with respect to r_2 in Example 2.

r_2	T_2^*	Q_2^*	$TC_2(T_2^*)$	$CE_2(T_2^*)/T_2^*$
0.05	0.2485	978.25	37300	21875
0.10	0.2534	999.14	35687	21917
0.20	0.2640	1045.30	32451	22014
0.30	0.2762	1098.80	29198	22130
0.40	0.2903	1161.50	25926	22271

(c) If $\Delta_{32} < 0$, then there exists a unique $T_{32}^* \in (0, M)$ such that $TC_{32}(T)$ is minimized.

Proof. . See Appendix H.

By using an analogous argument of Theorem 1, one can interpret the discrimination term Δ_{32}, and the results of Theorem 4.

From (24) - (26) and Lemma 1, we know that $\Delta_{311} = \Delta_{32} > \Delta_{312}$. Applying Theorems 3 and 4, one can obtain the solution of $TC_3(T)$ in

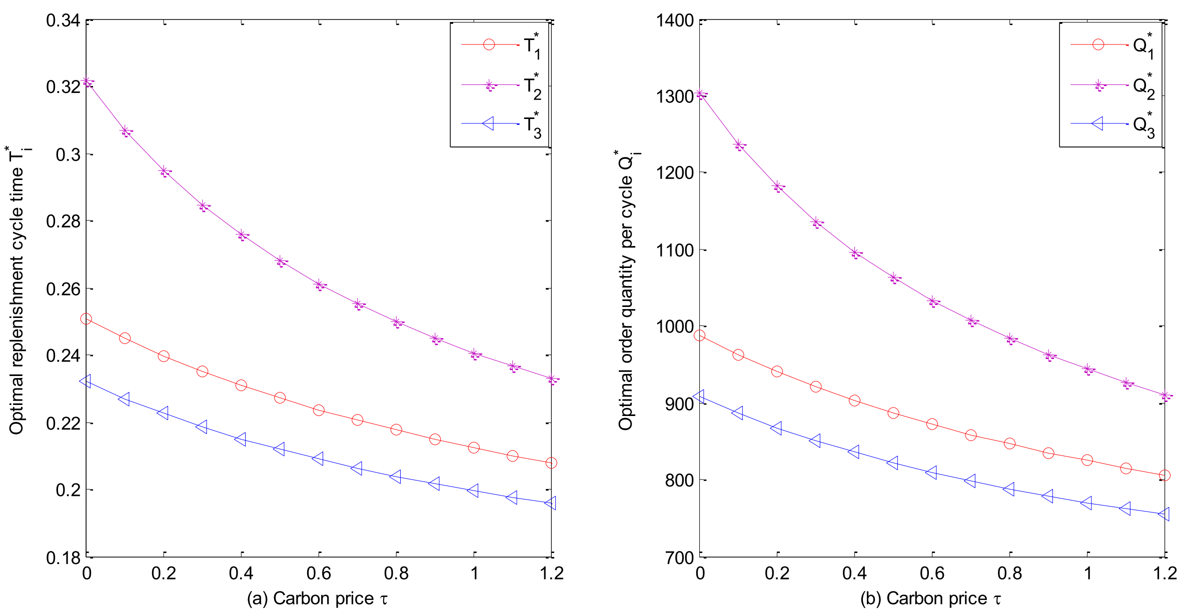

Fig. 4. Cycle time and order quantity with respect to carbon unit price.

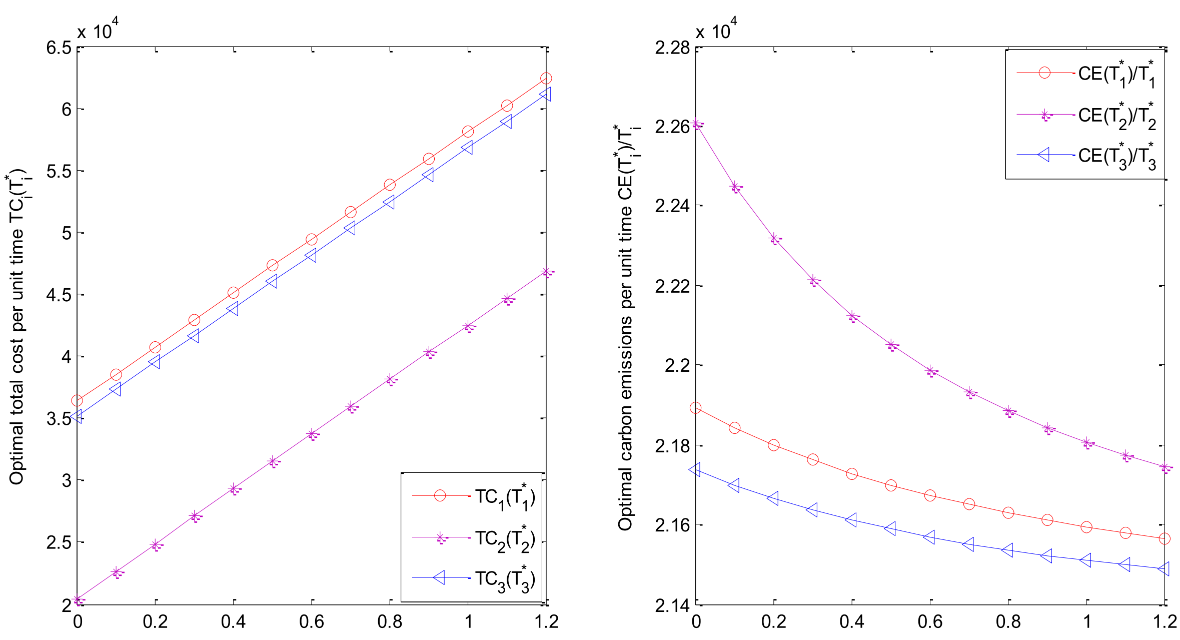

Fig. 5. Total cost and total carbon emission with respect to carbon unit price.

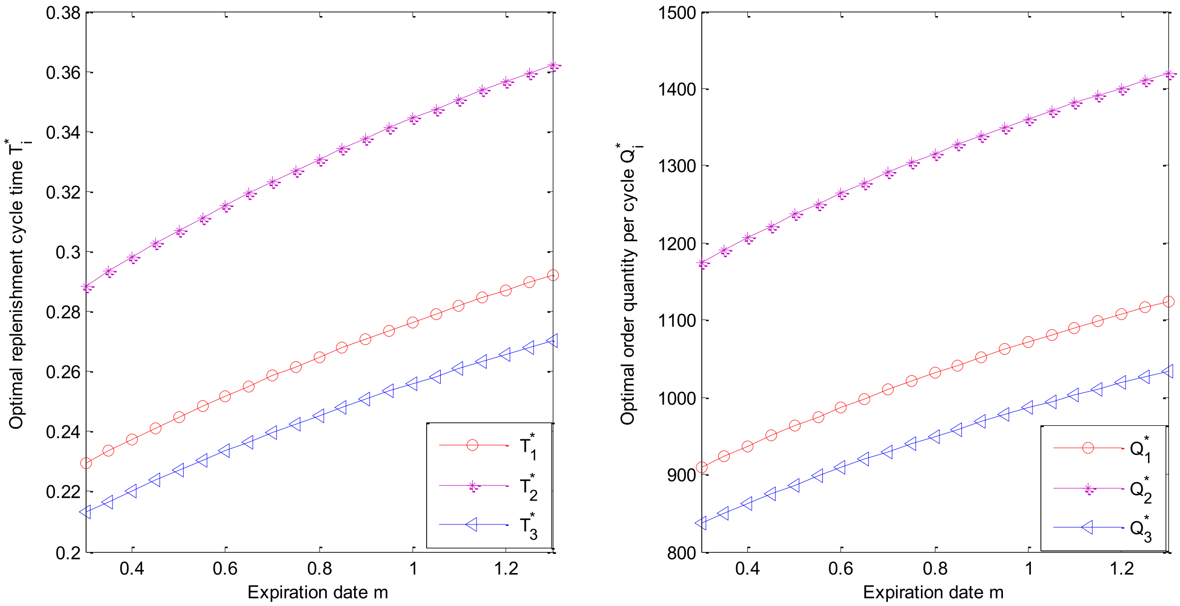

Fig. 6. Cycle time and order quantity with respect to expiration date.

