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Department of Economics

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Master Thesis

網紅行銷對電商產業的影響

The Effects of Online celebrity marketing on the Electronic Commerce Industry

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摘要

近年來在網路購物的快速興起下， 網紅行銷成為電子商務產業的一大重點，因為網紅在網際網路下受到許多消費者的認同與喜愛，許多電商皆選擇以網紅來推廣商品，增加商品能見度。網紅市場會存在網路效應，越多人參與會提高商品價值，此外網紅對於商品的推廣會提升網紅效應、學習效果，並降低不確定性。本文以兩種寡占模型分別分析電商在市場規模大或市場規模小時，引入網紅行銷後市場均衡的變化。研究結果發現，當廠商的競爭對手沒有採取網紅行銷時，在規模大的市場下，則該廠商有絕對的誘因採取網紅行銷以提高利潤；若在規模小的市場下，廠商採取網紅行銷反而會降低價格，因此需以提升網紅效應、學習效果，並降低不確定性提高自身商品價值，以此擴大市場份額。若是競爭對手採取網紅行銷時，不論市場規模大小，自身商品價值皆須高於對方價值才能使自身利潤高於對手利潤，因此廠商有誘因去提升網紅效應、學習效果，並降低不確定性以提高自身商品價值。本文研究中發現網紅行銷是很好去創造獲利的模式，當廠商引入網紅市場，便可通過提升網紅效應、學習效果，或降低不確定性來擴大市場份額、價格與利潤差距。

關鍵詞: 網紅市場、網紅行銷、網紅效果、網路效果、學習效果 、寡占

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Abstract

More and more people shopping online recently, and many online platforms choose the online celebrity to promote their products, so the scale of online celebrity marketing has been developed rapidly. Online celebrity marketing exists a network effect, which means when more people join the network, the product value will become higher. It can also increase the celebrity effect, learning effect and reduce uncertainty. In our paper, we use two models to analyze the equilibrium in the large market and the small market. When the firm's opponent didn't adopt online celebrity marketing, in the large market, the firm has an incentive to adopt online celebrity marketing; in the small market, its price may decrease, so it should increase the celebrity effect, learning effect, or decrease uncertainty to increase its market share. If the firm's opponent adopts online celebrity marketing, then in both large and small markets, the firm should ensure its product value would higher than its opponent. So that it will have a profit incentive to increase the celebrity effect, learning effect, or decrease uncertainty.

Keywords: Online celebrity marketing, Learning effect, Network effect, Imperfect competition

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1. Introduction

Online celebrity marketing is a form of an advertising campaign or marketing strategy which uses a celebrity's fame or social status to promote a product, brand, or service, or to raise awareness about an issue. This form become popular recently and is usually used on the one-commerce platform. The influence from online celebrity marketing is many, the first one is the celebrity effect, if the celebrity is famous or suitable to advertise the product, it may increase the consumer's purchasing intention. The second is the learning effect. When the celebrity promotes the product, they can use E-commerce live streaming or community platform to promote it. The consumer can interact with another consumer, and get more information. The third one is that it may reduce uncertainty. For example, when a celebrity uses E-commerce live streaming to promote a product. The consumer can see the product comprehensively and can ask questions about the products directly. That reason is why online celebrity marketing has developed rapidly.

We want to know how online celebrity marketing changes the market equilibrium and market structure. We proceed with our analyses in three steps. First, in a duopoly market, we consider that only one firm uses online celebrity marketing. This also shows individual firms' incentive to adopt online celebrity marketing. Second, we consider that both firms adopt online celebrity marketing. In this way, we can investigate how a firm's adoption incentive changes in the response to the opponents' online celebrity marketing and what factors drive this change. Third, we consider a new entrant with online celebrity marketing, competing with two existing traditional firms (with no internet marketing). This helps us clarify how internet online celebrity marketing changes the incumbent firms and traditional market.

In this paper, we use two models to analyze the marketing equilibrium. The first model is quantity competition. We follow Belleflamme and Peitz's (2015) by assuming that prices will adjust to the levels where the marginal consumer is "indifferent between buying the two

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products" and "indifferent between buying and not buying. This implies that we are considering a large market; consumers have sufficiently heterogeneous preferences such that some consumers would rather not buy anything. The second model follows Oz Shy(1995),whicht used price competition. we assume the market size is small such that the marginal consumer of a product is indifferent between buying this product and another product, and we set the restriction equation as $n_1 + n_2 = 1$.

The result we find in the first model is that if firm1's opponent doesn't adopt online celebrity marketing, firm1 's has market share and profit incentive to adopt online celebrity marketing. if firm1's opponent adopts online celebrity marketing, then firm1 adopts online celebrity marketing, it has a profit incentive to increase k_1 and l_1 or to decrease σ_1.

The result we find in the second model is that if firm1's opponent doesn't adopt online celebrity marketing, firm1 's price will decrease, only if $\mathrm{A} - \mathrm{B} > \tau$, firm1 can increase its market share, so it has an incentive to increase k_1 and l_1 or to decrease σ_1. If firm1's opponent adopts online celebrity marketing, when $\mathrm{A} > \mathrm{B}$ firm1 's best strategy is adopting online celebrity marketing, it has a profit incentive to increase k_1 and l_1 or to decrease σ_1. When we consider a new entrant with online celebrity marketing, competing with two traditional incumbent firms, and we find only when A>3B firm1 's profit will be higher than its competitor, it has a profit incentive to increase k_1 and l_1 or decreaseσ_1.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature on the online celebrity marketing. In Section 3, we analyze when in the large market, the equilibrium of internet online celebrity marketing in oligopolistic market. In section 4, we analyze when in the small market, the equilibrium of internet online celebrity marketing in an oligopolistic market. In Section 5, we describe the conclusion of the paper.

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2. Related Literature

1) The rise of online celebrity marketing The scale of online celebrity marketing has been developed rapidly (Abbas, 2018). With the increasing normalization of social media platforms, web celebrities quickly occupy a place in the social networking field due to the interactivity, experience, and low entry barriers of the Internet.

Online celebrity marketing is a form of online marketing. Social media have created new opportunities for marketers to reach the potential customers beyond traditional mass-media channels (Sundermann, 2019). Celebrities can be hired by firms or create their personal brands. The interactive and personal nature of social media helps their followers clarify uncertainty about product characteristics (quality or function) and their purchase decisions. (Kadekova, 2019)

Online celebrity marketing is an online marketing model that web celebrities use to recommend endorsement products to fans through videos, images, and texts (Zabel and Pagel, 2017). The proposed marketing model has become an emerging mode of China's online consumption growth. It is essentially virtual WOM that has become stronger with the strengthening of social media (Wang, 2019)

2) How does Internet online celebrity marketing influence consumers? Internet online celebrity marketing become a popular word in recent years, internet online celebrity marketing means "wanghong" in Chinese. It refers to professionals who use their popularity to get their fans to purchase goods or services that they endorse. They publish stylish photos, videos, and other content online for their fans. Internet online celebrity marketing typically has eye-catching appearances, with which they present fashion, accessories, and cosmetics.

Most of the e-commerce platforms are now using celebrity endorsers to promote brands (Geng, 2020). The increase in popularity of celebrity endorsement can be attributed to the ability to give messages to consumers. The use of celebrities is believed to help consumers remember the message of the advertisement and the brand name the celebrity is endorsing enables to create of the personality of a brand because when a celebrity is paired with a brand, this image helps shape the image of that brand in the minds of consumers (Agrawal and Kamakura, 1995).

Internet online celebrity marketing effect affects consumers' purchase intention in many ways. The first one is internet celebrity's personal stature (Liu,2021). The e-commerce seller will choose the suitable internet online celebrity marketing to advertise their product, for example, they may choose a mother internet online celebrity marketing to help them sell some baby care product. So choosing a suitable celebrity to sell their product can make the consumer have a higher trust in that product. It may lead to a positive effect on consumers' purchase willingness. (Chan, 2022) The second effect of internet online celebrity marketing is the fans economy (Geng, 2020) every internet online celebrity marketing has its fan community. Those fans are attracted by internet online celebrity marketing in different ways like appearance, personality, or specialty. When internet online celebrity marketing sells the product from Live streaming, there is some reason that makes the fans willing to buy the product. One is emotional consumption, the fans can enjoy the happiness of spending money and maybe they will be praised by the streamers, so it may satisfy their vanity. The other reason is event consumers. when the e-commerce holiday 11/11 comes, the platform will have some activity for example if a streamer sells more than 100 products, he can get a bonus, so their fans are willing to help the internet online celebrity marketing they like to achieve the task, and it may enhance the sales. The fourth reason is conformity, which means when the celebrity promotes the product, the product will become more popular, and much more people will buy it. This phenomenon is because people often choose to conform to society rather than pursue personal desires since it is often easier to follow the path others have made already, rather than forging a new one, so they prefer to buy the product which is more people used.

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3) How does interactive learning effect influence consumer's purchase willingness? The interactive learning effect (Kang, 2021) means people will make the decision on the information they get, and there are many gateways to gain the information. For example, we can get information from online information, from our friends, or from any way that is in our life. In most cases, we can use the information to reduce uncertainty (Cenfetelli, 2019). If we put the interactive learning effect into online celebrity marketing when the celebrity help to sell the product, he may write a post or post a video, so the consumer can interact with other under the post or video. They can share their suggestion and opinion or even ask some questions, so they may have more understanding of the product. It will increase consumers' willingness to buy the product. And another effect of interaction is that people may feel happy to interact with others. For Example, if a celebrity use lives streaming to sell his product, the consumer can interact with the streamer or other consumers, it can enhance their feeling of social relations (Bruce, 2018) viewer motivations to engage in live-stream entertainment appear to have a stronger social and community basis. Those two ways influence the consumer's willingness to buy the products.

4) Network effect Network effects exist in any network, whether it's on the internet or platforms. Network effects are the incremental benefit gained by an existing user for each new user that joins the network. (Si, 2018) The network effect has been a key factor in the competition in the information and telecommunication industries. It is the general principle that the value of connecting to a network depends on the number of existing customers in the network. Usually, it is positive feedback from consumers, and the self-reinforcement nature of the network effect makes the strong firms stronger and the weak firms weaker (Tseng, 2008) many theses set the two cases: Heterogeneous network effect and Heterogeneous stand-alone benefit. (Belleflamme, 2010) The heterogeneous network effect means that different has a different valuation of the network effects, simply to say we can write the equation as $U(\theta) = a + \theta v n^e$, which θ measures the consumer's valuation of the

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network effects. Another case is the Heterogeneous stand-alone benefit, we can write the equation as $\mathrm{U}(\theta) = \theta \mathrm{a} + \mathrm{v}n^{e}$, θ measures the preference for the stand-alone benefit itself. In the presence of network effects and heterogeneous stand-alone utilities, the fulfilled expectations demand is monotone and strictly decreases if network effects are not too strong. Otherwise, if network effects are sufficiently strong, there might exist multiple consumer equilibria for the given price of the network industry.

3. Online celebrity marketing in Large Oligopolistic Market

We incorporate the two properties of online celebrities marketing into Belleflamme and Peitz's (2015) model of competition between incompatible networks.

In particular, we assume a continuum of consumers who are identified by a taste parameter $\theta \in [0,1]$. This parameter indicates the consumer's favor or taste for a product. There are two firms: 1 and 2. Sequentially, we consider three setups. (1) Only firm 1 uses online celebrity marketing. This can show an individual firm's incentive to adopt the online celebrity marketing. (2) Both firms adopt online celebrity marketing. This setup allows us to investigate how a firm's adoption incentive changes in response of the opponents' online celebrity marketing and what factors drives this change. (3) We consider a new entrant with online celebrity marketing, competing with two traditional incumbent firms (with no internet marketing). This helps us clarify how internet online celebrity marketing can change the incumbent firms and the traditional market.

3.1 One-sided Online celebrity marketing in Duopoly

In this section, we assume that only firm 1 uses online celebrity marketing, while firm 2 does not. Given firm i's price p_i, consumer θ receives a net surplus for purchasing product i:

$$u_i(\theta, g_i) = \theta + g_i - p_i,$$

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where g_i indicates the expected value from good i. g_1 and g_2 are different because the online celebrity marketing will create a network effect for product 1. For both firms, we use a mean-variance setup to describe product value, where v is the common mean value and σ_i denotes the firm i's specific product uncertainty (standard deviation). Let k_i indicate the degree that consumers are affected by the product value. Hence, for firm 2 which does not adopt online celebrity marketing,

$$g_2 = k_2(v - \sigma_2).$$

For firm 1, as described, (i) consumers' valuation toward the product might be emotionally affected by their recognition or support for the celebrities as well as the sense of belonging to the group. (ii) Through interactive online conversations, anonymous consumers are more likely to respond to celebrates, who can act like informed consumers and hence can help clarify consumers' doubts about product quality.

We add these two properties into the mean-variance utility. Let n_1^e indicate the expected number of consumers who will purchase product 1. The expected value of product 1 is hence given by:

$$g_1 = k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1^e,$$

where k_1 indicates the degree that consumers are affected by the network effect. n_1^e captures the emotional herding effect for following this celebrity. The parameter $l_1 > 0$ When $l_1 > 1$, the interactive conversations can help reduce the uncertainty, which we address as a "learning effect". However, as not all celebrities are trained marketers, it is also possible that the question and answer process leads customers to be more dubious about product quality. We address this influence as the "misleading effect", which is denoted $0 <$

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$l_1 < 1$.

$$l_1 \begin{cases} > 1, & \text{learning effect,} \\ < 1, & \text{misleading effect.} \end{cases}$$

Firms compete in quantities, so we follow Belleflamme and Peitz's (2015) by assuming that prices will adjust to the levels where the marginal consumer is "indifferent between buying the two products" and "indifferent between buying and not buying". This implies that we are considering a large market; consumers have sufficiently heterogenous preferences such that some consumers would rather not buy anything. Specifically, for the marginal consumer $\hat{\theta}$,

$$\hat{\theta} + g_1 - p_1 = \hat{\theta} + g_2 - p_2,$$

which indicates that

$$p_1 - g_1 = p_2 - g_2 \equiv \hat{p}. \tag{0}$$

In other words, consumers with θ lower than $\hat{p}$ will not buy any product. Let n_i indicate the number of consumers who buy product i. Equation (0) implies that

$$n_1 + n_2 = 1 - \hat{p}.$$

Substitute the definition of $\hat{p}$ from equation (0), we have

$$p_i = \hat{p} + g_i$$

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$$= 1 - (n_i + n_j) + g_i.$$

In a self-fulfilled equilibrium, $n_i^e = n_i$, so $g_1 = k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1$. Substituting the definition of g_i into the above equation gives the two firms' inverse demand functions, where

$$p_1 = 1 - (1 - k_1(v - \frac{\sigma_1}{l_1}))n_1 - n_2.$$

$$p_2 = 1 + k_2(v - \sigma_2) - n_1 - n_2.$$

Let A= $k_1(v - \frac{\sigma_1}{l_1})$ and B=$k_2(v - \sigma_2)$, so

$$p_1 = 1 - (1 - A)n_1 - n_2.$$

$$p_2 = 1 + B - n_1 - n_2.$$

3.1.1 Market Equilibrium with one-sided online celebrity marketing

Firms compete in quantities ($i.\text{e.}, n_i$), so the two firms choose their quantities n_i simultaneously to maximize their profits, where

$$\pi_1 = \{1 - (1 - A)n_1 - n_2 - c_1\}n_1.$$

$$\pi_2 = \{1 + B - n_1 - n_2 - c_2\}n_2.$$

To simplify the analysis, we assume that both firms have the same technology, and the marginal production costs are assumed to be zero, and we assume $c_1 = c_2 = c$. Hence, the first order conditions of maximization are:

$$1 - 2(1 - A)n_1 - n_2 - c\text{=0.}$$

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$$1 + B - n_1 - 2n_2 - c = 0.$$

And the second order conditions for profit maximization are:

$$\frac{\partial^2(\pi_1)}{\partial(n_1)^2} = -2(1-A) < 0 \quad \text{and} \quad \frac{\partial^2(\pi_1)}{\partial(n_2)^2} = -1 < 0,$$

which are satisfied if $1 > A$. To simplify the analysis, we will make this assumption henceforth.