(20) as shown in Theorem 5 below.

Theorem 5. (a) If $\Delta_{311} = \Delta_{32} < 0$, then $TC_3(T)$ is minimized in the sub-case of $M \geq T$, and $T_3^* = T_{32}^*$.

(b) If $\Delta_{311} = \Delta_{32} > 0$, and $\Delta_{312} < 0$, then $TC_3(T)$ is minimized in the sub-case of $M \leq T$, and $T_3^* = T_{31}^*$.

(c) If $\Delta_{312} > 0$, then $TC_3(T)$ is minimized in both sub-cases, and $T_3^* = M$.

Proof. . See Appendix I.

5. Numerical analysis

To illustrate the proposed model under cash, advance, and credit payments, we adopt the data similar to those in Dye and Yang (2015), and study the problem under different payment types according to their order.

Example 1. . For a cash payment, consider an inventory system where $K = 1000$, $c_p = 8$, $c_h = 1$, $\widehat{K} = 250$, $\widehat{c}_p = 5$, $\widehat{c}_h = 2.5$, $\tau = 0.1$, $I_c = 0.1$, $D = 3600$, and $m = 0.5$. Computing $\Delta_1 = -4560.3 < 0$, and applying Theorem 1, one can have the unique optimal solution as follows:

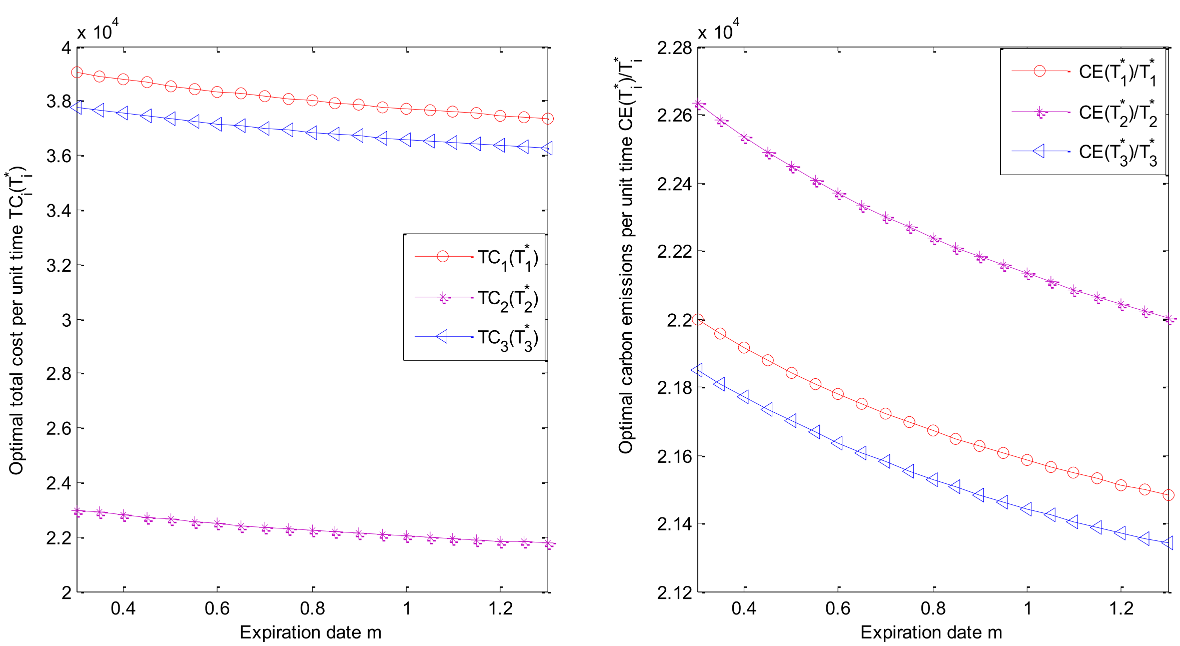

Fig. 7. Total cost and total carbon emission with respect to expiration date.

Table 2
Sensitivity analysis on cash, advance, and credit payments.

Parameters	Cash payment			Advance payment			Credit payment		
	T_1^*	Q_1^*	$TC_1(T_1^*)$	T_2^*	Q_2^*	$TC_2(T_2^*)$	T_3^*	Q_3^*	$TC_3(T_3^*)$
$K = 500$	0.181	692	36208	0.228	890	20768	0.157	595	34709
$K = 1000$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$K = 1500$	0.292	1169	40414	0.364	1502	24120	0.279	1111	39313
$c_p = 5$	0.288	1149	26520	0.349	1430	16377	0.265	1048	25538
$c_p = 8$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$c_p = 11$	0.217	845	50410	0.278	1106	28790	0.203	785	48998
$c_h = 0.5$	0.252	992	38316	0.321	1300	22327	0.234	914	37119
$c_h = 1$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$c_h = 1.5$	0.239	935	38784	0.295	1181	22924	0.221	861	37552
$\widehat{K} = 100$	0.243	955	38492	0.305	1228	22583	0.225	879	37272
$\widehat{K} = 250$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$\widehat{K} = 400$	0.247	969	38614	0.309	1245	22681	0.229	894	37405
$\widehat{c}_p = 1$	0.249	980	36980	0.315	1275	21019	0.231	902	35776
$\widehat{c}_p = 5$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$\widehat{c}_p = 9$	0.241	946	40124	0.299	1202	24241	0.223	871	38899
$\widehat{c}_h = 1.5$	0.246	968	38506	0.310	1249	22572	0.228	892	37295
$\widehat{c}_h = 2.5$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$\widehat{c}_h = 3.5$	0.244	957	38600	0.304	1225	22692	0.226	881	37382
$I_c = 0.05$	0.250	986	38364	0.313	1263	22420	0.229	895	37328
$I_c = 0.1$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$I_c = 0.15$	0.240	941	38739	0.302	1212	22842	0.225	878	37349
$D = 2600$	0.283	815	28922	0.353	1047	17207	0.269	771	28114
$D = 3600$	0.245	962	38553	0.307	1237	22632	0.227	886	37339
$D = 4600$	0.219	1090	48036	0.276	1401	27942	0.198	976	46371

$T_1^* = 0.2449 \in (0, m)$, $Q_1^* = 962.4244$, $TC_1(T_1^*) = 38553$, and $CE_1(T_1^*)/T_1^* = 21843$.

Example 2. . For an advance payment, in addition to those parameters in Example 1, we assume $L = 0.17$, $n = 3$, and $r_2 = 0.5$. Since $\Delta_2 = -2298.3 < 0$, by applying Theorem 2, one can obtain the unique optimal solution as follows:

$T_2^* = 0.3070 \in (0, m)$, $Q_2^* = 1236.6$, $TC_2(T_2^*) = 22632$, and $CE_2(T_2^*)/T_2^* = 22447$.

Similarly, one can obtain the optimal solutions for different values of r_2 as shown in Table 1.

Example 3. . For a credit payment, let $I_e = 0.08$, $M = 0.17$, and $s = 50$. Computing $\Delta_{311} = \Delta_{32} = 400.4875 > 0$ and $\Delta_{312} = -4662.9 < 0$, and then applying Theorem 5, one can get the unique optimal solution as follows:

$$T_3^* = 0.2270,\ Q_3^* = 886.2806,\ TC_3(T_3^*) = 37339, \text{ and } CE_3(T_3^*)/T_3^* = 21699.$$

Comparing the computational results in Examples 1–3, we know that the advance payment with a price discount (from 5% to 50%) is the least expensive for the buyer but creates the greatest damage to the environment due to its highest carbon emissions per unit time among all three payment types: cash, advance, and credit payments. On the other hand, the credit payment is the best of all three payments to curb carbon emissions and protect the climate and environment. Due to the complexity of the problem, we are unable to prove these two main findings mathematically. However, if demand rate of advance payment with a price discount is not significantly different from that of credit payment, then the advance payment with a price discount (which has interest earned and no default risks) is the least expensive to operate for the buyer.

We then investigate the impact of carbon unit price on the optimal solution under each of the three different payment terms. Figs. 4 and 5 reveal that the higher the carbon unit price τ, the lower the cycle time T_i^* ($i = 1,\ 2$, and 3) as well as the order quantity $Q_i^* (i = 1,\ 2$, and 3). In addition, the computational results also reveal that $T_3^* < T_1^* < T_2^*$ and $Q_3^* < Q_1^* < Q_2^*$. A simple economic interpretation is that if the seller offers a heavy price discount r_2 for an advance payment, then the buyer orders more quantity but less frequently (i.e., $Q_1^* < Q_2^*$ and $T_1^* < T_2^*$) to take the advantage of price discount. On the other hand, if the seller grants a permissible delay in payment, then the buyer orders less quantity but more frequently (i.e., $Q_3^* < Q_1^*$ and $T_3^* < T_1^*$) to take the benefits of trade credit often.

Furthermore, Fig. 5 shows that the total relevant cost per unit time $TC_i(T_i^*)$ linearly increases as carbon unit price τ increases with $TC_2(T_2^*) < TC_3(T_3^*) < TC_1(T_1^*)$. A simple economic interpretation of these results are as follows. If the seller offers a heavy price discount to encourage an advance payment, then the buyer reduces a significant amount of the purchase cost. On the other hand, when the seller grants a short-term interest-free loan for a credit payment, the buyer earns interest during credit period. As a result, the total relevant cost of credit payment $TC_3(T_3^*)$ is less than that of cash payment $TC_1(T_1^*)$, which has no interest earned. Combining the above two interpretations, it is obvious that $TC_2(T_2^*) < TC_3(T_3^*) < TC_1(T_1^*)$.

In contrast to the optimal relevant cost, the total carbon emissions per unit time decreases as carbon unit price τ increases with $CE(T_3^*)/T_3^* < CE(T_1^*)/T_1^* < CE(T_2^*)/T_2^*$. A simple economic interpretation is as follows. Since the quantities order is $Q_3^* < Q_1^* < Q_2^*$ as shown in Fig. 4, it is obvious that total carbon emissions per unit time follows the same order as $CE(T_3^*)/T_3^* < CE(T_1^*)/T_1^* < CE(T_2^*)/T_2^*$.

In summary, the buyer prefers an advance payment with a price discount, which results in the least total relevant cost to operate while producing the highest total carbon emissions. Conversely, to reduce carbon emissions and protect the environment, the buyer should select a credit payment, which has the lowest total carbon emissions per unit time among all three payment types.

Next, we study the impact of expiration date on the optimal solution among cash, advance, and credit payments. By assuming $0.3 \le m \le 1.3$, we obtain the computational results as stipulated in Figs. 6 and 7, which reveal that an increase in expiration date m elevates both cycle time and order quantity while reducing both total relevant cost and total carbon emissions.

Fig. 6 depicts the computational results that $T_3^* < T_1^* < T_2^*$ and $Q_3^* < Q_1^* < Q_2^*$, which are similar to the results in Fig. 4. Likewise, the computational result in Fig. 7 is similar to that in Fig. 5. Namely, $CE(T_3^*)/T_3^* < CE(T_1^*)/T_1^* < CE(T_2^*)/T_2^*$.

Example 4. . In this example, we conduct a sensitivity analysis to study how the optimal decision changes with respect to the change in each parameter value due to uncertainty in any decision-making situation. The computational results with respect to cash, advance and credit payments are shown in Table 2.

From Table 2, we obtain the following observations and then provide simple economic interpretations for managerial insights. For convenience, the subscript i represents 1, 2, or 3.

$T_3^* < T_1^* < T_2^*$, $Q_3^* < Q_1^* < Q_2^*$, and $TC_2(T_2^*) < TC_3(T_3^*) < TC_1(T_1^*)$.

(1) An increase in K, or $\widehat{K}$ elevates T_i^*, Q_i^*, and $TC_i(T_i^*)$. As a result, if the ordering cost per order K or the amount of carbon emissions per order $\widehat{K}$ increases, then the buyer should reduce the number of orders to lower expenses, which results in an increase in the order quantity Q_i^* as well as the replenishment cycle T_i^*.
(2) An increase in c_p, $\widehat{c}_p$, c_h, $\widehat{c}_h$, or I_c decreases T_i^*, and Q_i^*, while increasing $TC_i(T_i^*)$. In contrast to the previous observation, if the purchase cost c_p, the amount of carbon emissions associated with each unit purchased $\widehat{c}_p$, the holding cost c_h, the amount of carbon emissions associated with each inventory $\widehat{c}_h$, or the interest charged I_c increases, then the buyer should order less quantity Q_i^* in order to reduce the total purchase cost, the inventory holding cost, or the interest payable. The smaller the order quantity Q_i^*, the shorter the replenishment cycle T_i^*.
(3) An increase in D increases both Q_i^*, and $TC_i(T_i^*)$, while decreasing T_i^*. It is obvious the higher the demand the higher the order quantity Q_i^*, as well as the total relevant cost $TC_i(T_i^*)$. However, to reduce the total inventory holding cost due to higher order quantity Q_i^*, the buyer should shorten the replenishment cycle time T_i^*.
(4) Table 2 reveals that the optimal solutions with respect to cash, advance, and credit payments maintain the following relationship regardless any change in the value of parameter K, c_p, c_h, $\widehat{K}$, $\widehat{c}_p$, $\widehat{c}_h$, I_c, or D.