Figure 1 below shows the best replies for firm 1 and firm 2, where

$$n_1^*(n_2) = \frac{1}{2(1-A)}(1 - c - n_2), \tag{1}$$

$$n_2^*(n_1) = \frac{1}{2}(1 - c + B - n_1). \tag{2}$$

Notice that in the case without the network effect by online celebrity marketing, the best replies of firm 1 would be

$$n_1^0(n_2) = \frac{1}{2}(1 - c + A - n_2). \tag{3}$$

Comparing equations (1) and (3), we know that the one-sided online celebrity marketing has two effects on firm 1's best reply function. First, in equation (3), the product value A can shift the best reply up by $\frac{1}{2}A$, while equation (1) shows that the shift in the best reply is given by $(\frac{1-c}{2(1-A)} - \frac{1-c}{2})$, which is equal to $\frac{A(1-c)}{2(1-A)}$. Given that $1 > A$, the shift in $n_1^*(n_2)$ is higher if A>c. Second, the slope in $n_1^0(n_2)$ is -2, while the slope of $n_1^*(n_2)$ is -2(1-A). This indicates that the best reply function becomes flatter after considering the one-sided network effect from online celebrity marketing of firm 1. The two effects move the market equilibrium from

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E_0 to E_1. We have the following result.

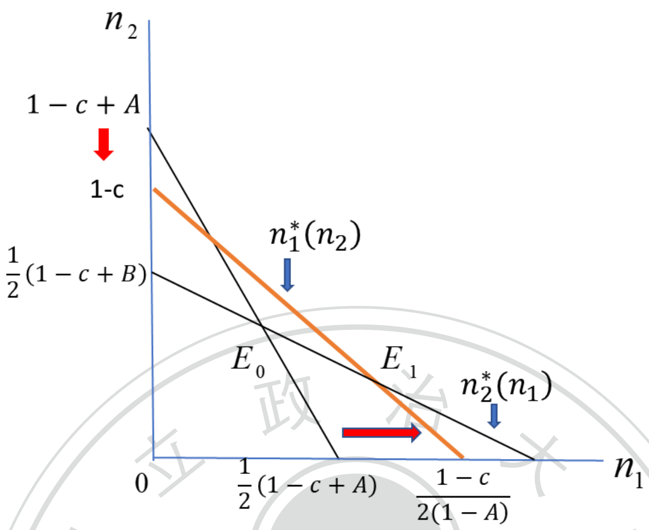

Figure 1 Best replies with one-sided celebrity marketing.

Proposition 1 *The one-sided online celebrity marketing can increase a firm's equilibrium output, while decreasing the opponent's output.*

From the best reply functions, we can solve the equilibrium outputs, where

$$n_1^* = \frac{1-B-c}{4(1-A)-1} \quad \text{and} \quad n_2^* = \frac{2AB-(1-c)(1-2A)}{4(1-A)-1},$$

The equilibrium prices are calculated as follows. First, firm 1's demand function is:

$$p_1 = 1 - (1 - A)n_1 - n_2,$$

and firm 1's FOC is:

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$$1-2(1-A)n_1-n_2-c=0.$$

Then, rewrite this condition using the definition of p_1, so we have:

$$p_1-(1-A)n_1-c=0.$$

Firm 1's equilibrium price is hence:

$$p_1^*=(1-A)n_1^*+c.$$

Similarly,

$$p_2^*=n_2^*+c.$$

Second, using the above equilibrium prices the equilibrium profits are given by

$$\pi_1^*=\{p_1^*-c\}n_1^*$$
$$=(1-\mathrm{A})(n_1^*)^2,$$

and

$$\pi_2^*=(n_2^*)^2.$$

3.1.2 Market Equilibrium without online celebrity marketing

As a benchmark of comparison, we describe the equilibrium without online celebrity marketing where firm 1's demand is given by

$$p_1=1-(1-A)n_1-n_2.$$

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and firm 1's FOC is given by equation (3). Together with firm 2's best reply in equation 2, we can solve the equilibrium output for no online celebrity marketing, where

$$n_1^0 = \frac{1+2A-B-c}{3} \quad \text{and} \quad n_2^0 = \frac{1-A+2B-c}{3}.$$

The equilibrium prices and profits are given by: for *i=1,2*

$$p_i^0 = n_i^0 + c, \quad \text{and} \quad \pi_i^0 = (n_i^0)^2. \tag{4}$$

3.1.3 Effects of one-sided online celebrity marketing

First, we show that online celebrity marketing will increase firm 1's market share. Recall that

$n_1^* = \frac{1-B-c}{3-4A}$ and $n_1^0 = \frac{1+2A-B-c}{3}$. To simplify, let X=$1-B-c$. Then

$$n_1^* = \frac{X}{3-4A} \text{ and } n_1^0 = \frac{2A+X}{3}.$$

So the condition for $n_1^* > n_1^0$ is:

$$3X>(3-4A)(2A+X). \tag{5}$$

Since n_1^* is smaller than one, so X<$3-4A$ and this condition becomes: 3X>X(2A+X). Moreover, since n_1^0 is also smaller than one, so $2A+X<3$. Hence this condition is satisfied and hence $n_1^* > n_1^0$.

Second, we compare the total output with one sided online celebrity marketing to the total output without the online celebrity marketing, to see if there is market expansion effect

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due to online celebrity marketing. However, since it is difficult to directly compare $n_1^* + n_2^*$ and $n_1^0 + n_2^0$, we proceed our discussion as follows. First, the FOCs without online celebrity marketing are:

$$1 + A - 2n_1 - n_2 - c = 0,$$
$$1 + B - n_1 - 2n_2 - c = 0,$$

and the sum of these two equations is:

$$2 + \mathrm{A} + \mathrm{B} - 2\mathrm{c} - 3(n_1 + n_2) = 0.$$

Second, the FOCs with one-sided online celebrity marketing are:

$$1 - 2(1 - A)n_1 - n_2 - c = 0,$$
$$1 + B - n_1 - 2n_2 - c = 0,$$

and the sum of these two equations is:

$$2 + \mathrm{A} + \mathrm{B} - 2\mathrm{c} - 3(n_1 + n_2) = A(1 - 2n_1).$$

So, if $2n_1^* < 1$, then the RHS of the above equation is higher. To decrease this value and according to the second order condition of profit maximization, $n_1^* + n_2^*$ will be higher than $n_1^0 + n_2^0$, which justifies the market expansion effect.

Third, the partial differentiation of $n_1^* - n_1^0$ w.r.t. A is given by

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$$\frac{\partial(n_1^* - n_1^0)}{\partial A} = \frac{4X}{[3-4A]^2} - \frac{2}{3}.$$

Given that $n_1^* = \frac{X}{3-4A}$<1, a sufficient condition for $\frac{\partial(n_1^* - n_1^0)}{\partial A}$>0 is $\frac{4}{[3-4A]}$>$\frac{2}{3}$, which implies that *12>6-8A*. This is satisfied for all A>0. So we have $\frac{\partial(n_1^* - n_1^0)}{\partial A}$>0. Since A=$k_1(v - \frac{\sigma_1}{l_1})$, we have the following result on firm 1's increase in market share.

Proposition 2: **(*i*)** *Firm 1's increase in market share due to online celebrity marketing is increasing in* k_1 *and* l_1*, but decreases with* σ_1*. (ii) Online celebrity marketing has a market expansion effect.*

Intuitively, as k_1 increases, the influence of the celebrity becomes higher, so consumers are more willing to buy the product, and the market share will increase. As l_1 increases, the learning effect becomes higher, which means if the celebrity promotes their product, the consumer can get more information about the product. They are more willing to buy the product. As σ_1 increases, the influence of uncertainty becomes higher. If the product's uncertainty is increasing, they may confuse about the reality of product value. It may reduce the consumer's intention to buy the product, the market share will decrease.

Third, firm 1's profit incentive to adopt online celebrity marketing is given by

$$\pi_1^* - \pi_i^0 = (1 - \text{A})(n_1^*)^2 - (n_1^0)^2.$$

The condition for $\pi_1^* - \pi_i^0 > 0$ is:

$$(1 - \text{A}) > \left(\frac{n_1^0}{n_1^*}\right)^2,$$

which becomes:

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$$1 - \mathrm{A} > \left(\frac{(2A+X)(3-4A)}{3X}\right)^2. \quad (6)$$

As presented in Figure 2, if A=0, then both dies of equation (6) are the same (=1); If A=$\frac{3}{4}$, then LHS is greater; If A=1, then RHS is greater. That is, there exists a critical value $\bar{A}$, and for $0<A<\bar{A}$, $\pi_1^* > \pi_i^0$.

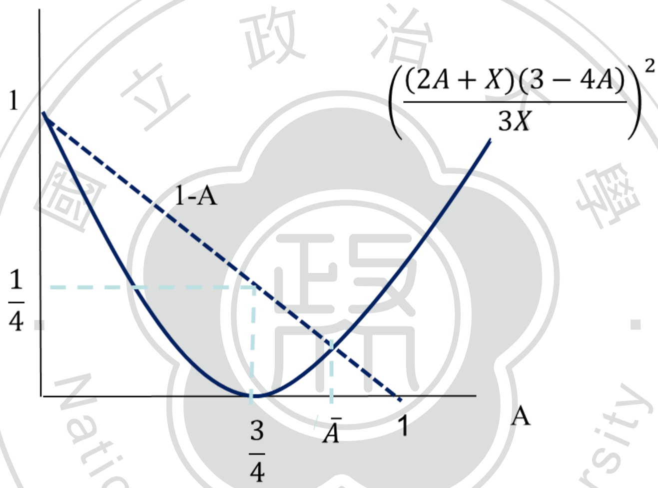

Figure 2. The condition for $\pi_1^* - \pi_i^0 > 0$.

Proposition 3: *F*irm 1's profit incentive to adopt online celebrity marketing is positive for *0<A<* $\bar{A}$*.*

Since A=$k_1(v - \frac{\sigma_1}{l_1})$, we know that A is increasing in both k_1 and l_1. So adopting online celebrity marketing will be profitable for three cases: (i) both k_1 and l_1 are low; (ii) k_1 is high but l_1 is low; (iii) k_1 is low but l_1 is high. For the case with both k_1 and l_1 are high, adopt online celebrity marketing is not better off. In this case, the profit loss due to greater

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elasticity is higher than gain from market share increase due to online celebrity marketing.

Corollary 1: *For the case with both* k_1 *and* l_1 *are high, adopt online celebrity marketing is not better off.*

Forth, the price dispersion is given by $(p_1^* - p_2^*)$, where

$$p_1^* - p_2^* = \frac{-(1-A) - 3(1-A)B + 1 - cA}{4(1-A) - 1},$$

Since $\frac{\partial(p_1 - p_2)}{\partial A} = \frac{3(1-B-c)}{[4(1-A)-1]^2} > 0$, and $\frac{\partial(p_1 - p_2)}{\partial B} = \frac{-3(1-A)}{4(1-A)-1} < 0$, we have the following result.

Proposition 4 The following describes the comparative statics on the price dispersion.
(i)As k_1 and l_1 *increase,* $p_1^* - p_2^*$ *will increase, and firm I's equilibrium price* (p_1^*) *will increase, while firm 2's price* p_2^* *will decrease.*
(ii)As σ_1 *increases,* $p_1^* - p_2^*$ *will decrease, and firm I's equilibrium price* (p_1^*) *will decrease, while firm 2's price* p_2^* *will increase.*

Proof. See the Appendix.

Intuitively, as k_1 increases, the influence of the celebrity becomes higher, so after the celebrity's advertising, the consumers are more willing to pay the higher price to buy the product, and the price dispersion will increase. As l_1 increases, the learning effect becomes higher, which means if the celebrity promotes their product, the consumers can get more information about the product. They are more willing to pay the higher price to buy the product. As σ_1 increases, the influence of uncertainty becomes higher. If the product's uncertainty is increasing, they may confuse about the reality of product value, so they reduce the willing price they pay, the price dispersion will decrease.

Fifthly, the profit difference is measured by π_1^*-π_2^*, where

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$$\pi_1^* - \pi_2^* = (1-\text{A})(n_1^*)^2 - (n_2^*)^2$$

$$= (1-\text{A})(n_1^*)^2[1-(\tfrac{n_2^*}{n_1^*})^2]. \qquad (7)$$

Notice that $\frac{n_2^*}{n_1^*}$=$\frac{2AB-(1-c)(1-2A)}{1-B-c}$, whose partial differentiation w.r.t. A is *2(B+1-c)>0*. Since both $(1-\text{A})$ *and* $[1-(\frac{n_2^*}{n_1^*})^2]$ are negatively related A and $(n_1^*)^2$ is positively related to A, we have we have $\frac{\partial(\pi_1^*-\pi_2^*)}{\partial A}$>0 (See the Appendix for the details). Hence we have the following result.

***Proposition* 5** *The following describes the comparative statics on the profit difference.*
(i)As k_1 and l_1 increase, π_1^*-π_2^* will increase, and firm I's equilibrium profit (π_1) will increase, while firm 2's profit (π_2) will decrease.
(ii)As σ_1 increases,π_1^*-π_2^* will decrease and firm I's equilibrium profit (π_1) will decrease, while firm 2's profit (π_2) will increase.

Proof. See the Appendix.

Intuitively, from proposition3 and proposition4, as k_1 increases, we can find the price and market share will increase, the firm can benefit from the online celebrity marketing, and can get higher profit dispersion. And as the learning effect increases, the consumer's willingness to pay and the purchase intention will increase at the same time, so it may increase the firm's profit. While the uncertainty will reduce the consumer's willingness to pay and the purchase intention, so when the uncertainty increases, the profit dispersion will decrease.

Finally, we use Herfindahl-Hershman Index (HHI) to measure the market concentration.

$$\text{HHI} = n_1^2 + n_2^2$$

$$= n_1^2[1+\left(\tfrac{n_2^*}{n_1^*}\right)^2\Big].$$

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As we just calculated, the partial differentiation of $\frac{n_2^*}{n_1^*}$=$\frac{2AB-(1-c)(1-2A)}{1-B-c}$ w.r.t. A is *2(B+1-c)>0*. HHI will be increasing in A. Hence we have the following result.

Proposition 6 *The following describes the comparative statics on the HHI.*
(i) As k_1 *and* l_1 *increase,* HHI *will increase,*
(ii) As σ_1 *increases,* HHI *will decrease,*

HHI can measure the market concentration and it is used to determine market competitiveness. Intuitively, as k_1 increasing, the influence of the celebrity will increase, the firm1's competitiveness will increase, and the market concentration will become higher. As firm1's learning effect increases, firm1's competitiveness will increase, and the market concentration will become higher. When the uncertainty increases, the market concentration will become lower.

3.2 Two-sided online celebrity marketing

In this section, we assume that both firm 1 and firm 2 use online celebrity marketing. The difference is now we assume that for both firms, i=1,2,

$$g_i = k_i\left(v - \frac{\sigma_i}{l_i}\right)n_i^e.$$

Substitute this definition and $n_i^e = n_i$ into:

$$p_i = 1 - (n_i + n_j) + g_i.$$

We have each firm's demand function:

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$$p_1 = 1 - (1 - A)n_1 - n_2.$$

$$p_2 = 1 - n_1 - (1 - B)n_2.$$

3.2.1 Market equilibrium with two-sided online celebrity marketing

Firms compete in quantities ($i.\text{e.}, n_i$), so the two firms choose their quantities n_i simultaneously to maximize their profits, where

$$\pi_1 = \{1 - (1 - A)n_1 - n_2 - c_1\}n_1.$$

$$\pi_2 = \{1 - n_1 - (1 - B)n_2 - c_2\}n_2.$$

Assuming symmetric cost: $c_1 = c_2 = c$, we have the following FOCs:

$$1 - 2(1 - A)n_1 - n_2 - c = 0,$$

$$1 - n_1 - 2(1 - B)n_2 - c = 0.$$

And the second order conditions for profit maximization are:

$$\frac{\partial^2(\pi_1)}{\partial(n_1)^2} = -2(1 - A) \text{ and } \frac{\partial^2(\pi_2)}{\partial(n_2)^2} = -2(1 - B),$$

which are satisfied if $1 > A$ and $1 > B$. To simplify the analysis, we will make this assumption henceforth.