6. Concluding remarks and future research

This paper has developed an EOQ inventory model in which: (1) The seller offers the buyer three payment schemes: cash payment, advance payment, or credit payment, (2) The degrading rate for a perishable product is linked to its expiration date, and (3) There is a fixed carbon tax to encourage firms to reduce carbon emissions and slow down global climate change. Next, the theoretical results have demonstrated that there exists a unique optimal solution to the problem under each of these three payment types. In addition, the relationships between an important parameter and optimal solution have been characterized. Furthermore, to illustrate the model under three different payment types, numerical examples have been conducted. Finally, sensitivity analyses to examine the impacts of critical parameters on ordering behaviors, optimal total cost per unit time and optimal carbon emissions per unit time have been performed. Comparing the results from three different payment types, several managerial insights have been obtained. For example, the advance payment with a price discount is the least expensive to operate while generating the highest carbon emissions per unit time, and creating the greatest damage to the environment. However, it is estimated by The World Health Organization (2016) that 3 million people die from ambient outdoor pollution every year. As a result, to a society or country, economic benefits of reducing carbon emissions outweigh the cost of mitigation carbon emissions. Therefore, the buyer as an individual should select a credit payment which has the lowest total carbon emissions per unit time among these three payment types.

This research can be extended in several directions in future studies. First, we may extend the carbon-tax policy examined here to include many other types of carbon-tax regulations (e.g., cap-and-trade regulation, carbon cap regulation, carbon offset regulation, etc.) which may affect buyers' ordering behaviors differently. Second, selling price and product freshness are two important factors that affect consumers purchasing decisions on perishable goods. As a result, the demand rate should be expanded to a dynamic function of selling price and product freshness. Third, there is only one objective in this research to minimize the total relevant cost per unit of time. It would be an interesting and relevant research to use a multi-criteria decision analysis minimizing both total relevant cost and carbon emissions simultaneously. Finally, the proposed model is built from the perspective of the buyer. In today's supply chain coordination, we could explore a win-win solution (e.g., Pareto or integrated solution) including both the seller's and the buyer's perspectives.

Acknowledgments

The authors are grateful to Editor Peter Kelle and two anonymous reviewers for their valuable comments and encouragement. The first and second authors' research is supported by the National Natural Science Foundation of China (No.71572058); MOE Project of Humanities and Social Sciences (No.17YJC630230); The Fundamental Research Funds for the Central Universities (No.2018JDXM05), Guangxi Colleges and Universities Key Laboratory of Symbolic Computation and Engineering Data Processing; Philosophy and Social Sciences Planning Research Project in Guangxi (No.15FGL008); Guangxi Colleges and Universities project (No.2018ky0563); Growing Spine Foundation (No.2018GXNSFBA281180). The third author gratefully acknowledges the funding support received from the National ChengChi University of the Republic of China under Grant NCCU Office of Research and Development 106DZ15-A3. The fourth author is supported by the Tecnológico de Monterrey Research Group in Optimization and Data Science 0822B01006.

Appendix A. Proof of Theorem 1

From (9), we define

$$f_1(T)=(K+\tau\widehat{K})+\left(c_p+\tau\widehat{c}_p\right)\int_0^T De^{\int_0^x \theta(\tau)d\tau}dx+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt,$$

and

$g_1(T)=T>0$. Taking the first-order and the second-order derivatives of $f_1(T)$ with respect to T, we obtain

$$f_1^{'}(T)=\left(c_p+\tau\widehat{c}_p\right)De^{\int_0^T \theta(v)dv}+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T De^{\int_t^T \theta(v)dv}dt, \tag{A1}$$

and

$$f_1^{''}(T)=\left(c_p+\tau\widehat{c}_p\right)\frac{1}{1+m-T}De^{\int_0^T \theta(v)dv}+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)D\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]>0. \tag{A2}$$

Applying Theorems 3.2.9 and 3.2.10 from Cambini and Martein (2009), we know that $TC_1(T)=f_1(T)/g_1(T)$ is a strictly pseudo-convex function in T. Hence, there exists a unique global minimum T_1^*. This completes the Proof of Part (a) of Theorem 1.

Taking the first-order derivative of $TC_1(T)$ in (9) with respect to T, and rearranging terms, we get

$$\begin{aligned}\frac{dTC_1(T)}{dT}=&-\frac{1}{T^2}\Bigg\{K+\tau\widehat{K}-\left(c_p+\tau\widehat{c}_p\right)\left[TDe^{\int_0^T \theta(v)dv}-\int_0^T De^{\int_0^x \theta(v)dv}dx\right]\\&-\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\left[T\int_0^T De^{\int_t^T \theta(v)dv}dt-\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right].\end{aligned} \tag{A3}$$

For simplicity, let

$$\begin{aligned}H_1(T)=&K+\tau\widehat{K}-\left(c_p+\tau\widehat{c}_p\right)\left[TDe^{\int_0^T \theta(v)dv}-\int_0^T De^{\int_0^x \theta(v)dv}dx\right]\\&-\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\left[T\int_0^T De^{\int_t^T \theta(v)dv}dt-\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right].\end{aligned} \tag{A4}$$

Then we know that $H_1(T)$ is decreasing in $T\in[0,m]$ because

$$H_1^{'}(T)=-\left(c_p+\tau\widehat{c}_p\right)T\frac{1}{1+m-T}e^{\int_0^T \theta(v)dv}-\left(c_h+\tau\widehat{c}_h+I_c c_p\right)DT\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]<0. \tag{A5}$$

Since $H_1(0)=K+\tau\widehat{K}>0$, and

$$\begin{aligned}H_1(m)=\Delta_1=&K+\tau\widehat{K}-\left(c_p+\tau\widehat{c}_p\right)\left[mDe^{\int_0^m \theta(v)dv}-\int_0^m De^{\int_0^x \theta(v)dv}dx\right]\\&-\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\left[m\int_0^m De^{\int_t^m \theta(v)dv}dt-\int_0^m\int_t^m De^{\int_t^x \theta(v)dv}dxdt\right].\end{aligned} \tag{A6}$$

Thus, if $\Delta_1\geq0$, then $H_1(T)\geq0$ for all $T\in[0,m]$. It is clear from (A3) and (A4) that $TC_1^{'}(T)<0$ for all $T\in[0,m]$, and hence $TC_1(T)$ is decreasing in $T\in[0,m]$. Therefore, $T_1^*=m$. This completes the Proof of Part (b) of Theorem 1. On the other hand, if $\Delta_1=H_1(m)<0$, then applying the Mean Value Theorem, we know that there exists a unique $T_1^*\in(0,m)$ such that $H_1(T_1^*)=0$. This completes the proof of Part (c) of Theorem 1.

Appendix B. Proof of Proposition 1

It is obvious that Part (a) of Proposition 1 immediately follows from (9). Next, we will provide the Proof of Part (b) of Proposition 1.

From Theorem 1 we know that there exists a unique minimum solution of $TC_1(T)$. Taking the first-order derivative of $TC_1(T)$ in (9) with respect to T, setting the result to zero,

we obtain:

$$\begin{aligned}&\frac{1}{T}\left[(K+\tau\widehat{K})+\left(c_p+\tau\widehat{c}_p\right)\int_0^T De^{\int_0^x \theta(v)dv}dx+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right]\\&=\left(c_p+\tau\widehat{c}_p\right)De^{\int_0^T \theta(v)dv}+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T De^{\int_t^T \theta(v)dv}dt.\end{aligned} \tag{B1}$$

Taking the implicit derivative of (B1) with respect to τ yields

$$\begin{aligned}&\frac{1}{T}\left[\widehat{K}+\widehat{c}_p\int_0^T De^{\int_0^x \theta(v)dv}dx+\left(c_p+\tau\widehat{c}_p\right)De^{\int_0^T \theta(v)dv}\frac{dT}{d\tau}+\widehat{c}_h\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right.\\&\left.+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T De^{\int_t^T \theta(v)dv}dt\frac{dT}{d\tau}\right]-\frac{1}{T^2}\left[(K+\tau\widehat{K})+\left(c_p+\tau\widehat{c}_p\right)\int_0^T De^{\int_0^x \theta(v)dv}dx\right.\end{aligned}$$

$$+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T\int_t^T De^{\int_t^x\theta(v)dv}dxdt\Big]\frac{dT}{d\tau}$$

$$=\widehat{c}_p De^{\int_0^T\theta(v)dv}+\left(c_p+\tau\widehat{c}_p\right)\frac{1}{1+m-T}De^{\int_0^T\theta(v)dv}\frac{dT}{d\tau}$$

$$+\widehat{c}_h\int_0^T De^{\int_t^T\theta(v)dv}dt+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)D\left(1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T\theta(v)dv}dt\right)\frac{dT}{d\tau}.$$

Rearranging terms, we get

$$\frac{CE}{T}+\frac{1}{T}\left\{\left(c_p+\tau\widehat{c}_p\right)De^{\int_0^T\theta(v)dv}+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T De^{\int_t^T\theta(v)dv}dt\right.$$

$$\left.-\frac{1}{T}\left[(K+\tau\widehat{K})+\left(c_p+\tau\widehat{c}_p\right)\int_0^T De^{\int_0^x\theta(v)dv}dx+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)\int_0^T\int_t^T De^{\int_t^x\theta(v)dv}dxdt\right]\right\}\frac{dT}{d\tau}$$

$$=\widehat{c}_p De^{\int_0^T\theta(v)dv}+\left(c_p+\tau\widehat{c}_p\right)\frac{1}{1+m-T}De^{\int_0^T\theta(v)dv}\frac{dT}{d\tau}$$

$$+\widehat{c}_h\int_0^T De^{\int_t^T\theta(v)dv}dt+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)D\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T\theta(v)dv}dt\right]\frac{dT}{d\tau}.$$

Applying (B1) and simplifying terms, we have

$$\frac{CE}{T}=\widehat{c}_p De^{\int_0^T\theta(v)dv}+\left(c_p+\tau\widehat{c}_p\right)\frac{1}{1+m-T}De^{\int_0^T\theta(v)dv}\frac{dT}{d\tau}$$
$$+\widehat{c}_h\int_0^T De^{\int_t^T\theta(v)dv}dt+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)D\left(1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T\theta(v)dv}dt\right)\frac{dT}{d\tau}. \quad (B2)$$

Finally, taking the first-order derivative of CE/T, and simplifying terms, we obtain

$$\frac{d}{d\tau}\left(\frac{CE}{T}\right)\Big|_{T=T_1^*}=\frac{1}{T}\left(\frac{dCE}{d\tau}-\frac{CE}{T}\frac{dT}{d\tau}\right)\Big|_{T=T_1^*}=-\frac{1}{T}\left\{\left(c_p+\tau\widehat{c}_p\right)\frac{1}{1+m-T}De^{\int_0^T\theta(v)dv}\left(\frac{dT}{d\tau}\right)^2\right.$$
$$\left.+\left(c_h+\tau\widehat{c}_h+I_c c_p\right)D\left(1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T\theta(v)dv}dt\right)\left(\frac{dT}{d\tau}\right)^2\right\}\Big|_{T=T_1^*}<0. \quad (B3)$$

This completes the Proof of Proposition 1.

Appendix C. Proof of Theorem 2

Applying (13), we define

$$f_2(T)=(K+\tau\widehat{K})+\left[(1-r_2)c_p+\tau\widehat{c}_p+\frac{1+n}{2n}I_cL(1-r_2)c_p\right]\int_0^T De^{\int_0^x\theta(v)dv}dx$$

$$+\left[c_h+\tau\widehat{c}_h+I_c(1-r_2)c_p\right]\int_0^T\int_t^T De^{\int_t^x\theta(v)dv}dxdt,$$

and

$$g_2(T)=T>0.$$

By taking the first-order and the second-order derivatives of $f_2(T)$ with respect to T, we derive as follows:

$$f_2'(T)=\left[(1-r_2)c_p+\tau\widehat{c}_p+\frac{1+n}{2n}I_cL(1-r_2)c_p\right]De^{\int_0^T\theta(v)dv}$$
$$+\left[c_h+\tau\widehat{c}_h+I_c(1-r_2)c_p\right]\int_0^T De^{\int_t^T\theta(v)dv}dt, \quad (C1)$$

and

$$f_2''(T)=\left[(1-r_2)c_p+\tau\widehat{c}_p+\frac{1+n}{2n}I_cL(1-r_2)c_p\right]\frac{1}{1+m-T}De^{\int_0^T\theta(v)dv}$$
$$+\left[c_h+\tau\widehat{c}_h+I_c(1-r_2)c_p\right]D\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T\theta(v)dv}dt\right]>0. \quad (C2)$$

Therefore, $TC_2(T)$ is a strictly pseudo-convex function in T, which implies that there exists a unique global minimum T_2^*. This completes the Proof of

Part (a) of Theorem 2.

For convenience, let us define

$$H_2(T) = K + \tau\widehat{K} - \left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL(1-r_2)c_p\right]\left[TDe^{\int_0^T \theta(v)dv} - \int_0^T De^{\int_0^x \theta(v)dv}dx\right]$$
$$-\left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]\left[T\int_0^T De^{\int_t^T \theta(v)dv}dt - \int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right]. \tag{C3}$$

Taking the first-order derivative of $TC_2(T)$ with respect to T and rearranging terms, we obtain

$$\frac{dTC_2(T)}{dT} = -\frac{1}{T^2}\Bigg\{K + \tau\widehat{K} - \left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL(1-r_2)c_p\right]\left[TDe^{\int_0^T \theta(v)dv} - \int_0^T De^{\int_0^x \theta(v)dv}dx\right]$$
$$-\left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]\left[T\int_0^T De^{\int_t^T \theta(v)dv}dt - \int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right]\Bigg\}$$
$$= -\frac{1}{T^2}H_2(T). \tag{C4}$$

Taking the first-order derivative of $H_2(T)$ with respect to T, we obtain

$$H_2'(T) = -\left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL(1-r_2)c_p\right]\frac{1}{1+m-T}DTe^{\int_0^T \theta(v)dv}$$
$$-\left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]DT\left[1 + \frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right] < 0. \tag{C5}$$

Hence, $H_2(T)$ is decreasing in $T \in (0,m)$. Note that $H_2(0) = K + \tau\widehat{K} > 0$, and $\Delta_2 = H_2(m)$. If $\Delta_2 \geq 0$, then $H_2(T) > 0$ for all $T \in [0,m]$, which implies that $TC_2(T)$ is decreasing in $[0,m]$. Thus, $T_2^* = m$. This completes the Proof of Part (b) of Theorem 2.

If $\Delta_2 < 0$, then $H_2(0) > 0$, and $H_2(m) < 0$. Applying the Intermediate Value Theorem, there exists a unique $T_2^* \in (0,m)$ such that $TC_2(T)$ is minimized. This completes the Proof of Part (c) of Theorem 2.