Figure 3 below shows the best replies for firm 1 and firm 2, where

$$n_1^*(n_2) = \frac{1}{2(1-A)}(1 - c - n_2), \quad \text{and} \quad n_2^*(n_1) = \frac{1}{2(1-B)}(1 - c - n_1).$$

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The equilibrium market shares are:

$$n_1 = \frac{(1-2B)(1-c)}{4(1-A)(1-B)-1} \quad \text{and} \quad n_2 = \frac{(1-2A)(1-c)}{4(1-A)(1-B)-1}.$$

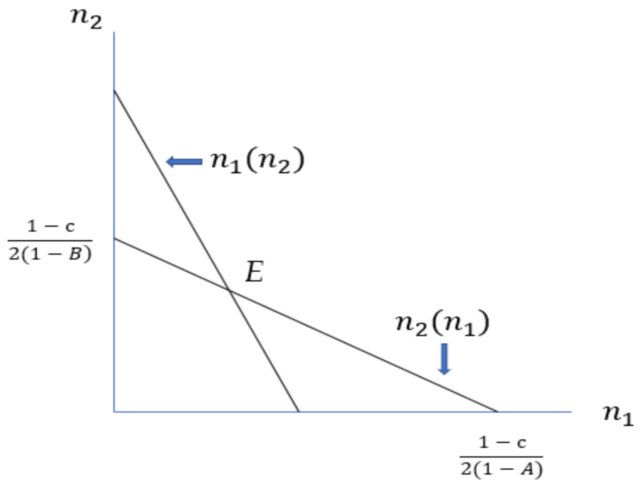

Figure3 Best replies with both-sided celebrity marketing

Next, the equilibrium prices are calculated as follows. First, firm 1's demand function is:

$$p_1 = 1-(1-A)n_1 - n_2,$$

and firm 1's FOC is:

$$1-2(1-A)n_1 - n_2 - c=0.$$

Then, rewrite this condition using the definition of p_1, so we have:

$$p_1 - (1-A)n_1 - c = 0.$$

Firm 1's equilibrium price is hence:

$$p_1^* = (1-A)n_1^* + c.$$

Similarly, $$p_2^* = (1-B)n_2^* + c.$$

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Second, using the above equilibrium prices the equilibrium profits are given by

$$\pi_1^* = \{p_1^* - c\}n_1^* = (1-\mathrm{A})(n_1^*)^2 = \frac{(1-2B)(1-c)}{4(1-A)(1-B)-1},$$

And

$$\pi_2^* = \{p_2^* - c\}n_2^* = (1-\mathrm{B})(n_2^*)^2.$$

3.2.2 Effects of two-sided online celebrity marketing

First, recall that $n_1^* = \frac{(1-2B)(1-c)}{4(1-A)(1-B)-1}$ and $n_2^* = \frac{(1-2A)(1-c)}{4(1-A)(1-B)-1}$ To simplify, let X=$\frac{(1-c)}{4(1-A)(1-B)-1}$. Therefore $n_1^* = (1-2B)X$ and $n_2^* = (1-2A)X$. So the condition for $n_1^* > n_2^*$ is: A>B.

Next, it can be calculated that

$$\frac{\partial(n_1^* - n_2^*)}{\partial A} = \frac{(1-c)(8(1-B)^2-2)}{[4(1-A)(1-B)-1]^2}.$$

Since the equilibrium output can be rewritten as $n_1^* = \frac{X}{3-4A}$, we have X>0. From $n_1^* = (1-2B)X$, we have 1-2B>0, which implies that B<0.5. Therefore, $(1-\mathrm{B})^2 > 0.25$, and we can conclude $\frac{\partial(n_1^*-n_2^*)}{\partial A} > 0$.

Proposition 7: *The following describes the comparative statics on the market share difference.*
(i)As k_1 and l_1 increase, n_1^*-n_2^* will increase, and firm I's equilibrium market share (n_1^*) will increase, while firm 2's market share (n_2^*) will decrease; As k_2 and l_2 increase, n_1^*-n_2^* will decrease, and firm I's equilibrium market share (n_1^*) will decrease, while firm 2's market share (n_2^*)

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will increase

(ii)As σ_1 increases,n_1^*-n_2^* will decrease and firm I's equilibrium market share (n_1^*) will decrease, while firm 2's equilibrium market share (n_2^*) will increase; As σ_2 increases,π_1^*-π_2^* will increase and firm I's equilibrium market share (n_1^*) will increase, while firm 2's equilibrium market share (n_2^*) will decrease

Proof. See the Appendix.

Intuitively, as k_1 increases, for both firms with online celebrity marketing, the influence of the celebrity is expected to become higher. The consumer has a higher confidence level of product 1. When they are more willing to buy product 1, the market share difference will increase. As the learning effect increases, the consumer can have more understanding of the product. They have a higher intention to buy the product, and the market share difference will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to buy the product, and the market share difference will decrease. The effect from k_2, l_2, σ_2 is the opposite.

Second, the price dispersion is given by $(p_1^* - p_2^*)$, where $p_1 = \frac{(1-A)(1-2B)(1-c)}{4(1-A)(1-B)-1}$, and $p_2 = \frac{(1-B)(1-2A)(1-c)}{4(1-A)(1-B)-1}$. Hence we have:

$$p_1^* - p_2^* = p_1 - p_2 = \frac{A - B - Ac + Bc}{4(1-A)(1-B) - 1},$$

Since $\frac{\partial(p_1-p_2)}{\partial A} = \frac{(1-c)(4(1-B)^2-1)}{(4(1-A)(1-B)-1)^2} > 0$, and $\frac{\partial(p_1-p_2)}{\partial B} = \frac{(1-c)(1-4(1-A)^2)}{(4AB-1)^2} < 0$, we have the following result.

Proposition 8 The following describes the comparative statics on the price dispersion.

(i)As k_1 and l_1 increase, $p_1^* - p_2^*$ will increase, and firm I's equilibrium price (p_1^*) will increase, while firm 2's price (p_2^*) will decrease; As k_2 and l_2 increase, $p_1^* - p_2^*$ will decrease, and firm I's equilibrium price (p_1^*) will decrease, while firm 2's price (p_2^*) will increase

(ii)As σ_1 increases, $p_1^* - p_2^*$ will decrease, and firm I's equilibrium price (p_1^*) will decrease, while

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firm 2's price (p_2^*) *will increase; As* σ_2 *increases,* $p_1^* - p_2^*$ *will increase, and firm I's equilibrium price* (p_1^*) *will increase, while firm 2's price* (p_2^*) *will decrease*

Proof. See the Appendix.

Intuitively, as k_1 increases, for both firms with online celebrity marketing, the influence of the celebrity is expected to become higher. The consumers have a higher confidence level of product 1. When they are more willing to pay a higher price to buy product 1, the price dispersion will increase. As the learning effect increases, the consumer can have more understanding of the product. They pay a higher price to buy the product, and the price dispersion will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products. It may reduce their willingness to pay for the product, and the price dispersion will decrease. The effect from k_2, l_2, σ_2 is the opposite.

Third, the profit difference is measured by π_1^*-π_2^*, where $\pi_1 = \frac{(1-A)(1-2B)^2(1-c)^2}{(4AB-1)^2}$, and $\pi_2 = \frac{(1-B)(1-2A)^2(1-c)^2}{(4AB-1)^2}$. Hence, we have:

$$\pi_1^* - \pi_2^* = (1-\mathrm{A})(n_1^*)^2 - (1-B)(n_2^*)^2, \tag{7}$$

Since $\frac{\partial(\pi_1-\pi_2)}{\partial A} = \frac{(4(1-B)^2-1)(1-c)^2}{(4(1-A)(1-B)-1)^2} > 0$, and $\frac{\partial(\pi_1-\pi_2)}{\partial B} = \frac{(1-4(1-A)^2)(1-c)^2}{(4(1-A)(1-B)-1)^2} < 0$, we have the following result.

Proposition 9 *The following describes the comparative statics on the profit difference.*
(i)As k_1 *and* l_1 *increase,* π_1^*-π_2^* *will increase, and firm I's equilibrium profit* (π_1) *will increase, while firm 2's profit* (π_2) *will decrease; as* k_2 *and* l_2 *increase,* π_1^*-π_2^* *will decrease, and firm I's equilibrium profit* (π_1) *will decrease, while firm 2's profit* (π_2) *will increase.*
(ii)As σ_1 *increases,*π_1^*-π_2^* *will decrease and firm I's equilibrium profit* (π_1) *will decrease, while firm 2's profit* (π_2) *will increase; As* σ_2 *increases,*π_1^*-π_2^* *will increase and firm I's equilibrium profit* (π_1) *will increase, while firm 2's profit* (π_2) *will decrease*

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Proof. See the Appendix.

Intuitively, from proposition 7 and proposition 8, we can find when k_1 increasing, the firm 1's price and market share will increase. It is because when the influence of the celebrity becomes higher, the consumer will have a higher confidence level in the product, so the consumer's willingness-to-pay and the purchase intention will increase, and profit dispersion increasing. And when the learning effect increases, the consumer's willingness to pay and the purchase intention will increase, so the profit dispersion increasing. While the uncertainty will reduce the consumer's willingness to pay and the purchase intention. when the uncertainty increases, profit dispersion will decrease. The effect from k_2, l_2, σ_2 will be the opposite.

Finally, we use Herfindahl-Hershman Index (HHI) to measure the market concentration.

$$\mathrm{HHI} = n_1^2 + n_2^2,$$

Take the partial differentiaon wrt A and B, and we have

$$\frac{\partial HHI}{\partial A} = \frac{8(1-B)(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2} * n_1 + \frac{-4(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2} * n_2,$$
$$\frac{\partial HHI}{\partial B} = \frac{-4(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2} * n_1 + \frac{8(1-A)(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2} * n_2.$$

We have the following result.

Proposition 10 *The following describes the comparative statics on the HHI.*
When $2(1-\mathrm{B})n_1 > n_2$, *(i) As* k_1 *and* l_1 *increase,* HHI *will increase. (ii) As* σ_1 *increases,* HHI *will decrease,*
When $2(1-\mathrm{A})n_2 > n_1$ *(iii) As* k_2 *and* l_2 *increase,* HHI *will increase, (iv) As* σ_2 *increases,* HHI *will decrease,*

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Proof. See the Appendix.

Intuitively, when $2(1-\mathrm{B})\mathrm{n}_1 > \mathrm{n}_2$, we can anticipate firm 2's product value is small, as k_1 increases HHI will decrease. It is because firm1 has higher competition, so the market concentration will increase. And when the learning effect increases, people can get more information from the product1, firm 1 's competitiveness will become higher, HHI will increase. when uncertainty increases, the consumer may confuse about product value, so firm1's competitive decreasing, HHI decreases.

3.3 Entrant with Online celebrity marketing vs Two Traditional Incumbents

In this section, we consider that only firm 1 uses online celebrity marketing, while firm 2 and firm 3 do not. As before, we assume that given firm i's price p_i, consumer θ obtains a net surplus for purchasing product i:

$$u_i(\theta, g_i) = \theta + g_i - p_i,$$

and that

$$g_1 = k_1\left(v - \frac{\sigma_1}{l_1}\right) n_1^e,$$

$$g_2 = k_2(v - \sigma_2),$$

$$g_3 = k_3(v - \sigma_3).$$

Only firm 1 uses online celebrity marketing that has a network effect. The difference setting as before is that in this case, firm 1 is a new entrant, so it has higher uncertainty than the other firms, the other parameters are interpreted as before.

$$\sigma_1 > \sigma_2 = \sigma_3$$

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Firms compete in quantities, so we follow Belleflamme and Peitz's (2015) by assuming that prices will adjust to the levels where the marginal consumer is "indifferent between buying the two products" and "indifferent between buying and not buying". This implies that we are considering a large market; consumers have sufficiently heterogenous preferences such that some consumers would rather not buy anything. Specifically, for the marginal consumer $\hat{\theta}$,

$$\hat{\theta} + g_1 - p_1 = \hat{\theta} + g_2 - p_2 = \hat{\theta} + g_3 - p_3,$$

which indicates that

$$p_1 - g_1 = p_2 - g_2 = p_3 - g_3 \equiv \hat{p}.$$

In other words, consumers with θ lower than $\hat{p}$ will not buy any product. Let n_i indicate the number of consumers who buy product i. The above equation implies that

$$n_1 + n_2 + n_3 = 1 - \hat{p}.$$

Substitute the definition of $\hat{p}$, we have

$$p_i = \hat{p} + g_i = 1 - \left(n_i + n_j - n_l\right) + g_i.$$

Substituting g_i and $n_i^e = n_i$ into this equation gives the two firms' inverse demand functions, where

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$$p_1 = 1 - (1 - \ k_1(v - \frac{\sigma_1}{l_1}))n_1 - n_2 - n_3.$$

$$p_2 = 1 - n_1 - n_2 - n_3 + k_2(v - \sigma_2),$$

$$p_3 = 1 - n_1 - n_2 - n_3 + k_3(v - \sigma_3).$$

To simplify the analysis, let A= $k_1\left(v - \frac{\sigma_1}{l_1}\right)$, B=$k_2(v - \sigma_2)$, and C=$k_3(v - \sigma_3)$. Hence

$$p_1 = 1 - (1 - A)n_1 - n_2 - n_3.$$

$$p_2 = 1 - n_1 - n_2 - n_3 + B,$$

$$p_3 = 1 - n_1 - n_2 - n_3 + C.$$

3.3.1 Market Equilibrium with only entrant using online celebrity marketing

Firms compete in quantities ($i.\text{e.}, n_i$), so the two firms choose their quantities n_i simultaneously to maximize their profits, where

$$\pi_1 = \{1 - (1 - A)n_1 - n_2 - c_1\}n_1.$$

$$\pi_2 = \{1 - n_1 - n_2 - n_3 + B - c_2\}n_2.$$

$$\pi_3 = \{1 - n_1 - n_2 - n_3 + C - c_3\}\, n_3.$$

To simplify the analysis, we assume symmetric cost: $c_1 = c_2 = c_3 = c$. Hence, the first order conditions of maximization are:

$$1 - 2(1 - A)n_1 - n_2 - n_3 - c\text{=}0,$$

$$1 + B - n_1 - 2n_2 - n_3 - c\text{=}0,$$

$$1 + C - n_1 - n_2 - 2n_3 - c\text{=}0,$$

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and the second order conditions for profit maximization are:

$$\frac{\partial^2(\pi_1)}{\partial(n_1)^2} = -2(1-A),\ \frac{\partial^2(\pi_2)}{\partial(n_2)^2} = -2,\ \text{and}\ \frac{\partial^2(\pi_3)}{\partial(n_3)^2} = -2.$$

We focus on a specific case where firm 2 and firm 3 have symmetric equilibrium. In what follows, we assume that C=B in firms' best reply functions and that $n_2 = n_3$. From the FOCs, we can solve the equilibrium outputs, where

$$n_1 = \frac{1-2B-c}{4-6A},$$

$$n_2 = n_3 = \frac{-1+2(1-A)+2(1-A)B+c(2A-1))}{4-6A}.$$

The equilibrium prices are calculated as follows. First, firm 1's demand function is:

$$p_1 = 1-(1-A)n_1 - 2n_2,$$

and firm 1's FOC is:

$$1-2(1-A)n_1-2n_2-c=0.$$

Then, rewrite this condition using the definition of p_1, so we have:

$$p_1-(1-A)n_1-c=0.$$

Firm 1's equilibrium price is hence:

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$$p_1^* = (1 - A)n_1^* + c.$$

Similarly, $$p_2^* = p_3^*. = n_2^* + c$$

Second, using the above equilibrium prices the equilibrium profits are given by

$$\pi_1^* = \{p_1^* - c\}n_1^*$$
$$= (1 - \mathrm{A})(n_1^*)^2,$$

And

$$\pi_2^* = (n_2^*)^2 = \pi_3^*$$

3.3.2 Effects of entrant with online celebrity marketing to the traditional market

First, recall that

$$n_1^* = \frac{1-2B-c}{4-6A} \text{ and } n_2^* = n_3^* = \frac{1-2A+2B-2AB+c(2A-1))}{4-6A}.$$

Therefore,

$$n_1^* - n_2^* = \frac{A - 2B + AB - cA}{2 - 3A},$$

Since $\frac{\partial(n_1^*-n_2^*)}{\partial A} = \frac{2(1-2B-c)}{(2-3A)^2} > 0$, and $\frac{\partial(n_1^*-n_2^*)}{\partial B} = \frac{A-2}{2-3A} < 0$, we have the following result.