Appendix D. Proof of Proposition 2

Part (a) of Proposition 2 immediately follows from (13), thus we omit it. Next, we will provide the Proof of Part (b) of Proposition 2.

From Theorem 2 we know that there exists a unique minimum solution of $TC_2(T)$. Taking the first-order derivative of $TC_2(T)$ in (13) with respect to T, setting the result to zero, we obtain:

$$\frac{1}{T}\Bigg\{(K + \tau\widehat{K}) + \left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL(1-r_2)c_p\right]\int_0^T De^{\int_0^x \theta(v)dv}dx$$
$$+\left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\Bigg\}$$
$$= \left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL(1-r_2)c_p\right]De^{\int_0^T \theta(v)dv}$$
$$+\left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]\int_0^T De^{\int_t^T \theta(v)dv}dt, \tag{D1}$$

Similar to the Proof of Proposition 1, taking the implicit derivative of (D1) with respect to τ, applying (D1) and simplifying terms, we have

$$\frac{CE}{T} = \widehat{c}_pDe^{\int_0^T \theta(v)dv} + \left[(1-r_2)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL(1-r_2)c_p\right]\frac{1}{1+m-T}De^{\int_0^T \theta(v)dv}\frac{dT}{d\tau}$$
$$+\widehat{c}_h\int_0^T De^{\int_t^T \theta(v)dv}dt + \left[c_h + \tau\widehat{c}_h + I_c(1-r_2)c_p\right]D\left[1 + \frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]\frac{dT}{d\tau}. \tag{D2}$$

Finally, taking the first-order derivative of CE/T, and simplifying terms, we obtain

$$\frac{d}{d\tau}\left(\frac{CE}{T}\right)\Bigg|_{T=T_2^*} = \frac{1}{T}\left(\frac{dCE}{d\tau} - \frac{CE}{T}\frac{dT}{d\tau}\right)\Bigg|_{T=T_2^*}$$
$$= -\frac{1}{T}\Bigg\{\left[\left(1-r_2\right)c_p + \tau\widehat{c}_p + \frac{1+n}{2n}I_cL\left(1-r_2\right)c_p\right]\frac{1}{1+m-T}De^{\int_0^T \theta(v)dv}\left(\frac{dT}{d\tau}\right)^2$$
$$+\left[c_h + \tau\widehat{c}_h + I_c\left(1-r_2\right)c_p\right]D\left[1 + \frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]\left(\frac{dT}{d\tau}\right)^2\Bigg\}\Bigg|_{T=T_2^*} < 0. \tag{D3}$$

This completes the Proof of Part (b) of Proposition 2.

Appendix E. Proof of Lemma 1

For simplicity, we define

$$H_{31}(T) = K + \tau\widehat{K} - \left(c_p + \tau\widehat{c}_p\right)\left[TDe^{\int_0^T \theta(v)dv} - \int_0^T De^{\int_0^x \theta(v)dv}dx\right]$$
$$-(c_h + \tau\widehat{c}_h)\left[T\int_0^T De^{\int_t^T \theta(v)dv}dt - \int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right] \tag{E1}$$
$$-I_c c_p\left[T\int_M^T De^{\int_t^T \theta(v)dv}dt - \int_M^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right] - \frac{1}{2}I_e sDM^2.$$

Taking the first-order derivative of $H_{31}(T)$ with respect to T yields

$$H'_{31}(T) = -\left(c_p + \tau\widehat{c}_p\right)\frac{1}{1+m-T}DTe^{\int_0^T \theta(v)dv} - (c_h + \tau\widehat{c}_h)DT\left[1 + \frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]$$
$$-I_c c_p DT\left[1 + \frac{1}{1+m-T}\int_M^T e^{\int_t^T \theta(v)dv}dt\right] < 0, \tag{E2}$$

Consequently, $H_{31}(T)$ is a strictly decreasing function in T, which implies that $\Delta_{311} = H_{31}(M) > H_{31}(m) = \Delta_{312}$. This completes the Proof of Lemma 1.

Appendix F. Proof of Theorem 3

From (17), we define

$$f_{31}(T) = (K + \tau\widehat{K}) + \left(c_p + \tau\widehat{c}_p\right)\int_0^T De^{\int_0^x \theta(v)dv}dx + (c_h + \tau\widehat{c}_h)\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt$$
$$+I_c c_p\int_M^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt - \frac{1}{2}I_e sDM^2,$$

and

$g_{31}(T) = T.$

Taking the first-order and second-order derivatives of $f_{31}(T)$ with respect to T, we obtain

$$f'_{31}(T) = \left(c_p + \tau\widehat{c}_p\right)De^{\int_0^T \theta(v)dv} + (c_h + \tau\widehat{c}_h)\int_0^T De^{\int_t^T \theta(v)dv}dt + I_c c_p\int_M^T De^{\int_t^T \theta(v)dv}dt, \tag{F1}$$

and

$$f''_{31}(T) = \left(c_p + \tau\widehat{c}_p\right)D\frac{1}{1+m-T}e^{\int_0^T \theta(v)dv} + (c_h + \tau\widehat{c}_h)D\left[1 + \frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]$$
$$+I_c c_p D\left[1 + \frac{1}{1+m-T}\int_M^T e^{\int_t^T \theta(v)dv}dt\right] > 0, \tag{F2}$$

Therefore, $TC_{31}(T)$ is a strictly pseudo-convex function in T, and hence there exists a unique global minimum T^*_{31} such that $TC_{31}(T)$ is minimized. This completes the Proof of Part (a) of Theorem 3.

By taking the first-order derivative of $TC_{31}(T)$ in (19) with respect to T, and rearranging terms, we have $dTC_{31}(T)/dT = -H_{31}(T)/T$ as shown in (E1). We know from Appendix E that $H_{31}(T)$ is a strictly decreasing function in T, and $\Delta_{311} = H_{31}(M) > H_{31}(m) = \Delta_{312}$, for all $M < m$. If $\Delta_{312} \geq 0$, then $H_{31}(T)$ is positive for all $T \in [M, m]$. Hence, $TC_{31}(T)$ is decreasing in $[M, m]$, and $T^*_{31} = m$. This completes the Proof of Part (b) of Theorem 3.

$H_{31}(T)$ is a decreasing function in T. If $\Delta_{311} > 0$ and $\Delta_{312} < 0$, then $H_{31}(T) = 0$ has a unique solution $T^*_{31} \in [M, m]$. From Part (a), we know that $TC_{31}(T)$ is a strictly pseudo-convex function in T. Therefore, there exists a unique $T^*_{31} \in (M, m)$ such that $TC_{31}(T)$ is minimized. This completes the Proof of Part (c) of Theorem.

If $\Delta_{311} < 0$ and $\Delta_{312} < \Delta_{311} < 0$, then $H_{31}(T)$ is negative for all $T \in [M, m]$. Conversely, $TC_{31}(T)$ is increasing in $T \in [M, m]$, which implies $TC_{31}(T)$ is minimized at $T^*_{31} = M$. This completes the Proof of Part (d) of Theorem 3.

Appendix G. Proof of Proposition 3

Part (a) of Proposition 3 immediately follows from (17) and (19), thus we omit it. Next, we will provide the Proof of Part (b) of Proposition 3.