Proposition 11 *The following describes the comparative statics on the market share difference.*
(i)As k_1 and l_1 *increase,* $n_1 - n_2$ *will increase, and firm I's equilibrium output* (n_1) *will increase,*

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while firm 2's and firm 3's output will decrease.
(ii)As σ_1 *increases,* $n_1 - n_2$ *will decrease, and firm I's equilibrium output* (n_1) *will decrease, while firm 2's and firm 3's output will increase.*

Proof. See the Appendix.

Intuitively, when a firm that uses online celebrity marketing enters the traditional market, it can use celebrity influence to increase the consumer's purchase intention. If the celebrity influence becomes higher, people have higher confidence in the product, so they are more willing to buy it. The market share difference will increase. As the learning effect increases, the consumer can have more understanding of the product. They have a higher intention to buy the product, and the market share difference will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to buy the product, and the market share difference will decrease.

Second, the price dispersion is given by $(p_1^* - p_2^*)$, where

$$p_1 = \frac{(1-A)(1-2B-c)}{4-6A} + \text{c}, \quad \text{and} \quad p_2 = p_3 = \frac{1-2A+2B-2AB+c(3-4A)}{4-6A}.$$

Therefore,

$$p_1^* - p_2^* = \frac{A - 4 + AB - cA}{4 - 6A}.$$

Since $\frac{\partial(p_1 - p_2)}{\partial A} = \frac{1-2B-c}{(2-3A)^2} > 0$, and $\frac{\partial(p_1 - p_2)}{\partial B} = \frac{2(A-1)}{2-3A} < 0$, we have the following result.

Proposition 12 *The following describes the comparative statics on the price dispersion.*
(i)As k_1 and l_1 *increase,* $p_1^* - p_2^*$ *will increase, and firm I's equilibrium price* (p_1^*) *will increase, while firm 2's and firm 3's price will decrease.*
(ii)As σ_1 *increases,* $p_1^* - p_2^*$ *will decrease, and firm I's equilibrium price* (p_1^*) *will decrease, while*

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firm 2's and firm 3's price will increase.

Proof. See the Appendix.

Intuitively, when a firm that has a celebrity market enters the traditional market, it can use celebrity influence to increase the consumer's willingness to pay. If the celebrity influence becomes higher, people have higher confidence in the product, so they are more willing to pay the higher price to buy it. The price dispersion will increase. When the learning effect increases, the consumer can have more understanding of the product. They will pay a higher price, and the price dispersion will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to pay for the product, and the price dispersion will decrease.

Third, the profit difference is measured by π_1^*-π_2^*, where

$$\pi_1 = \frac{(1-A)(1-2B-c)^2}{4(2-3A)^2}, \text{ and } \quad \pi_2 = \pi_3 = \frac{\left(1-2A+2B-2AB+c(2A-1)\right)^2}{4(2-3A)^2}$$

Therefore,

$$\pi_1^* - \pi_2^* = (1-\text{A})(n_1^*)^2 - (n_2^*)^2.$$

Let $\text{x} = \frac{1-2B-c}{2(3A-1)^2}$, and $\text{y} = \frac{1}{2-3A}$. The partial differentiation wrt A and B are:

$$\frac{\partial(\pi_1-\pi_2)}{\partial A} = \text{x}n_1 + 3xp_1 + \text{x}n_2 + xp_2 - c4x > 0.$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial B} = -(1-\text{A})yn_1 - yp_1 - yn_2 - (1-A)yp_2 + cy(2-A) < 0.$$

Proposition 13 *The following describes the comparative statics on the profit difference.*

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(i)As k_1 and l_1 increase, π_1^*-π_2^* will increase, and firm I's equilibrium profit (π_1) will increase, while firm 2's and firm 3's profit will decrease.
(ii)As σ_1 increases,π_1^*-π_2^* will decrease and firm I's equilibrium profit (π_1) will decrease, while firm 2's and firm 3's profit will increase.

Proof. See the Appendix.

Intuitively, from proposition 11 and proposition 12 we can find when a firm that uses online celebrity marketing enters the traditional market, it can use celebrity influence to increase the consumer's willingness to pay and the purchase intention, so the profit dispersion will increase. When the learning effect increases, the consumer can have more understanding of the product, so they are more willing to buy the product and pay a higher price, and the profit dispersion will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce t the consumer's willingness to pay and the purchase intention, and the profit dispersion will decrease.

Finally, the Herfindahl-Hershman Index (HHI) is.

$$\begin{aligned}\text{HHI} &= n_1^2 + n_2^2 + n_3^2 \\ &= \left(\frac{1-2B-c}{4-6A}\right)^2 + 2\left(\frac{-1+2(1-A)+2(1-A)B+c(2A-1))}{4-6A}\right)^2,\end{aligned}$$

Since $\frac{\partial HHI}{\partial A} = \frac{6(1-2B-c)}{2(2-3A)^2} * n_1 + \frac{-4(1-2B-c)}{2(2-3A)^2} * n_2$, and $\frac{\partial HHI}{\partial B} = \frac{-4}{2-3A} * n_1 + \frac{8-8A}{2-3A} * n_2$, we have the following result.

Proposition 14 *The following describes the comparative statics on the HHI.*
(i) if $3n_1 > 2n_2$, then as k_1 and l_1 increase, HHI will increase,
(ii) if $3n_1 > 2n_2$, then as σ_1 increases, HHI will decrease,
(iii) if $n_1 < 2(1-A)n_2$, then as k_2 increase, HHI will increase,
(iv) if $n_1 < 2(1-A)n_2$, then as σ_2 increases, HHI will decrease,

Proof. See the Appendix.

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Intuitively, when $3n_1 > 2n_2$, as k_1 increasing, HHI will increase. It is because firm1 has higher competition, so the market concentration will increase. And when the learning effect increases, people can get more information from the product1, firm 1 's competitiveness will become higher, HHI will increase. when uncertainty increases, the consumer may confuse about product value, so firm1's competitive decreasing, HHI decreases.

4. Online celebrity marketing in Small Oligopolistic Market

In this chapter, we assume the market size is small such that $\sum_i n_i = 1$. The biggest difference between the large and small markets is as follows.

In the large market, the consumer heterogeneity is big enough, and some consumers choose not to buy anything, so the marginal consumer is indifferent between buying and not buying. For example, in China, the consumer heterogeneity is high, so some consumers are not willing to buy any products, even if the product's quality is high. In a small market, consumer heterogeneity is low. In this case, the consumers must buy one of the products, so it had a restriction function $\sum_i n_i = 1$.

The presence of a network effect from online celebrity marketing will increase the marginal utility of consuming one unit of product. Hence, in the Hotelling model, the indifferent consumer will be smaller than the traditional model. This causes the market share to be smaller with online celebrity marketing. Moreover, we will show that higher network effect will decrease the marketing firm's price. As a result, unless this network effect is sufficiently high, the firm adopting the online celebrity marketing will receive a smaller profit. This is totally different from the result for a lager market.

4-1 Incentive for Online celebrity marketing in Duopoly

To simplify, we assume that there are two firms located on the two extremes of a unit street. We assume that only firm 1 adopts the online celebrity marketing, while firm 2 does

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not. The notations of g_1 *and* g_1 are the same as Section 3.1; Namely,

$$g_1 = k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1^e,$$

$$g_2 = k_2(v - \sigma_2).$$

Since g_1 involves the expectation of market share for firm 1, we focus on a self-fulfilled equilibrium: $n_1^e = n_1$.

There is a continuum of consumers uniformly distributed on this street. By slightly abusing the notations, let n_1 be the indifferent consumer for whom the net values of the two products are the same. That is,

$$k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1 - \tau n_1 - p_1 = k_2(v - \sigma_2) - \tau(1 - n_1) - p_2,$$

p_1 and p_2 are the prices for firm 1 and 2, respectively. τn_1 and $\tau(1 - n_1)$ denote the respective transportation cost for this indifferent consumer travelling to firm 1 and firm 2.

From the indifferent condition, we can calculate

$$n_1 = \frac{p_1 - p_2 + k_2(v - \sigma_2) - \tau}{k_1\left(v - \frac{\sigma_1}{l_1}\right) - 2\tau},$$

$$n_2 = (1 - n_1).$$

To simplify the analysis, let A= $k_1(v - \frac{\sigma_1}{l_1})$ and B=$k_2(v - \sigma_2)$, so

$$n_1 = \frac{p_1 - p_2 + B - \tau}{A - 2\tau},$$

$$n_2 = \frac{-p_1 + p_2 + A - B - \tau}{A - 2\tau}.$$

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To satisfy the law of demand, $A - 2\tau < 0$. In other words, the size of network effect is smaller than the degree of horizontal heterogeneity among consumers. If otherwise, the size of network effect will dominate and drive the opponent firm out of the market.

4.1.1 Market Equilibrium with one-sided online celebrity marketing

Different form Section 3, here firms compete in price, so the two firms choose their price p_i simultaneously to maximize their profits, where

$$\pi_1 = (p_1 - c_1) * \frac{p_1 - p_2 + B - \tau}{A - 2\tau},$$

$$\pi_2 = (p_2 - c_2) * \frac{-p_1 + p_2 + A - B - \tau}{A - 2\tau}.$$

To simplify the analysis, we assume symmetric cost: $c_1 = c_2 = c$. Hence, the first order conditions of maximization are:

$$\frac{p_1 - p_2 + B - \tau + (p_1 - c)}{A - 2\tau} = 0,$$

$$\frac{p_2 - p_1 + A - B - \tau + (p_2 - c)}{A - 2\tau} = 0$$

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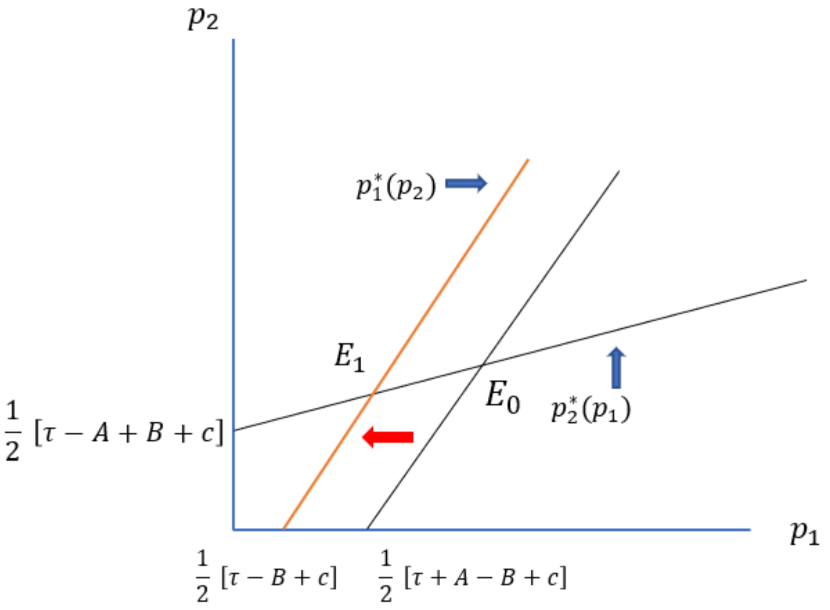

Figure4 Best replies with one-sided celebrity marketing

Figure 4 shows the best replies for firm 1 and firm 2, where

$$p_1^*(p_2) = \frac{1}{2}[p_2 + \tau - B + c],$$

$$p_2^*(p_1) = \frac{1}{2}[p_1 + \tau - A + B + c].$$

Notice that in the case without such a network effect by online celebrity marketing, the best reply for firm 1 would be

$$p_1^{'}(p_2) = \frac{1}{2}[p_2 + \tau + A - B + c].$$

We know the one-sided online celebrity marketing influences firm 1's best reply function. The slope in $p_1^*(p_2)$ is 2, and the slope of $p_2^*(p_1)$ is $\frac{1}{2}$. When the firm adopt online celebrity marketing, $p_1(p_2)$ will shift to $p_1^*(p_2)$, both equilibrium price will decrease. The effect to p_1 is bigger than p_2. When A increases, $p_1^*(p_2)$ doesn't change, $p_2^*(p_1)$ will shift

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downward$\frac{1}{2}A$, the decrease to p_2 is higher than the decrease to p_1.

The second order conditions for profit maximization are:

$$\frac{\partial^2(\pi_1)}{\partial(p_1)^2} = \frac{2}{A-2\tau} < 0 \text{ and } \frac{\partial^2(\pi_1)}{\partial(n_2)^2} = \frac{2}{A-2\tau} < 0,$$

From the best replies functions, we can calculate the equilibrium price:

$$p_1^* = \frac{3\tau-A-B+3c}{3} \text{ and } p_2^* = \frac{3\tau-2A+B+3c}{3}.$$

The equilibrium market shares are:

$$n_1^* = \frac{3\tau-A-B}{3(2\tau-A)} \text{ and } n_2^* = (1-n_1^*) = \frac{3\tau-2A+B}{3(2\tau-A)}.$$

The equilibrium profits are

$$\pi_1^* = \frac{(3\tau-A-B)^2}{9(2\tau-A)} \text{ and } \pi_2^* = \frac{(3\tau-2A+B)^2}{9(2\tau-A)}.$$

4.1.2 Market Equilibrium without online celebrity marketing

As a benchmark of comparison, we describe the equilibrium without online celebrity marketing. In this case, firm 1's demand is given by

$$n_1 = \frac{p_1 - p_2 - A + B - \tau}{-2\tau}$$

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The equilibrium prices are determined by $p_1^1(p_2)$ and $p_2^*(p_1)$ in Section 4.1.1, where

$$p_1^0 = \frac{A - B + 3\tau + 3c}{3},$$
$$p_2^0 = \frac{-A + B + 3\tau + 3c}{3}.$$

.

The equilibrium output and profits are given by: for *i=1,2*

$$n_1^0 = \frac{3\tau + A - B}{6\tau} \text{ and } n_2^0 = \frac{3\tau - A + B}{6\tau},$$
$$\pi_1^0 = \frac{(3\tau - A + B)^2}{6\tau} \text{ and } \pi_2^0 = \frac{(3\tau + A - B)^2}{6\tau}$$

4.1.3 Effects of one-sided online celebrity marketing

First, we examine how online celebrity marketing changes firm 1's market share. Recall that

$n_1^* = \frac{3\tau - A - B}{3(2\tau - A)}$ and $n_1^0 = \frac{3\tau + A - B}{6\tau}$. To simplify, let X=$3\tau + A - B$. Then

$$n_1^* = \frac{X - 2A}{6\tau - 3A} \text{ and } n_1^0 = \frac{X}{6\tau}.$$

The condition for $n_1^* - n_1^0 \geq 0$ is hence: $A - B \geq \tau$. In other words, when the network effect from online celebrity marketing is higher than the impact on horizontal differentiation, then online celebrity marketing can create a higher market share. In the special case with A=B, we actually have $n_1^* < n_1^0$.

Second, the price dispersion is given by $(p_1^* - p_2^*)$, where

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$$p_1^* - p_2^* = \frac{A - 2B}{3}.$$

In other words, whether $p_1^* \geq p_2^*$ depends on $A \geq 2B$. Actually, this condition can be derived from $\mathrm{A} - \mathrm{B} \geq \tau$ and $A - 2\tau < 0$. Again, if the network effect from online celebrity marketing is higher than the impact on horizontal differentiation, then online celebrity marketing can raise its equilibrium price. Otherwise, the price will decrease.

Proposition 15: *Online celebrity marketing may decrease firm 1's equilibrium price.*

Since $\frac{\partial(p_1 - p_2)}{\partial A} = \frac{1}{3} > 0$, and $\frac{\partial(p_1 - p_2)}{\partial B} = \frac{-2}{3} < 0$, we have the following result.

Proposition 16 The following describes the comparative statics on the price dispersion.
(i)As k_1 and l_1 *increase,* $p_1^* - p_2^*$ *will increase*
(ii)As σ_1 *increases,* $p_1^* - p_2^*$ *will decrease*

Proof. See the Appendix.

In the small market, the influence to $p_1^* - p_2^*$ will be the same as in the large market. Intuitively, as k_1 increasing, the influence of the celebrity is more important, so after the celebrity's advertising, the consumers are more willing to pay the higher price to buy the product, and the price dispersion will increase. When the learning effect exists, it means when the celebrities advertise their product, the consumer can Let more information about the product, so when the learning effect increases, people can have more understanding of the product, and they are more willing to pay the higher price to buy the product, the price dispersion will increase. If the product's uncertainty is increasing, the consumer may consider the product value is decreasing, so they reduce the price they pay, and the price dispersion will decrease.

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Third, the market share difference is given by $(n_1^* - n_2^*)$, where

$$n_1^* - n_2^* = \frac{A - 2B}{3(2\tau - A)},$$

Since $\frac{\partial(n_1 - n_2)}{\partial A} = \frac{2\tau - 2B}{3(2\tau - A)^2} > 0$, and $\frac{\partial(n_1 - n_2)}{\partial B} = \frac{-2}{3(2\tau - A)} < 0$, we have the following result.

Proposition **17** The following describes the comparative statics on the market share difference.

(i)As k_1 and l_1 *increase,* $n_1^* - n_2^*$ *will increase*

(ii)As σ_1 *increases,* $n_1^* - n_2^*$ *will decrease*

Proof. See the Appendix.

In the small market, the influence on $n_1^* - n_2^*$ will be the same as in the large market. Intuitively, as k_1 increasing, it means the influence of the celebrity is become higher, from the literature review, we can find celebrities will lead to positive influence on the purchase intention, so the consumers are more willing to buy the product, and the market share will increase. When the learning effect exists, it means when the celebrities advertise their product, the consumer can Let more information about the product, so when the learning effect increases, people can have more understanding of the product, and they are more willing to buy the product. If the product's uncertainty is increasing, it may reduce the consumer's intention to buy the product, the market share will decrease.

Forth, the profit difference is measured by π_1^*-π_2^*, where

$$\pi_1^* - \pi_2^* = \frac{A - 2B}{3},$$

Again, whether adopting online celebrity marketing is better will depend on whether the network effect is higher than the impact on horizontal differentiation. Since $\frac{\partial(\pi_1 - \pi_2)}{\partial A} = \frac{1}{3} > 0$,

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and $\frac{\partial(\pi_1-\pi_2)}{\partial B}=\frac{-2}{3}<0$, we have the following result.

Proposition 18 The following describes the comparative statics on the market share difference.

(i)As k_1 and l_1 *increase,* $\pi_1^*-\pi_2^*$ *will increase*

(ii)As σ_1 *increases,* $\pi_1^*-\pi_2^*$ *will decrease*

Proof. See the Appendix.

In a small market, the influence to $\pi_1^*-\pi_2^*$ will be the same as in large market. Intuitively, from proposition 17 and proposition 18, as k_1 increases the price and market share will increase, so the firm can benefit from the celebrity influence, and can Let a higher profit. And when the learning effect increases, the consumer's willingness to pay and the purchase intention will increase at the same time, so it may increase the firm's profit. While the uncertainty will reduce the consumer's willingness to pay and the purchase intention, so when the uncertainty increases, the profit will decrease.

Finally, we use Herfindahl-Hershman Index (HHI) to measure the market concentration.

$$\text{HHI}=n_1^2+n_2^2,$$

Since $\frac{\partial HHI}{\partial A}=\frac{\partial HHI}{\partial A}=\frac{(2A-4B)(\tau-B)}{9(2\tau-A)^3}$and $\frac{\partial HHI}{\partial B}=\frac{(4B-2A)}{9(2\tau-A)^2}$ we have the following result.

Proposition 19 *The following describes the comparative statics on the HHI.*

When $\text{A}>2\text{B}$

(i) As k_1 *and* l_1 *increase,* HHI *will increase*

(ii) As σ_1 *increases,* HHI *will decrease,*

When $2\text{B}>\text{A}$

(iii) As k_2 *and* l_2 *increase,* HHI *will increase*

(iv) As σ_2 *increases,* HHI *will decrease,*

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Proof. See the Appendix.

Intuitively, in the small market, when A>2B, it means that product 1's value is high. As k_1 increasing, the firm1 will have higher competitiveness, the market concentration will become higher. When firm1's learning effect increases, firm1's competitiveness will increase, and the market concentration will become higher. When the uncertainty increases, the market concentration will become lower.

4.2 Two-sided online celebrity marketing

In this section, we assume that both firm 1 and firm 2 use online celebrity marketing. We retain most of the assumptions in Section 4.1, but now we assume that:

$$g_2 = k_2\left(v - \frac{\sigma_2}{l_2}\right)n_2^e.$$

Then by the indifferent condition,

$$k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1 - \tau n_1 - p_1 = k_2\left(v - \frac{\sigma_2}{l_2}\right)n_2 - \tau n_2 - p_2,$$

We have:

$$n_1 = \frac{p_2 - p_1 + \tau - k_2\left(v - \frac{\sigma_2}{l_2}\right)}{2\tau - k_1\left(v - \frac{\sigma_1}{l_1}\right) - k_2\left(v - \frac{\sigma_2}{l_2}\right)}.$$

$$n_2 = (1 - n_1).$$

Let A= $k_1(v - \frac{\sigma_1}{l_1})$ and B=$k_2\left(v - \frac{\sigma_2}{l_2}\right)$, so

$$n_1 = \frac{p_2 - p_1 - B + \tau}{2\tau - A - B}.$$

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$$n_2 = (1 - n_1) = \frac{-p_2 + p_1 - A + \tau}{2\tau - A - B}.$$

4.2.1 Market Equilibrium with two-sided online celebrity marketing

Firms compete in prices, so the two firms choose their price p_i simultaneously to maximize their profits, where

$$\pi_1 = (p_1 - c_1) * \frac{p_2 - p_1 - B + \tau}{2\tau - A - B}.$$

$$\pi_2 = (p_2 - c_2) * \frac{-p_2 + p_1 - A + \tau}{2\tau - A - B}.$$

To simplify the analysis, we assume $c_1 = c_2 = c$. Hence, the first order conditions of maximization are:

$$\frac{p_2 - p_1 - B + \tau - (p_1 - c)}{2\tau - A - B} = 0,$$

$$\frac{-p_2 + p_1 - A + \tau - (p_2 - c)}{2\tau - A - B} = 0.$$

Hence the best replies for firm 1 and firm 2 are:

$$p_1^*(p_2) = \frac{1}{2}[p_2 + \tau - B + c].$$

$$p_2^*(p_1) = \frac{1}{2}\ [p_1 + \tau - A + c].$$

And the second order conditions for profit maximization are:

$$\frac{\partial^2(\pi_1)}{\partial(p_1)^2} = \frac{-2}{2\tau - A - B} < 0 \ \text{ and } \ \frac{\partial^2(\pi_1)}{\partial(n_2)^2} = \frac{-2}{2\tau - A - B} < 0,$$

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The equilibrium prices are:

$$p_1 = \frac{3\tau - A - 2B + 3c}{3} \text{ and } p_2 = \frac{3\tau - 2A - B + 3c}{3}.$$

The equilibrium market shares are:

$$n_1 = \frac{3\tau - A - 2B}{3(2\tau - A - B)} \quad \text{and} \quad n_2 = (1 - n_1) = \frac{3\tau - 2A - B}{3(2\tau - A - B)}.$$

The equilibrium profits are

$$\pi_1 = \frac{(3\tau - A - 2B)^2}{9(2\tau - A - B)} \text{ and } \pi_2 = \frac{(3\tau - 2A - B)^2}{9(2\tau - A - B)}.$$

4.2.2 Effects of two-sided online celebrity marketing

First, the price dispersion is given by $(p_1^* - p_2^*)$, where

$$p_1^* - p_2^* = \frac{A - B}{3},$$

Since $\frac{\partial(p_1 - p_2)}{\partial A} = \frac{1}{3} > 0$, and $\frac{\partial(p_1 - p_2)}{\partial B} = \frac{-1}{3} < 0$, we have the following result.

Proposition 20 The following describes the comparative statics on the price dispersion.
(i)As k_1 and l_1 *increase,* $p_1^* - p_2^*$ *will increase*
(ii)As σ_1 *increases,* $p_1^* - p_2^*$ *will decrease*
(i)As k_2 and l_2 *increase,* $p_1^* - p_2^*$ *will decrease*
(ii)As σ_2 *increases,* $p_1^* - p_2^*$ *will increase*

Proof. See the Appendix.

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In a small market, the influence on $p_1^* - p_2^*$ will be the same as in a large market. Intuitively, when both firms have a celebrity market, as k_1 increases, firm 1's celebrity influence becomes higher. The consumers have a higher confidence level of product 1, and they are willing to pay a higher price to buy product 1, so the price dispersion will increase. When the learning effect increases, the consumer can have more understanding of the product, so they pay a higher price to buy the product, and the price dispersion will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to pay for the product, and the price dispersion will decrease. The effect from k_2, l_2, σ_2 will be the opposite.

Second, the market share difference between is given by $(n_1^* - n_2^*)$, where

$$n_1^* - n_2^* = \frac{A - B}{3(2\tau - A - B)},$$

Since $\frac{\partial(n_1 - n_2)}{\partial A} = \frac{2\tau - 2B}{3(2\tau - A - B)^2} > 0$, and $\frac{\partial(n_1 - n_2)}{\partial B} = \frac{2A - 2\tau}{3(2\tau - A - B)^2} < 0$, we have the following result.

Proposition 21 The following describes the comparative statics on the market share difference.
(i)As k_1 and l_1 *increase,* $n_1^* - n_2^*$ *will increase.*
(ii)As σ_1 *increases,* $n_1^* - n_2^*$ *will decrease*
(iii)As k_2 and l_2 *increase,* $n_1^* - n_2^*$ *will decrease*
(iv)As σ_2 *increases,* $n_1^* - n_2^*$ *will increase,*

Proof. See the Appendix.

In a small market, the influence on $n_1^* - n_2^*$ will be the same as in a large market. Intuitively, when both firms have a celebrity market, as k_1 increases, firm 1's celebrity influence becomes higher. The consumer has a higher confidence level of product 1, they are more willing to buy product 1, so the market share difference will increase. When the learning

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effect increases, the consumer can have more understanding of the product, so they have a higher intention to buy the product, and the market share difference will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to buy the product, and the market share difference will decrease. The effect from k_2, l_2, σ_2 will be the opposite

Third, the market share difference between is given by $(\pi_1^* - \pi_2^*)$, where

$$\pi_1^* - \pi_2^* = \frac{A - B}{3},$$

Since $\frac{\partial(\pi_1 - \pi_2)}{\partial A} = \frac{1}{3} > 0$, and $\frac{\partial(\pi_1 - \pi_2)}{\partial B} = \frac{-1}{3} < 0$, we have the following result.

Proposition 22 The following describes the comparative statics on the profit dispersion.
(i)As k_1 and l_1 *increase,* $\pi_1^* - \pi_2^*$ *will increase*
(ii)As σ_1 *increases,* $\pi_1^* - \pi_2^*$ *will decrease*
(iii)As k_2 and l_2 *increase,* $\pi_1^* - \pi_2^*$ *will decrease*
(iv)As σ_2 *increases,* $\pi_1^* - \pi_2^*$ *will increas*

Proof. See the Appendix.

In the small market, the influence to $\pi_1^* - \pi_2^*$ will be the same as in the large market. Intuitively, from proposition 20 and proposition 21, we can find when $k_1$increases, the firm 1's price and market share will increase, it is because when the influence of the celebrity becomes higher, the consumer will have a higher confidence level in the product, so the consumer's willingness-to-pay and the purchase intention will increase, profit dispersion increasing. And when the learning effect increases, the consumer's willingness to pay and the purchase intention will increase, so it may increase the firm's profit, and profit dispersion increasing. While the uncertainty will reduce the consumer's willingness to pay and the purchase intention, so when the uncertainty increases, profit dispersion will decrease, and

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the effect from k_2, l_2, σ_2 will be the opposite.

Finally, we use Herfindahl-Hershman Index (HHI) to measure the market concentration.

$$\text{HHI} = n_1^2 + n_2^2,$$

Since $\frac{\partial HHI}{\partial A} = \frac{(2A-2B)(\tau-B)}{9(2\tau-A-B)^3}$ and $\frac{\partial HHI}{\partial B} = \frac{(-2A+2B)(\tau-B)}{9(2\tau-A-B)^3}$, we have the following result.

***Proposition* 23** *The following describes the comparative statics on the HHI.*
When $A > B$
(i) As k_1 *and* l_1 *increase,* HHI *will increase*
(ii) As σ_1 *increases,* HHI *will decrease,*
(iii) As k_2 *and* l_2 *increase,* HHI *will decrease*
(iv) As σ_2 *increases,* HHI *will increase,*

Proof. See the Appendix.

Intuitively, in the small market, when 2A>2B, it means that product 1's value is higher than product2's value. As k_1 increases, the firm1 will have higher competitiveness, the market concentration will become higher. When firm1's learning effect increases, firm1's competitiveness will increase, and the market concentration will become higher. When the uncertainty increases, the market concentration will become lower.