From Theorem 3 we know that there exists a unique minimum solution of $TC_{31}(T)$. Taking the first-order derivative of $TC_{31}(T)$ in (17) with respect to T, setting the result to zero, we obtain:

$$\frac{1}{T}\left[(K+\tau\widehat{K})+\left(c_p+\tau\widehat{c}_p\right)\int_0^T De^{\int_0^x \theta(v)dv}dx+\left(c_h+\tau\widehat{c}_h\right)\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right.$$
$$\left.+I_c c_p\int_M^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt-\frac{1}{2}I_e sDM^2\right] \tag{G1}$$
$$=\left(c_p+\tau\widehat{c}_p\right)De^{\int_0^T \theta(v)dv}+\left(c_h+\tau\widehat{c}_h\right)\int_0^T De^{\int_t^T \theta(v)dv}dt+I_c c_p\int_M^T De^{\int_t^T \theta(v)dv}dt,$$

Similar to the Proof of Proposition 1, taking the implicit derivative of (G1) with respect to τ, applying (G1) and simplifying terms, we have

$$\frac{CE}{T}=\widehat{c}_p De^{\int_0^T \theta(v)dv}+\left(c_p+\tau\widehat{c}_p\right)D\frac{1}{1+m-T}e^{\int_0^T \theta(v)dv}\frac{dT}{d\tau}$$
$$+\widehat{c}_h\int_0^T De^{\int_t^T \theta(v)dv}dt+\left(c_h+\tau\widehat{c}_h\right)D\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]\frac{dT}{d\tau} \tag{G2}$$
$$+I_c c_p D\left[1+\frac{1}{1+m-T}\int_M^T e^{\int_t^T \theta(v)dv}dt\right]\frac{dT}{d\tau}.$$

Finally, taking the first-order derivative of CE/T, and simplifying terms, we obtain

$$\frac{d}{d\tau}\left(\frac{CE}{T}\right)\bigg|_{T=T_{31}^*}=\frac{1}{T}\left(\frac{dCE}{d\tau}-\frac{CE}{T}\frac{dT}{d\tau}\right)\bigg|_{T=T_{31}^*}$$
$$=-\frac{1}{T}\left\{\left(c_p+\tau\widehat{c}_p\right)D\frac{1}{1+m-T}e^{\int_0^T \theta(v)dv}\left(\frac{dT}{d\tau}\right)^2+\left(c_h+\tau\widehat{c}_h\right)D\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right]\left(\frac{dT}{d\tau}\right)^2\right. \tag{G3}$$
$$\left.+I_c c_p D\left[1+\frac{1}{1+m-T}\int_M^T e^{\int_t^T \theta(v)dv}dt\right]\left(\frac{dT}{d\tau}\right)^2\right\}\bigg|_{T=T_{31}^*}<0.$$

This completes the Proof of Part (b) of Proposition 3.

Appendix H. Proof of Theorem 4

Applying (19), we define

$$f_{32}(T)=(K+\tau\widehat{K})+\left(c_p+\tau\widehat{c}_p\right)\int_0^T De^{\int_0^x \theta(v)dv}dx+\left(c_h+\tau\widehat{c}_h\right)\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt$$
$$-I_e sDT\left(M-\frac{1}{2}T\right),$$

and

$$g_{32}(T)=T.$$

Taking the first-order and second-order derivatives of $f_{31}(T)$ with respect to T, we obtain

$$f_{32}^{'}(T)=\left(c_p+\tau\widehat{c}_p\right)De^{\int_0^T \theta(v)dv}+\left(c_h+\tau\widehat{c}_h\right)\int_0^T De^{\int_t^T \theta(v)dv}dt-I_e sD(M-T), \tag{H1}$$

and

$$f_{32}^{''}(T)=\left(c_p+\tau\widehat{c}_p\right)D\frac{1}{1+m-T}e^{\int_0^T \theta(v)dv}+\left(c_h+\tau\widehat{c}_h\right)D\left[1+\frac{1}{1+m-T}\int_0^T e^{\int_t^T \theta(v)dv}dt\right] \tag{H2}$$
$$+I_e sD>0.$$

Therefore, $TC_{32}(T)$ is a strictly pseudo-convex function in T, and hence there exists a unique global minimum T_{32}^* such that $TC_{32}(T)$ is minimized. This completes the Proof of Part (a) of Theorem 4.

By taking the first-order derivative of $TC_{32}(T)$ in (21) with respect to T, and rearranging terms, we have

$$\frac{dTC_{32}(T)}{dT}=-\frac{1}{T^2}\left\{K+\tau\widehat{K}-\left(c_p+\tau\widehat{c}_p\right)\left[TDe^{\int_0^T \theta(v)dv}-\int_0^T De^{\int_0^x \theta(v)dv}dx\right]\right.$$
$$\left.-\left(c_h+\tau\widehat{c}_h\right)\left[T\int_0^T De^{\int_t^T \theta(v)dv}dt-\int_0^T\int_t^T De^{\int_t^x \theta(v)dv}dxdt\right]-\frac{1}{2}I_e sDT^2\right\} \tag{H3}$$

Let $\frac{dTC_{32}(T)}{dT}=-\frac{1}{T^2}H_{32}(T)$, taking the first-order derivative of $H_{32}(T)$, yield

$$H_{32}^{'}(T) = -\left(c_p + \tau\widehat{c}_p\right)DT\frac{1}{1+m-T}e^{\int_0^T\theta(v)dv} - \left(c_h + \tau\widehat{c}_h\right)DT\left[1 + \frac{1}{1+m-T}\int_0^T e^{\int_t^T\theta(v)dv}dt\right] \tag{H4}$$

$-I_e sDT < 0.$

Hence, $H_{32}(T)$ is decreasing in $T \in (0, M)$. Note that $H_{32}(0) = K + \tau\widehat{K} > 0$, and $\Delta_{32} = H_{32}(M)$. If $\Delta_{32} \geq 0$, then $H_{32}(T) > 0$ for all $T \in [0, M]$, which implies that $TC_{32}(T)$ is decreasing in $[0, M]$. Thus, $T_{32}^* = M$. This completes the Proof of Part (b) of Theorem 4.

If $\Delta_{32} < 0$, then $H_{32}(0) > 0$, and $\Delta_{32} = H_{32}(M) < 0$. Applying the Intermediate Value Theorem, there exists a unique $T_{32}^* \in (0, M)$ such that $TC_{32}(T)$ is minimized. This completes the Proof of Part (c) of Theorem 4.

Appendix I. Proof of Theorem 5

We know from (20) that $TC_{31}(M) = TC_{32}(M)$. Applying the fact that $\Delta_{311} = \Delta_{32} > \Delta_{312}$, and Theorems 3 and 4, we have the following results:

If $\Delta_{311} = \Delta_{32} < 0$, then $\Delta_{312} < \Delta_{311} = \Delta_{32} < 0$, $TC_{31}(T)$ is increasing in $[M, m]$ and $TC_{32}(T)$ is decreasing in $[0, T_{32}^*]$ but increasing in $[T_{32}^*, M]$. Consequently, $TC_{32}(T_{32}^*) < TC_{31}(M) = TC_{32}(M)$. As a result, the optimal replenishment cycle time is $T_3^* = T_{32}^*$. This completes the Proof of Part (a) of Theorem 5.

If $\Delta_{311} = \Delta_{32} > 0$, and $\Delta_{312} < 0$, then $TC_{31}(T)$ is decreasing in (M, T_{31}^*) while increasing in (T_{31}^*, m) and $TC_{32}(T)$ is decreasing in $(0, M)$. Thus, $TC_{31}(T_{31}^*) < TC_{31}(M) = TC_{32}(M)$ which results in $T_3^* = T_{31}^*$. This completes the Proof of Part (b) of Theorem 5.