4.3 Entrant with Online celebrity marketing vs Two Traditional Incumbents

In this section, we consider that only firm 1 uses online celebrity marketing, while firm 2 and firm 3 do not. As before, we assume that the two traditional firms are located at the two extremes of the street. Now, we assume that the new entrant is located at the middle. The location here should be interpreted as the loyalty toward a firm. Consumers in the middle are not so loyal to either firm, so are prone for the new product. The difference setting as before is that in this case, firm 1 is a new entrant, so it has higher uncertainty than the other

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firms.

$$\sigma_1 > \sigma_2 = \sigma_3$$

Specifically, we assume that

$$g_1 = k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1^e,$$

$$g_2 = k_2(v - \sigma_2),$$

$$g_3 = k_3(v - \sigma_3).$$

Hence, the indifferent conditions are:

$$k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1 - \tau n_1 - p_1 = k_2(v - \sigma_2) - \tau n_2 - p_2 = k_3(v - \sigma_3) - \tau n_3 - p_3.$$

To simplify the analysis, we assume that the two traditional firms behave similarly, so they have the same demand: $n_2 = n_3 = \frac{1-n_1}{2}$, and $k_3(v - \sigma_3) = k_2(v - \sigma_2)$. Hence the indifferent condition becomes:

$$k_1\left(v - \frac{\sigma_1}{l_1}\right)n_1 - \tau n_1 - p_1 = k_2(v - \sigma_2) - \tau\left(\frac{1 - n_1}{2}\right) - p_2.$$

Solve the equation we can find

$$n_1 = \frac{2p_2 - 2p_1 + \tau - 2k_2(v - \sigma_2)}{3\tau - 2k_1\left(v - \frac{\sigma_1}{l_1}\right)},$$

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$$n_2 = n_3 = \frac{(1 - n_1)}{2}.$$

Let A= $k_1(v - \frac{\sigma_1}{l_1})$ and B=$k_2(v - \sigma_2)$, so

$$n_1 = \frac{2p_2 - 2p_1 - 2B + \tau}{3\tau - 2A},$$

$$n_2 = n_3 = \frac{(1 - n_1)}{2} = \frac{-p_2 + p_1 - A + B + \tau}{(3\tau - 2A)}.$$

4.3.1 Market equilibrium with only entrant using online celebrity marketing

Firms compete in price, so the two firms choose their price p_i simultaneously to maximize their profits, where

$$\pi_1 = (p_1 - c_1) * \frac{2p_2 - 2p_1 - 2B + \tau}{3\tau - 2A},$$

$$\pi_2 = \pi_3 = (p_2 - c_2) * \frac{-p_2 + p_1 - A + B + \tau}{(3\tau - 2A)}.$$

To simplify the analysis, we assume $c_1 = c_2 = c$. Hence, the first order conditions of maximization are:

$$\frac{2p_2 - 2p_1 - 2B + \tau - (p_1 - c)}{3\tau - 2A} = 0,$$

$$\frac{-p_2 + p_1 - A + B + \tau - (p_2 - c)}{(3\tau - 2A)} = 0.$$

The second order conditions for profit maximization are:

$$\frac{\partial^2(\pi_1)}{\partial(p_1)^2} = \frac{-3}{3\tau-2A} < 0 \text{ and } \frac{\partial^2(\pi_1)}{\partial(n_2)^2} = \frac{-2}{3\tau-2A} < 0,$$

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The equilibrium prices are

$$p_1 = \frac{2\tau - A - B + 2c}{2} \text{ and } p_2 = p_3 = \frac{4\tau - 3A + B + 4c}{4}.$$

The equilibrium market shares are

$$n_1 = \frac{2\tau - A - B}{2(3\tau - 2A)} \quad \text{and} \quad n_2 = n_3 = \frac{1}{2}(1 - n_1) = \frac{4\tau - 3A + B}{4(3\tau - 2A)}.$$

Finally the equilibrium profits are

$$\pi_1 = \frac{(2\tau - A - B)^2}{4(3\tau - 2A)} \text{ and } \pi_2 = \pi_3 = \frac{(4\tau - 3A + B)^2}{16(3\tau - 2A)}.$$

4.3.2 Effects of online celebrity marketing entrant on traditional market

First, the price dispersion is given by $(p_1^* - p_2^*)$, where

$$p_1^* - p_2^* = \frac{A - 3B}{6}.$$

Since $\frac{\partial(p_1 - p_2)}{\partial A} = \frac{1}{6} > 0$, and $\frac{\partial(p_1 - p_2)}{\partial B} = \frac{-1}{2} < 0$, we have the following result.

Proposition 24 The following describes the comparative statics on the price dispersion.
(i)As k_1 and l_1 *increase,* $p_1^* - p_2^*$ *will increase*
(ii)As σ_1 *increases,* $p_1^* - p_2^*$ *will decrease*
(iii)As k_2 *increase,* $p_1^* - p_2^*$ *will decrease*
(iv)As σ_2 *increases,* $p_1^* - p_2^*$ *will increase*

Proof. See the Appendix.

In the small market, the influence to $p_1^* - p_2^*$ will be the same as in the large market.

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Intuitively, when a firm that uses online celebrity marketing enters the traditional market, it can use celebrity influence to increase the consumer's willingness to pay. If the celebrity influence becomes higher, people have higher confidence in the product, so they are more willing to pay the higher price to buy it, and the price dispersion will increase. When the learning effect increases, the consumer can have more understanding of the product, so they will pay a higher price, and the price dispersion will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to pay for the product, and the price dispersion will decrease

Second, the market share difference between is given by $(n_1^* - n_2^*)$, where

$$n_1^* - n_2^* = \frac{A - 3B}{4(3\tau - 2A)},$$

Since $\frac{\partial(n_1 - n_2)}{\partial A} = \frac{3\tau - 6B}{4(3\tau - 2A)^2} > 0$, $\frac{\partial(n_1 - n_2)}{\partial B} = \frac{-3}{4(3\tau - 2A)} < 0$we have the following result.

Proposition 25 The following describes the comparative statics on the market share difference.
(i)As k_1, l_1 increase $n_1^* - n_2^*$ will increase
(ii)As σ_2 increases, $n_1^* - n_2^*$ will decrease
(i)As k_2 increase, $n_1^* - n_2^*$ will decrease
(ii)As σ_2 increases, $n_1^* - n_2^*$ will increase

Proof. See the Appendix.

In the small market, the influence to $n_1^* - n_2^*$ will be the same as in a large market. Intuitively, when a firm that uses online celebrity marketing enters the traditional market, it can use celebrity influence to increase the consumer's purchase intention. If the celebrity influence becomes higher, people have higher confidence in the product. When they are more willing

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to buy it. the market share difference will increase. When the learning effect increases, the consumer can have more understanding of the product, so they have a higher intention to buy the product, and the market share difference will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce their willingness to buy the product, and the market share difference will decrease.

Third, the market share difference between is given by $(\pi_1^* - \pi_2^*)$, where

$$\pi_1^* - \pi_2^* = p_1 n_1 - p_2 n_2 - c(n_1 - n_2),$$

Since $\frac{\partial(\pi_1 - \pi_2)}{\partial A} = \left[\frac{-1}{2} n_1 + \frac{\tau - 2B}{2(3\tau - 2A)^2} p_1 + \frac{3}{4} n_2 + \frac{\tau - 2B}{2(3\tau - 2A)^2} p_2\right] > 0$

, and $\frac{\partial(\pi_1 - \pi_2)}{\partial B} = \left[\frac{-1}{2} n_1 + \frac{-1}{2(3\tau - 2A)} p_1 - \frac{1}{4} n_2 - \frac{1}{4(3\tau - 2A)} p_2\right] < 0$, we have the following result.

Proposition 26 The following describes the comparative statics on the profit dispersion.
(i)As k_1 and l_1 *increase,* $\pi_1^* - \pi_2^*$ *will increase,*
(ii)As σ_1 *increases,* $\pi_1^* - \pi_2^*$ *will decrease*
(iii)As k_2 *increase,* $\pi_1^* - \pi_2^*$ *will decrease*
(iv)As σ_2 *increases,* $\pi_1^* - \pi_2^*$ *will increase*

Proof. See the Appendix.

In the small market, the influence to $\pi_1^* - \pi_2^*$ will be the same as in the large market. Intuitively, from proposition 24 and proposition 25 we can find when a firm that uses online celebrity marketing enters the traditional market, it can use celebrity influence to increase the consumer's willingness to pay and the purchase intention, so the profit dispersion will increase. When the learning effect increases, the consumer can have more understanding of the product, so they are more willing to buy the product and pay a higher price, and the profit dispersion will increase. When the uncertainty increases, the consumer may have some misunderstanding of the products, so it may reduce t the consumer's willingness to pay and

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the purchase intention, and the profit dispersion will decrease.

Finally, we use Herfindahl-Hershman Index (HHI) to measure the market concentration.

$$\text{HHI} = n_1^2 + n_2^2 + n_3^2,$$

Since $\frac{\partial HHI}{\partial A} = 4n_1 \frac{\tau - 2B}{2(3\tau - 2A)} - 4n_2 \frac{\tau - 2B}{2(3\tau - 2A)}$ and $\frac{\partial HHI}{\partial B} = 4n_1 \frac{-1}{2(3\tau - 2A)} + 4n_2 \frac{1}{2(3\tau - 2A)}$, we have the following result.

Proposition 27 *The following describes the comparative statics on the HHI.*
When $n_1 > n_2$
(i) As k_1 *and* l_1 *increase,* HHI *will increase*
(ii) As σ_1 *increases,* HHI *will decrease,*
(iii) As k_2 *and* l_2 *increase,* HHI *will decrease*
(iv) As σ_2 *increases,* HHI *will increase,*

Proof. See the Appendix.

Intuitively, in the small market, when $n_1 > n_2$, as k_1 increases, the firm1 will have higher competitiveness, and the market concentration will become higher. When firm1's learning effect increases, firm1's competitiveness will increase, and the market concentration will become higher. When the uncertainty increases, the market concentration will become lower.

5. Conclusion

In our paper, we use two models to analyze the influence of online celebrity marketing. When in a large market, we assume three cases. The first one is in a duopoly market, we consider that only one firm uses online celebrity marketing. We can find when firm1 's competitor doesn't adopt online celebrity marketing, Firm 1's will be profitable for three cases: (i) both k_1 and l_1 are low; (ii) k_1 is high but l_1 is low; (iii) k_1 is low but l_1

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is high. And another finding is that when firm1 increases k_1 and l1 or decreases σ_1, it can increase the market share, price, and profit dispersion. In the second case, we assume both firms adopt online celebrity marketing, we find that when A>B, firm1 's best strategy is adopting online celebrity marketing, so it has a profit incentive to increase k_1 and l_1 or decrease σ_1. In the third case, we consider a new entrant with online celebrity marketing, competing with two traditional incumbent firms, and we find the firm which has online celebrity marketing has an incentive to increase k_1 and l_1 or decrease σ_1. But the new entrant has higher uncertainty, which may decrease its product value, so the new entrant firm 1 should make more effort to increase the profit dispersion.

When in a small market, we also analyze those three cases. The first one is in a duopoly market, we consider that only one firm uses online celebrity marketing. We can find when firm1 's competitor doesn't adopt online celebrity marketing, online celebrity marketing may decrease firm 1's equilibrium price. Another finding is the same as in the large market, when firm1 increases k_1 and l_1 or decreases σ_1, it can increase the market share, price, and profit dispersion. In the second case, we assume both firms adopt online celebrity marketing, we find that when A>B firm1 's best strategy is adopting online celebrity marketing, it has a profit incentive to increase k_1 and l_1 or decrease σ_1. In the third case, we consider a new entrant with online celebrity marketing, competing with two traditional incumbent firms, and we find when A>3B firm1 's profit will be higher than its competitor, it has a profit incentive to increase k_1 and l_1 or decreaseσ_1. But the new entrant has higher uncertainty, which may decrease its product value, so the new entrant firm 1 should make more effort to increase the profit dispersion.

The biggest difference between the two models is that in the small market we set the restriction $n_1 + n_2 = 1$ and use price competition to circulate the equilibrium. We find the higher network effect will decrease the marketing firm's price, but it will decrease the opponent's price more. As a result, unless this network effect is sufficiently high, the firm

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adopting online celebrity marketing will receive a smaller profit. In the large market, we assume that prices will adjust to the levels where the marginal consumer is "indifferent between buying the two products" and "indifferent between buying and not buying. It uses quantity competition to circulate the equilibrium, and we can find that increasing k_1 and l_1 or decreasing σ_1 can increase profit in every case.

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Appendix

The definition of the notations:

Notation	Definition
$\boldsymbol{u_i}$	*utility function of buying product i*
$\boldsymbol{g}$	*expected product value*
$\boldsymbol{k}$	*celebrity effect*
$\boldsymbol{v}$	*common mean value*
$\boldsymbol{\sigma}$	*uncertainty*
$\boldsymbol{l}$	*learning effect*
$\boldsymbol{n}$	*market share*

Proof of Proposition 4.

$p_1 = \frac{(1-A)-(1-A)B-c(1-3(1-A))}{4(1-A)-1}$, $p_2 = \frac{2(1-A)+2(1-A)B-1+2(1-A)c}{4(1-A)-1}$

Since $\frac{\partial(p_1-p_2)}{\partial A} = \frac{3(1-B-c)}{[4(1-A)-1]^2} > 0$, and $\frac{\partial(p_1-p_2)}{\partial B} = \frac{-3(1-A)}{4(1-A)-1} < 0$, we have the following result.

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$$\frac{\partial(p_1-p_2)}{\partial k_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{3(1-B-c)}{[4(1-A)-1]^2}*\left(v-\frac{\sigma_1}{l_1}\right)=\frac{3\left(v-\frac{\sigma_1}{l_1}\right)(1-k_2(v-\sigma_2)-c)}{\left[3-4k_1\left(v-\frac{\sigma_1}{l_1}\right)\right]^2}$$

$$>0$$

$$\frac{\partial(p_1-p_2)}{\partial k_2}=\frac{\partial(p_1-p_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\frac{-3(1-A)}{4(1-A)-1}*(v-\sigma_2)=\frac{3(v-\sigma_2)\left(k_1\left(v-\frac{\sigma_1}{l_1}\right)-1\right)}{3-4k_1\left(v-\frac{\sigma_1}{l_1}\right)}<0$$

$$\frac{\partial(p_1-p_2)}{\partial l_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{3(1-B-c)}{[4(1-A)-1]^2}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{3(1-B-c)}{4(1-A)-1}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_2}=\frac{\partial(p_1-p_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{-3(1-A)}{[4(1-A)-1]^2}*-k_2>0$$

Proof of Proposition 5.

$$\pi_1=\frac{(1-B-c)(2(1-A)+2(1-A)B-1+c(2A-1))}{[4A-1]^2}$$

$$\pi_2=\frac{[2A+2AB-1+c(2A-1)]^2}{[4A-1]^2}$$

$$\pi_1-\pi_2$$

$$=\frac{[(2(1-A)+2(1-A)B-1+c(2A-1)][2-2(1-A)-B-2(1-A)B-2c(1-A)]}{[4(1-A)-1]^2}$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\frac{\partial[p_1n_1-p_2n_2-c(n_1-n_2)]}{\partial A}$$

$$=\frac{(1-B-c)}{[4(1-A)-1]^2}n_1+\frac{4(1-B-c)}{[4(1-A)-1]^2}p_1+\frac{2(1-B-c)}{[4(1-A)-1]^2}n_2$$

$$+\frac{2(1-B-c)}{[4(1-A)-1]^2}p_2-c\frac{6[1-B-c]}{[4(1-A)-1]^2}$$

Let $\frac{(1-B-c)}{[4(1-A)-1]^2}=\mathrm{x}$ we can write

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\mathrm{x}n_1+4xp_1+2xn_2+2xp_2-6xc>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial B}=\frac{\partial[p_1n_1-p_2n_2-c(n_1-n_2)]}{B}$$

$$=\frac{-(1-A)}{4(1-A)-1}n_1-\frac{1}{4(1-A)-1}p_1-\frac{2(1-A)}{4(1-A)-1}n_2-\frac{2(1-A)}{4(1-A)-1}p_2-\mathrm{c}$$

$$*\frac{-1-2(1-A)}{4(1-A)-1}$$

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Let $\frac{1}{4(1-A)-1} = \mathrm{y}$ we can write

$$\frac{\partial(\pi_1-\pi_2)}{\partial B} = -(1-\mathrm{A})yn_1 - yp_1 - 2(1-A)yn_2 - 2(1-A)yp_2 + cy(-3+2A) < 0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_1} = \frac{\partial(\pi_1-\pi_2)}{\partial A} * \frac{\partial A}{\partial k_1} = (\mathrm{x}n_1 + 4xp_1 + 2xn_2 + 2xp_2 - 6xc) * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_2} = \frac{\partial(\pi_1-\pi_2)}{\partial B} * \frac{\partial B}{\partial k_2}$$
$$= -(1-\mathrm{A})yn_1 - yp_1 - 2(1-A)yn_2 - 2(1-A)yp_2 + cy(-3+2A) * (v-\sigma_2) < 0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial l_1} = \frac{\partial(\pi_1-\pi_2)}{\partial A} * \frac{\partial A}{\partial l_1} = (\mathrm{x}n_1 + 4xp_1 + 2xn_2 + 2xp_2 - 6xc) * \frac{k_1\sigma_1}{l_1^2} > 0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_1} = \frac{\partial(\pi_1-\pi_2)}{\partial A} * \frac{\partial A}{\partial \sigma_1} = (\mathrm{x}n_1 + 4xp_1 + 2xn_2 + 2xp_2 - 6xc) * -\frac{k_1}{l_1} < 0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_2} = \frac{\partial(\pi_1-\pi_2)}{\partial B} * \frac{\partial B}{\partial \sigma_2}$$
$$= -(1-\mathrm{A})yn_1 - yp_1 - 2(1-A)yn_2 - 2(1-A)yp_2 + cy(-3+2A) * -k_2 > 0$$

Proof of Proposition 7.

$$\frac{\partial(n_1-n_2)}{\partial A} = \frac{(1-c)(8(1-B)^2-2)}{[4(1-A)(1-B)-1]^2} > 0$$

$$\frac{\partial(n_1-n_2)}{\partial B} = \frac{(1-c)(2-8(1-A)^2)}{[4(1-A)(1-B)-1]^2} < 0$$

$$\frac{\partial(n_1-n_2)}{\partial k_1} = \frac{\partial n_1 - n_2)}{\partial A} * \frac{\partial A}{\partial k_1} = \frac{(1-c)(8(1-B)^2-2)}{[4(1-A)(1-B)-1]^2} * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(n_1-n_2)}{\partial k_2} = \frac{\partial(n_1-n_2)}{\partial B} * \frac{\partial B}{\partial k_1} = \frac{(1-c)(2-8(1-A)^2)}{[4(1-A)(1-B)-1]^2} * \left(v - \frac{\sigma_2}{l_2}\right) < 0$$

$$\frac{\partial(n_1-n_2)}{\partial l_1} = \frac{\partial n_1 - n_2)}{\partial A} * \frac{\partial A}{\partial l_1} = \frac{(1-c)(8(1-B)^2-2)}{[4(1-A)(1-B)-1]^2} * \frac{k_1\sigma_1}{l_1^2} > 0$$

$$\frac{\partial(n_1-n_2)}{\partial l_2} = \frac{\partial(n_1-n_2)}{\partial B} * \frac{\partial B}{\partial l_1} = \frac{(1-c)(2-8(1-A)^2)}{[4(1-A)(1-B)-1]^2} * \frac{k_2\sigma_2}{l_2^2} < 0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_1} = \frac{\partial(n_1-n_2)}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \frac{(1-c)(8(1-B)^2-2)}{[4(1-A)(1-B)-1]^2} * -\frac{k_1}{l_1} < 0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_2} = \frac{\partial(n_1-n_2)}{\partial B} * \frac{\partial B}{\partial \sigma_2} = \frac{(1-c)(2-8(1-A)^2)}{[4(1-A)(1-B)-1]^2} * -\frac{k_2}{l_2} > 0$$

Proof of Proposition 8.

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$$p_1 = \frac{2(1-A)(1-B) - (1-A) + (1-A)c + 2(1-A)(1-B)c - c}{4(1-A)(1-B) - 1}$$

$$p_2 = \frac{2(1-A)(1-B) - (1-B) + (1-B)c + 2(1-A)(1-B)c - c}{4(1-A)(1-B) - 1}$$

$$p_1 - p_2 = \frac{A - B - Ac + Bc}{4(1-A)(1-B) - 1}$$

$$\frac{\partial(p_1 - p_2)}{\partial A} = \frac{(1-c)(4(1-B)^2 - 1)}{(4(1-A)(1-B)-1)^2} > 0$$

$$\frac{\partial(p_1 - p_2)}{\partial B} = \frac{(1-c)(1 - 4(1-A)^2)}{(4(1-A)(1-B)-1)^2} < 0$$

$$\frac{\partial(p_1 - p_2)}{\partial k_1} = \frac{\partial(p_1 - p_2)}{\partial A} * \frac{\partial A}{\partial k_1} = \frac{(1-c)(4(1-B)^2 - 1)}{(4(1-A)(1-B)-1)^2} * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(p_1 - p_2)}{\partial k_2} = \frac{\partial(p_1 - p_2)}{\partial B} * \frac{\partial B}{\partial k_1} = \frac{(1-c)(1 - 4(1-A)^2)}{(4(1-A)(1-B)-1)^2} * \left(v - \frac{\sigma_2}{l_2}\right) < 0$$

$$\frac{\partial(p_1 - p_2)}{\partial l_1} = \frac{\partial(p_1 - p_2)}{\partial A} * \frac{\partial A}{\partial l_1} = \frac{(1-c)(4(1-B)^2 - 1)}{(4(1-A)(1-B)-1)^2} * \frac{k_1\sigma_1}{l_1^2} > 0$$

$$\frac{\partial(p_1 - p_2)}{\partial l_2} = \frac{\partial(p_1 - p_2)}{\partial B} * \frac{\partial B}{\partial l_1} = \frac{(1-c)(1 - 4(1-A)^2)}{(4(1-A)(1-B)-1)^2} * \frac{k_2\sigma_2}{l_2^2} < 0$$

$$\frac{\partial(p_1 - p_2)}{\partial \sigma_1} = \frac{\partial(p_1 - p_2)}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \frac{(1-c)(4(1-B)^2 - 1)}{(4(1-A)(1-B)-1)^2} * -\frac{k_1}{l_1} < 0$$

$$\frac{\partial(p_1 - p_2)}{\partial \sigma_2} = \frac{\partial(p_1 - p_2)}{\partial B} * \frac{\partial B}{\partial \sigma_2} = \frac{(1-c)(1 - 4(1-A)^2)}{(4(1-A)(1-B)-1)^2} * -\frac{k_2}{l_2} > 0$$

Proof of Proposition 9.

$$\pi_1 = \frac{(1-A)(1-2B)^2(1-c)^2}{(4(1-A)(1-B)-1)^2}$$

$$\pi_2 = \frac{(1-B)(1-2A)^2(1-c)^2}{(4(1-A)(1-B)-1)^2}$$

$$\pi_1 - \pi_2 = \frac{(1-c)^2[(1-A)(1-2B)^2 - (1-B)(1-2A)^2]}{(4(1-A)(1-B)-1)^2}$$

$$\frac{\partial(\pi_1 - \pi_2)}{\partial A} = \frac{(4(1-B)^2 - 1)(1-c)^2}{(4(1-A)(1-B)-1)^2} > 0$$

$$\frac{\partial(\pi_1 - \pi_2)}{\partial B} = \frac{(1 - 4(1-A)^2)(1-c)^2}{(4(1-A)(1-B)-1)^2} < 0$$

$$\frac{\partial(\pi_1 - \pi_2)}{\partial k_1} = \frac{\partial(\pi_1 - \pi_2)}{\partial A} * \frac{\partial A}{\partial k_1} = \frac{(4(1-B)^2 - 1)(1-c)^2}{(4(1-A)(1-B)-1)^2} * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(\pi_1 - \pi_2)}{\partial k_2} = \frac{\partial(\pi_1 - \pi_2)}{\partial B} * \frac{\partial B}{\partial k_2} = \frac{(1 - 4(1-A)^2)(1-c)^2}{(4(1-A)(1-B)-1)^2} * \left(v - \frac{\sigma_2}{l_2}\right) < 0$$

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$$\frac{\partial(\pi_1-\pi_2)}{\partial l_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{(4(1-B)^2-1)(1-c)^2}{(4(1-A)(1-B)-1)^2}*\frac{k_1\sigma_1}{l_1^2}>0$$
$$\frac{\partial(\pi_1-\pi_2)}{\partial l_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial l_2}=\frac{(1-4(1-A)^2)(1-c)^2}{(4(1-A)(1-B)-1)^2}*\frac{k_2\sigma_2}{l_2^2}<0$$
$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{(4(1-B)^2-1)(1-c)^2}{(4(1-A)(1-B)-1)^2}*-\frac{k_1}{l_1}<0$$
$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{(1-4(1-A)^2)(1-c)^2}{(4(1-A)(1-B)-1)^2}*-\frac{k_2}{l_2}>0$$

Proof of Proposition 10.

$$\text{HHI}=n_1^2+n_2^2$$

$$=\left(\frac{(1-2B)(1-c)}{4(1-A)(1-B)-1}\right)^2+\left(\frac{(1-2A)(1-c)}{4(1-A)(1-B)-1}\right)^2$$
$$=\frac{2(c-1)^2[2(1-A)^2+2(1-B)^2-2(1-A)-2(1-B)+1]}{(4(1-A)(1-B)-1)^2}$$
$$\frac{\partial HHI}{\partial A}=\frac{\partial n_1^2}{\partial A}+\frac{\partial n_2^2}{\partial A}=\frac{8(1-B)(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{-4(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_2$$
$$\frac{\partial HHI}{\partial B}=\frac{\partial n_1^2}{\partial B}+\frac{\partial n_2^2}{\partial B}=\frac{-4(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{8(1-A)(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_2$$
$$\frac{\partial HHI}{\partial k_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial k_1}$$
$$=\left[\frac{8(1-B)(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{-4(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_2\right]*\left(v-\frac{\sigma_1}{l_1}\right)$$
$$\frac{\partial HHI}{\partial k_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial k_2}$$
$$=\left[\frac{-4(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{8(1-A)(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_2\right]*\left(v-\frac{\sigma_2}{l_2}\right)$$

$$\frac{\partial HHI}{\partial l_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial l_1}=\left[\frac{8(1-B)(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{-4(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_2\right]*\frac{k_1\sigma_1}{l_1^2}$$
$$\frac{\partial HHI}{\partial l_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial l_2}=\left[\frac{-4(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{8(1-A)(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_2\right]*\frac{k_2\sigma_2}{l_2^2}$$

$$\frac{\partial HHI}{\partial \sigma_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\left[\frac{8(1-B)(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{-4(1-c)(1-2B)}{(4(1-A)(1-B)-1)^2}*n_2\right]*-\frac{k_1}{l_1}$$
$$\frac{\partial HHI}{\partial \sigma_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial \sigma_1}=\left[\frac{-4(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_1+\frac{8(1-A)(1-c)(1-2A)}{(4(1-A)(1-B)-1)^2}*n_2\right]*-\frac{k_2}{l_2}$$

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Proof of Proposition 11.

$$\frac{\partial(n_1-n_2)}{\partial A}=\frac{2-4B-2c}{(2-3A)^2}>0$$

$$\frac{\partial(n_1-n_2)}{\partial B}=\frac{A-2}{2-3A}<0$$

$$\frac{\partial(n_1-n_2)}{\partial k_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{2-4B-2c}{(2-3A)^2}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(n_1-n_2)}{\partial k_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial k_1}=\frac{A-2}{2-3A}*(v-\sigma_2)<0$$

$$\frac{\partial(n_1-n_2)}{\partial l_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{2-4B-2c}{(2-3A)^2}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{2-4B-2c}{(2-3A)^2}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{A-2}{2-3A}*-k_2>0$$

Proof of Proposition 12.

$$\frac{\partial(p_1-p_2)}{\partial A}=\frac{1-2B-c}{(2-3A)^2}>0$$

$$\frac{\partial(p_1-p_2)}{\partial B}=\frac{-2(1-A)}{2-3A}<0$$

$$\frac{\partial(p_1-p_2)}{\partial k_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{1-2B-c}{(2-3A)^2}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(p_1-p_2)}{\partial k_2}=\frac{\partial(p_1-p_2)}{\partial B}*\frac{\partial B}{\partial k_1}=\frac{-2(1-A)}{2-3A}*(v-\sigma_2)<0$$

$$\frac{\partial(p_1-p_2)}{\partial l_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{1-2B-c}{(2-3A)^2}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{1-2B-c}{(2-3A)^2}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_2}=-1*\frac{\partial B}{\partial \sigma_2}=\frac{-2(1-A)}{2-3A}*-k_2>0$$

Proof of Proposition 13.

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\frac{\partial[p_1n_1-p_2n_2-c(n_1-n_2)]}{\partial A}$$

$$=\frac{1-2B-c}{2(2-3A)^2}*n_1+\frac{3(1-2B-c)}{2(2-3A)^2}*p_1+\frac{1-2B-c}{2(2-3A^2}*n_2+\frac{1-2B-c}{2(2-3A)^2}*p_2-\mathrm{c}$$
$$*\frac{4(1-2B-c)}{2(2-3A)^2}$$

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Let $\frac{1-2B-c}{2(3A-1)^2}=$ x, we know that x>0 we can write

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\mathrm{x}n_1+3xp_1+\mathrm{x}n_2+xp_2-c4x>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial B}=\frac{\partial[p_1n_1-p_2n_2-c(n_1-n_2)]}{\partial B}$$

$$=\frac{-(1-A)}{2-3A}*n_1+\frac{-1}{2-3A}*p_1-\frac{1}{2-3A}*n_2-\frac{(1-A)}{2-3A}*p_2+\mathrm{c}*\frac{2-A}{2-3A}$$

Let $\frac{1}{2-3A}=$ y, we know that y>1 we can write

$$\frac{\partial(\pi_1-\pi_2)}{\partial B}=-(1-\mathrm{A})\mathrm{y}n_1-\mathrm{y}p_1-yn_2-(1-A)yp_2+cy(2-A)<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial k_1}=[\mathrm{x}n_1+3xp_1+\mathrm{x}n_2+xp_2-c4x]*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial k_2}$$

$$=[-(1-\mathrm{A})\mathrm{y}n_1-yp_1-yn_2-(1-A)yp_2+cy(2-A)]*(v-\sigma_2)<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial l_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial l_1}=[\mathrm{x}n_1+3xp_1+\mathrm{x}n_2+xp_2-c4x]*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=[\mathrm{x}n_1+3xp_1+\mathrm{x}n_2+xp_2-c4x]*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=[-(1-\mathrm{A})\mathrm{y}n_1-\mathrm{y}p_1-yn_2-(1-A)yp_2+cy(2-A)]*-k_2$$

Proof of Proposition 14.

$$\frac{\partial HHI}{\partial A}=\frac{\partial n_1^2}{\partial A}+2\frac{\partial n_2^2}{\partial A}=\frac{6(1-2B-c)}{2(2-3A)^2}*n_1+\frac{-4(1-2B-c)}{2(2-3A)^2}*n_2$$

$$\frac{\partial HHI}{\partial B}=\frac{\partial n_1^2}{\partial B}+2\frac{\partial n_2^2}{\partial B}=\frac{-2}{2-3A}*n_1+\frac{4A}{2-3A}*n_2$$

$$\frac{\partial HHI}{\partial k_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial k_1}=\left[\frac{6(1-2B-c)}{2(2-3A)^2}*n_1+\frac{-4(1-2B-c)}{2(2-3A)^2}*n_2\right]*-(v-\frac{\sigma_1}{l_1})$$

$$\frac{\partial HHI}{\partial k_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial k_2}=\left[\frac{-2}{2-3A}*n_1+\frac{4A}{2-3A}*n_2\right]*(v-\sigma_2)$$

$$\frac{\partial HHI}{\partial l_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial l_1}=\left[\frac{6(1-2B-c)}{2(2-3A)^2}*n_1+\frac{-4(1-2B-c)}{2(2-3A)^2}*n_2\right]*\frac{k_1\sigma_1}{l_1^2}$$

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$$\frac{\partial HHI}{\partial \sigma_1} = \frac{\partial HHI}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \left[\frac{6(1-2B-c)}{2(2-3A)^2} * n_1 + \frac{-4(1-2B-c)}{2(2-3A)^2} * n_2\right] * -\frac{k_1}{l_1}$$

$$\frac{\partial HHI}{\partial \sigma_2} = \frac{\partial HHI}{\partial B} * \frac{\partial B}{\partial \sigma_1} = \left[\frac{-2}{2-3A} * n_1 + \frac{4A}{2-3A} * n_2\right] * -k_2$$