If $\Delta_{312} > 0$, then $\Delta_{311} = \Delta_{32} > \Delta_{312} > 0$, $TC_{31}(T)$ is decreasing in (M, m) and $TC_{32}(T)$ is decreasing in $(0, M)$. Therefore, $TC_{31}(m) < TC_{31}(M) = TC_{32}(M)$ and $T_3^* = m$. This completes the Proof of Part (c) of Theorem 5.

References

Aljazzar, S.M., Gurtu, A., Jaber, M.Y., 2018. Delay-in-payments-A strategy to reduce carbon emissions from supply chains. J. Clean. Prod. 170, 636–644.

Benjaafar, S., Li, Y., Daskin, M., 2013. Carbon footprint and the management of supply chains: insights from simple models. IEEE Trans. Autom. Sci. Eng. 10 (1), 99–116.

Cambini, A., Martein, L., 2009. Generalized Convexity and Optimization: Theory and Application. Springer-Verlag Berlin Heidelberg, USA.

Chang, C.T., Ouyang, L.Y., Teng, J.T., Lai, K.K., Cárdenas-Barrón, L.E., 2019. Manufacturer's pricing and lot-sizing decisions for perishable goods under various payment terms by a discounted cash flow analysis. Int. J. Prod. Econ. 218 (12), 83–95.

Chen, S.C., Teng, J.T., 2014. Retailer's optimal ordering policy for deteriorating items with maximum lifetime under supplier's trade credit financing. Appl. Math. Model. 38 (15–16), 4049–4061.

Chen, Z., 2018. Optimization of production inventory with pricing and promotion effort for a single-vendor multi-buyer system of perishable products. Int. J. Prod. Econ. 203, 333–349.

Dye, C.Y., Yang, C.T., 2015. Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints. Eur. J. Oper. Res. 244, 187–200.

Feng, L., Chan, Y.L., 2019. Joint pricing and production decisions for new products with learning curve effects under upstream and downstream trade credits. Eur. J. Oper. Res. 272 (3), 905–913.

Feng, L., Chan, Y.L., Cárdenas-Barrón, L.E., 2017. Pricing and lot-sizing polices for perishable goods when the demand depends on selling price, displayed stocks, and expiration date. Int. J. Prod. Econ. 185, 11–20.

Goyal, S.K., 1985. Economic order quantity under conditions of permissible delay in payments. J. Oper. Res. Soc. 36 (4), 335–338.

Harris, F.W., 1913. How many parts to make at once, Factory. The Magazine of Management 10 (2), 135-136, and 152.

He, P., Zhang, W., Xu, X., Bian, Y., 2015. Production lot-sizing and carbon emissions under cap-and-trade and carbon tax regulations. J. Clean. Prod. 103, 241–248.

Li, R., Chan, Y.L., Chang, C.T., Cárdenas-Barrón, L.E., 2017. Pricing and lot-sizing policies for perishable products with advance-cash-credit payments by a discounted cash-flow analysis. Int. J. Prod. Econ. 193, 578–589.

Li, R., Liu, Y., Teng, J.T., Tsao, Y.C., 2019a. Optimal pricing, lot-sizing and backordering decisions when a seller demands for an advance-cash-credit payment scheme. Eur. J. Oper. Res. 278 (1), 283–295.

Li, R., Skouri, K., Teng, J.T., Yang, W.G., 2018. Seller's optimal replenishment policy and payment term among advance, cash, and credit payments. Int. J. Prod. Econ. 197, 35–42.

Li, R., Teng, J.T., 2018. Pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks. Eur. J. Oper. Res. 270 (3), 1099–1108.

Li, R., Teng, J.T., Zheng, Y., 2019b. Optimal credit term, order quantity and pricing policies for perishable products when demand depends on price, expiration date, and credit period. Ann. Oper. Res. 280 (1–2), 377–405.

Sarkar, B., 2012. An EOQ model with delay in payments and time varying deterioration rate. Math. Comput. Model. 55 (3), 367–377.

Taleizadeh, A.A., 2014. An EOQ model with partial backordering and advance payments for an evaporating item. Int. J. Prod. Econ. 155, 185–193.

Teng, J.T., 2009. Optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers. Int. J. Prod. Econ. 119, 415–423.

Teng, J.T., Cárdenas-Barrón, L.E., Chang, H.J., Wu, J., Hu, Y., 2016. Inventory lot-size policies for deteriorating items with expiration dates and advance payments. Appl. Math. Model. 40 (19–20), 8605–8616.

The World Health Organization, 2016. Ambient Air Pollution: A Global Assessment of Exposure and Burden of Disease. World Health. Organization, pp. 1–131. Available at: doi:9789241511353.

Tsao, Y.C., Lee, P.L., Chen, C.H., Liao, Z.W., 2017. Sustainable newsvendor models under trade credit. J. Clean. Prod. 141, 1478–1491.

Tiwari, S., Cárdenas-Barrón, L.E., Goh, M., Shaikh, A.A., 2018. Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. Int. J. Prod. Econ. 200, 16–36.

Wang, W.C., Teng, J.T., Lou, K.R., 2014. Seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime. Eur. J. Oper. Res. 232 (2), 315–321.

Wu, J., Al-khateeb, F.B., Teng, J.T., Cárdenas-Barrón, L.E., 2016. Inventory models for deteriorating items with maximum life time under downstream partial trade credits to credit-risk customers by discounted cash-flow analysis. Int. J. Prod. Econ. 171, 105–115.

Wu, J., Chang, C.T., Teng, J.T., Lai, K.K., 2017. Optimal order quantity and selling price over a product life cycle with deterioration rate linked to expiration date. Int. J. Prod. Econ. 193, 343–351.

Wu, J., Ouyang, L.Y., Cárdenas-Barrón, L.E., Goyal, S.K., 2014a. Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. Eur. J. Oper. Res. 237 (3), 898–908.

Wu, J., Skouri, K., Teng, J.T., Ouyang, L.Y., 2014b. A note on optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment. Int. J. Prod. Econ. 155, 324–329.

Wu, J., Teng, J.T., Chan, Y.L., 2018a. Inventory policies for perishable products with expiration dates and advance-cash-credit payment schemes. Int. J. Syst. Sci.: Operations & Logistics 5 (4), 310–326.

Wu, J., Teng, J.T., Skouri, K., 2018b. Optimal inventory policies for deteriorating items with trapezoidal-type demand patterns and maximum lifetimes under upstream and downstream trade credits. Ann. Oper. Res. 264 (1–2), 459–476.

Xu, X., Xu, X., He, P., 2016. Joint production and pricing decisions for multiple products with cap-and-trade and carbon tax regulations. J. Clean. Prod. 112, 4093–4106.

Xiang, D., Lawley, C., 2018. The impact of British Columbia's carbon tax on residential natural gas consumption. Energy Econ. https://doi.org/10.1016/j.eneco.2018.12.004.

Zhang, A.X., 1996. Optimal advance payment scheme involving fixed per-payment costs. Omega 24 (5), 577–582.

Zhang, Q., Tsao, Y.C., Chen, T.H., 2014. Economic order quantity under advance payment. Appl. Math. Model. 38 (24), 5910–5921.