Proof of Proposition 15

$X=3\tau + A - B$

$n_1^* = \frac{X-2A}{6\tau-3A}$ and $n_1^0 = \frac{X}{6\tau}$.

$$\frac{X-2A}{6\tau-3A} > \frac{X}{6\tau}$$

$$X - 2A > \frac{X}{6\tau}(6\tau - 3A) = X - \frac{X3A}{6\tau}$$

$$-2A > -\frac{X3A}{6\tau}$$

$$4\tau < X = 3\tau + A - B$$

$$\tau < A - B$$

Proof of Proposition 16

$$\frac{\partial(p_1-p_2)}{\partial A} = \frac{1}{3} > 0$$

$$\frac{\partial(p_1-p_2)}{\partial B} = \frac{-2}{3} < 0$$

$$\frac{\partial(p_1-p_2)}{\partial k_1} = \frac{\partial(p_1-p_2)}{\partial A} * \frac{\partial A}{\partial k_1} = \frac{1}{3} * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(p_1-p_2)}{\partial k_2} = \frac{\partial(p_1-p_2)}{\partial B} * \frac{\partial B}{\partial k_2} = \frac{-2}{3} * (v - \sigma_2) < 0$$

$$\frac{\partial(p_1-p_2)}{\partial l_1} = \frac{\partial(p_1-p_2)}{\partial A} * \frac{\partial A}{\partial l_1} = \frac{1}{3} * \frac{k_1\sigma_1}{l_1^2} > 0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_1} = \frac{\partial(p_1-p_2)}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \frac{1}{3} * -\frac{k_1}{l_1} < 0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_2} = \frac{\partial(p_1-p_2)}{\partial B} * \frac{\partial B}{\partial \sigma_2} = \frac{-2}{3} * -k_2 > 0$$

Proof of Proposition 17.

$$\frac{\partial(n_1-n_2)}{\partial A} = \frac{2\tau - 2B}{3(2\tau - A)^2} > 0$$

$$\frac{\partial(n_1-n_2)}{\partial B} = \frac{-2}{3(2\tau - A)} < 0$$

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$$\frac{\partial(n_1-n_2)}{\partial k_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{2\tau-2B}{3(2\tau-A)^2}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(n_1-n_2)}{\partial k_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\frac{-2}{3(2\tau-A)}*(v-\sigma_2)<0$$

$$\frac{\partial(n_1-n_2)}{\partial l_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{2\tau-2B}{3(2\tau-A)^2}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{2\tau-2B}{3(2\tau-A)^2}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{-2}{3(2\tau-A)}*-k_2>0$$

Proof of Proposition 18

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\frac{1}{3}>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial B}=\frac{-2}{3}<0$$

$$\frac{\partial\left(\pi_1-\pi_{2_2}\right)}{\partial k_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{1}{3}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\frac{-2}{3}*(v-\sigma_2)<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial l_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{1}{3}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{1}{3}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{-2}{3}*-k_2>0$$

Proof of Proposition 19

$$\frac{\partial HHI}{\partial A}=\frac{(2A-4B)(\tau-B)}{9(2\tau-A)^3}$$

$$\frac{\partial HHI}{\partial B}=\frac{(4B-2A)}{9(2\tau-A)^2}$$

$$\frac{\partial HHI}{\partial k_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial k_1}=\left[\frac{(2A-4B)(\tau-B)}{9(2\tau-A)^3}\right]*(v-\frac{\sigma_1}{l_1})$$

$$\frac{\partial HHI}{\partial k_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial k_2}=\left[\frac{(4B-2A)}{9(2\tau-A)^2}\right]*(v-\sigma_2)$$

$$\frac{\partial HHI}{\partial l_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial l_1}=\left[\frac{(2A-4B)(\tau-B)}{9(2\tau-A)^3}\right]*\frac{k_1\sigma_1}{l_1^2}$$

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$$\frac{\partial HHI}{\partial \sigma_1} = \frac{\partial HHI}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \left[\frac{(2A-4B)(\tau-B)}{9(2\tau-A)^3}\right] * -\frac{k_1}{l_1}$$

$$\frac{\partial HHI}{\partial \sigma_2} = \frac{\partial HHI}{\partial B} * \frac{\partial B}{\partial \sigma_1} = \left[\frac{(4B-2A)}{9(2\tau-A)^2}\right] * -k_2$$

Proof of Proposition 20

$$\frac{\partial(p_1-p_2)}{\partial A} = \frac{1}{3} > 0$$

$$\frac{\partial(p_1-p_2)}{\partial B} = \frac{-1}{3} < 0$$

$$\frac{\partial(p_1-p_2)}{\partial k_1} = \frac{\partial(p_1-p_2)}{\partial A} * \frac{\partial A}{\partial k_1} = \frac{1}{3} * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(p_1-p_2)}{\partial k_2} = \frac{\partial(p_1-p_2)}{\partial B} * \frac{\partial B}{\partial k_2} = \frac{-1}{3} * \left(v - \frac{\sigma_2}{l_2}\right) < 0$$

$$\frac{\partial(p_1-p_2)}{\partial l_1} = \frac{\partial(p_1-p_2)}{\partial A} * \frac{\partial A}{\partial l_1} = \frac{1}{3} * \frac{k_1\sigma_1}{l_1^2} > 0$$

$$\frac{\partial(p_1-p_2)}{\partial l_2} = \frac{\partial(p_1-p_2)}{\partial B} * \frac{\partial B}{\partial l_2} = \frac{-1}{3} * \frac{k_2\sigma_2}{l_2^2} < 0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_1} = \frac{\partial(p_1-p_2)}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \frac{1}{3} * -\frac{k_1}{l_1} < 0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_2} = \frac{\partial(p_1-p_2)}{\partial B} * \frac{\partial B}{\partial \sigma_2} = \frac{-1}{3} * -\frac{k_2}{l_2} > 0$$

Proof of Proposition 21

$$\frac{\partial(n_1-n_2)}{\partial A} = \frac{2\tau-2B}{3(2\tau-A-B)^2} > 0$$

$$\frac{\partial(n_1-n_2)}{\partial B} = \frac{2A-2\tau}{3(2\tau-A-B)^2} < 0$$

$$\frac{\partial(n_1-n_2)}{\partial k_1} = \frac{\partial(n_1-n_2)}{\partial A} * \frac{\partial A}{\partial k_1} = \frac{2\tau-2B}{3(2\tau-A-B)^2} * \left(v - \frac{\sigma_1}{l_1}\right) > 0$$

$$\frac{\partial(n_1-n_2)}{\partial k_2} = \frac{\partial(n_1-n_2)}{\partial B} * \frac{\partial B}{\partial k_2} = \frac{2A-2\tau}{3(2\tau-A-B)^2} * \left(v - \frac{\sigma_2}{l_2}\right) < 0$$

$$\frac{\partial(n_1-n_2)}{\partial l_1} = \frac{\partial(n_1-n_2)}{\partial A} * \frac{\partial A}{\partial l_1} = \frac{2\tau-2B}{3(2\tau-A-B)^2} * \frac{k_1\sigma_1}{l_1^2} > 0$$

$$\frac{\partial(n_1-n_2)}{\partial l_2} = \frac{\partial(n_1-n_2)}{\partial B} * \frac{\partial B}{\partial l_1} = \frac{2A-2\tau}{3(2\tau-A-B)^2} * \frac{k_2\sigma_2}{l_2^2} < 0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_1} = \frac{\partial(n_1-n_2)}{\partial A} * \frac{\partial A}{\partial \sigma_1} = \frac{2\tau-2B}{3(2\tau-A-B)^2} * -\frac{k_1}{l_1} < 0$$

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$$\frac{\partial(n_1-n_2)}{\partial\sigma_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial\sigma_2}=\frac{2A-2\tau}{3(2\tau-A-B)^2}*-\frac{k_2}{l_2}>0$$

Proof of Proposition 22

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\frac{1}{3}>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial B}=\frac{-1}{3}<0$$

$$\frac{\partial(\pi_1-\pi_{2_2})}{\partial k_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{1}{3}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\frac{-1}{3}*\left(v-\frac{\sigma_2}{l_2}\right)<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial l_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{1}{3}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial l_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial l_2}=\frac{-1}{3}*\frac{k_2\sigma_2}{l_2^2}<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial\sigma_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial\sigma_1}=\frac{1}{3}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial\sigma_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial\sigma_2}=\frac{-1}{3}*-\frac{k_2}{l_2}>0$$

Proof of Proposition 23

$$\frac{\partial HHI}{\partial A}=\frac{(2A-2B)(\tau-B)}{9(2\tau-A-B)^3}$$

$$\frac{\partial HHI}{\partial B}=\frac{(-2A+2B)(\tau-B)}{9(2\tau-A-B)^3}$$

$$\frac{\partial HHI}{\partial k_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial k_1}=\left[\frac{(2A-2B)(\tau-B)}{9(2\tau-A-B)^3}\right]*(v-\frac{\sigma_1}{l_1})$$

$$\frac{\partial HHI}{\partial k_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial k_2}=\left[\frac{(-2A+2B)(\tau-B)}{9(2\tau-A-B)^3}\right]*(v-\frac{\sigma_2}{l_2})$$

$$\frac{\partial HHI}{\partial l_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial l_1}=\left[\frac{(2A-2B)(\tau-B)}{9(2\tau-A-B)^3}\right]*\frac{k_1\sigma_1}{l_1^2}$$

$$\frac{\partial HHI}{\partial l_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial l_2}=\left[\frac{(-2A+2B)(\tau-B)}{9(2\tau-A-B)^3}\right]*\frac{k_2\sigma_2}{l_2^2}$$

$$\frac{\partial HHI}{\partial\sigma_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial\sigma_1}=\left[\frac{(2A-2B)(\tau-B)}{9(2\tau-A-B)^3}\right]*-\frac{k_1}{l_1}$$

$$\frac{\partial HHI}{\partial\sigma_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial\sigma_1}=\left[\frac{(-2A+2B)(\tau-B)}{9(2\tau-A-B)^3}\right]*-\frac{k_2}{l_2}$$

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Proof of Proposition 24

$$\frac{\partial(p_1-p_2)}{\partial A}=\frac{1}{6}>0$$

$$\frac{\partial(p_1-p_2)}{\partial B}=\frac{-1}{2}<0$$

$$\frac{\partial(p_1-p_2)}{\partial k_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{1}{6}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(p_1-p_2)}{\partial k_2}=\frac{\partial(p_1-p_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\frac{-1}{2}*(v-\sigma_2)<0$$

$$\frac{\partial(p_1-p_2)}{\partial l_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{1}{6}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_1}=\frac{\partial(p_1-p_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{1}{6}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(p_1-p_2)}{\partial \sigma_2}=\frac{\partial(p_1-p_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{-1}{2}*-k_2>0$$

Proof of Proposition 25

$$\frac{\partial(n_1-n_2)}{\partial A}=\frac{3\tau-6B}{4(3\tau-2A)^2}>0>0$$

$$\frac{\partial(n_1-n_2)}{\partial B}=\frac{-3}{4(3\tau-2A)}<0$$

$$\frac{\partial(n_1-n_2)}{\partial k_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial k_1}=\frac{3\tau-6B}{4(3\tau-2A)^2}*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(n_1-n_2)}{\partial k_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\frac{-3}{4(3\tau-2A)}*(v-\sigma_2)<0$$

$$\frac{\partial(n_1-n_2)}{\partial l_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\frac{3\tau-6B}{4(3\tau-2A)^2}*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_1}=\frac{\partial(n_1-n_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\frac{3\tau-6B}{4(3\tau-2A)^2}*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(n_1-n_2)}{\partial \sigma_2}=\frac{\partial(n_1-n_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\frac{-3}{4(3\tau-2A)}*-k_2>0$$

Proof of Proposition 26

$$\frac{\partial(\pi_1-\pi_2)}{\partial A}=\left[\frac{-1}{2}n_1+\frac{\tau-2B}{2(3\tau-2A)^2}p_1+\frac{3}{4}n_2+\frac{\tau-2B}{2(3\tau-2A)^2}p_2\right]>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial B}=\left[\left[\frac{-1}{2}n_1+\frac{-1}{2(3\tau-2A)}p_1-\frac{1}{4}n_2-\frac{1}{4(3\tau-2A)}p_2\right]\right]<0$$

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$$\frac{\partial(\pi_1-\pi_{2_2})}{\partial k_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial k_1}$$
$$=\left[\frac{-1}{2}n_1+\frac{\tau-2B}{2(3\tau-2A)^2}p_1+\frac{3}{4}n_2+\frac{\tau-2B}{2(3\tau-2A)^2}p_2\right]*\left(v-\frac{\sigma_1}{l_1}\right)>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial k_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial k_2}=\left[\left[\frac{-1}{2}n_1+\frac{-1}{2(3\tau-2A)}p_1-\frac{1}{4}n_2-\frac{1}{4(3\tau-2A)}p_2\right]\right]*(v-\sigma_2)$$
$$<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial l_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial l_1}=\left[\frac{-1}{2}n_1+\frac{\tau-2B}{2(3\tau-2A)^2}p_1+\frac{3}{4}n_2+\frac{\tau-2B}{2(3\tau-2A)^2}p_2\right]*\frac{k_1\sigma_1}{l_1^2}>0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_1}=\frac{\partial(\pi_1-\pi_2)}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\left[\frac{-1}{2}n_1+\frac{\tau-2B}{2(3\tau-2A)^2}p_1+\frac{3}{4}n_2+\frac{\tau-2B}{2(3\tau-2A)^2}p_2\right]*-\frac{k_1}{l_1}<0$$

$$\frac{\partial(\pi_1-\pi_2)}{\partial \sigma_2}=\frac{\partial(\pi_1-\pi_2)}{\partial B}*\frac{\partial B}{\partial \sigma_2}=\left[\left[\frac{-1}{2}n_1+\frac{-1}{2(3\tau-2A)}p_1-\frac{1}{4}n_2-\frac{1}{4(3\tau-2A)}p_2\right]\right]*-k_2>0$$

Proof of Proposition 27

$$\frac{\partial HHI}{\partial A}=4n_1\frac{\tau-2B}{2(3\tau-2A)}-4n_2\frac{\tau-2B}{2(3\tau-2A)}$$

$$\frac{\partial HHI}{\partial B}=4n_1\frac{-1}{2(3\tau-2A)}+4n_2\frac{1}{2(3\tau-2A)}$$

$$\frac{\partial HHI}{\partial k_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial k_1}=\left[4n_1\frac{\tau-2B}{2(3\tau-2A)}-4n_2\frac{\tau-2B}{2(3\tau-2A)}\right]*(v-\frac{\sigma_1}{l_1})$$

$$\frac{\partial HHI}{\partial k_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial k_2}=\left[4n_1\frac{-1}{2(3\tau-2A)}+4n_2\frac{1}{2(3\tau-2A)}\right]*(v-\sigma_2)$$

$$\frac{\partial HHI}{\partial l_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial l_1}=\left[4n_1\frac{\tau-2B}{2(3\tau-2A)}-4n_2\frac{\tau-2B}{2(3\tau-2A)}\right]*\frac{k_1\sigma_1}{l_1^2}$$

$$\frac{\partial HHI}{\partial \sigma_1}=\frac{\partial HHI}{\partial A}*\frac{\partial A}{\partial \sigma_1}=\left[4n_1\frac{\tau-2B}{2(3\tau-2A)}-4n_2\frac{\tau-2B}{2(3\tau-2A)}\right]*-\frac{k_1}{l_1}$$

$$\frac{\partial HHI}{\partial \sigma_2}=\frac{\partial HHI}{\partial B}*\frac{\partial B}{\partial \sigma_1}=\left[4n_1\frac{-1}{2(3\tau-2A)}+4n_2\frac{1}{2(3\tau-2A)}\right]*-k_2$$

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