考試科目	微積分(一)	系所別	應用數學系二年級	考試時間	7月5日(三) 第二節

Show all your work to earn the credits.

1. Evaluate each of the following limits. If the limit does NOT exist, then state "the limit does not exist" and provide the reason.
   (a) (10 points) $\lim_{x\to-\infty} \frac{\sqrt{4x^2+x+7}}{5x-1}$
   (b) (10 points) $\lim_{x\to\infty} \frac{x}{x-\sin x}$
   (c) (10 points) $\lim_{(x,y)\to(0,0)} \frac{3y^2-x^2}{x^2+y^2}$

2. (10 points) Let $(\cos x)^{\sin y} = 1$. Find $\frac{dy}{dx}$.

3. (10 points) Let $f(x) = x^3 \arctan(x)$. Find the 64th derivative $f^{(64)}(0)$ of $f(x)$ at 0.

4. (10 points) Let $F(x, y, z)$ be a smooth function of three variables for which the $\nabla F(1, -1, \sqrt{2}) = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Evaluate $\frac{\partial F}{\partial \phi}$ at the point whose spherical coordinates are $(\rho, \theta, \phi) = (2, -\frac{\pi}{4}, \frac{\pi}{4})$.

5. (10 points) The temperature distribution on the surface $x^2 + y^2 + z^2 = 1$ is given by $T(x, y, z) = xz + yz$. Find the hottest spot.

6. (10 points) Find the absolute maximum and minimum values of $f(x, y) = 2x^2 + y^2 - 4y$ on the closed triangular region R in the xy-plane, bounded by the lines $y = 4$, $y = x$ and $y = -x$.

7. (10 points) Let f be a function such that $f'(x) = e^{x^2}$ and $f(0) = 2$. Show that $3 < f(1) < 2 + e$.

8. (10 points) Using the definition of the derivative to show that

$$\frac{d}{dx}[a^x] = (\ln a) \cdot a^x$$

where $a > 0$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分(二)	系所別	應用數學系二年級	考試時間	7月5日(三) 第四節

Show all your work to earn the credits.

1. Evaluate each of the following:

   (a) (10 points) $\lim_{x\to 0} \dfrac{\int_2^{3x+2} \ln(t-1)dt}{x^2}$

   (b) (10 points) $\int \dfrac{\sin x}{1+\cos^2 x}dx$

   (c) (10 points) $\int_0^1 4\arctan(x)dx$

   (d) (10 points) $\int \dfrac{4}{\sin^3(x)}dx$

2. Find each of the following:

   (a) (10 points) Find the value(s) for p such that the series $\sum_{n=1}^{\infty} \dfrac{n}{\sqrt{3+n^p}}$. converges.

   (b) (10 points) Evaluate the limit: $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{3\pi}{4n} \sec^2\left(\dfrac{i\pi}{4n}\right)$.

3. (10 points) Solve the initial value problem: $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{1+x^2}$, $y(0) = 0$.

4. (10 points) Find the volume of the solid in the first octant bounded above by $x^2+y^2+z^2 = 12$ and bounded below by $z = \sqrt{x^2+y^2}$.

5. (10 points) Let $F(x,y) = (xy^2+2y)\mathbf{i} + (x^2y+2x+2)\mathbf{j}$ be a vector field. Evaluate the line integral $\int_C F\cdot d\mathbf{r}$, where C is a curve defined by $\mathbf{r}(t) = e^t\mathbf{i} + (1+t)\mathbf{j}$, $0 \le t \le 1$.

6. (10 points) Use Green's Theorem to evaluate $\int_C -y^3dx + x^3dy$, where C is the curve defined by $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j}$, $0 \le t \le 2\pi$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	應用數學系三年級	考試時間	7月5日(三)第二節

Show all your work to earn the credits.

1. Evaluate each of the following limits. If the limit does NOT exist, then state "the limit does not exist".

   (a) (10 points) $\lim_{n\to\infty} \frac{(-1)^n n^2}{n^2+n+1}$.

   (b) (10 points) $\lim_{x\to-\infty} \frac{\sqrt{4x^6+7}}{x^3+1}$

   (c) (10 points) $\lim_{(x,y)\to(0,0)} \frac{2xy+3x^3}{\sqrt{x^2+y^2}}$.

2. (10 points) Let $xy + e^{xz} + yz^2 = 0$. Find $\frac{\partial z}{\partial x}(0,1,2)$.

3. Find each of the following integrals:

   (a) (10 points) $\int \frac{2}{\cos^3(x)}dx$.

   (b) (10 points) $\int e^{x+e^x}dx$

4. (10 points) Suppose $f(x)$ is differentiable for all real numbers. If $-1 \le f'(x) \le 3$ and $f(5) = 4$. What is the largest value $f(x)$ can be at $x = 1$?

5. (10 points) Find the maximum and minimum of the function $f(x,y) = x^2y$ on the region $x^2+y^2 \le 1$.

6. (10 points) Show that $\sum_{n=0}^{\infty}(-1)^n x^{2n} = \frac{1}{1+x^2}$ provided that $|x| < 1$ and use this result to find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

7. (10 points) Use Green's Theorem to evaluate $\int_C (e^x + y^2)dx + (e^y + x^2)dy$, where C is the positively oriented boundary of the region in the first quadrant bounded by $y = x^2$, $y = 4$ and the y-axis.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

國立政治大學 112 學年度 轉學生 招生考試試題

考試科目	線性代數	系所別	應用數學系三年級	考試時間	7月5日(三) 第四節

注意事項：

- 作答時，請於答案卷上標明題號，並請勿任意更改題目符號，且請詳列過程，只有答案不給分。
- 本試題共有 6 個問題, 總計 100 分。

1. (16 %) Let $\mathbb{R}^2$ be the x-y-plane. Then $\mathbb{R}^2$ is a vector space. A line $\ell \subset \mathbb{R}^2$ with slope m and y-intercept b is defined by
$$\ell = \{(x, y) \in \mathbb{R}^2 : y = mx + b\}.$$
Prove that ℓ is a subspace of $\mathbb{R}^2$ if and only if $b = 0$.

2. (16 %) Let
$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$
Determine whether the matrix A is diagonalizable? If it is diagonalizable, then diagonalize A.

3. (16 %) Consider two matrices
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$
Are the two matrices similar? If so, find a matrix P such that $B = P^{-1}AP$.

4. (18 %) Let n be a positive integer. Let $T : \mathbb{R}^n \to \mathbb{R}$ be a non-zero linear transformation. Prove the followings:
   (a) (6 %) The dimension of the nullspace of T is $n - 1$.
   (b) (6 %) Let $B = \{\mathrm{v}_1, \ldots, \mathrm{v}_{n-1}\}$ be a basis of the nullspace $\mathcal{N}(T)$ of T. Let w be the n-dimensional vector that is not in $\mathcal{N}(T)$. Then $B' = \{\mathrm{v}_1, \ldots, \mathrm{v}_{n-1}, \mathrm{w}\}$ is a basis of $\mathbb{R}^n$.
   (c) (6 %) Each vector $\mathrm{u} \in \mathbb{R}^n$ can be expressed as
$$\mathrm{u} = \mathrm{v} + \frac{T(\mathrm{u})}{T(\mathrm{w})}\mathrm{w}$$
   for some vector $\mathrm{v} \in \mathcal{N}(T)$.

5. (18 %) Consider the matrix
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 4 \\ 1 & 1 & 0 \end{bmatrix}.$$
   (a) (6%) Calculate the inverse matrix A^{-1}. If You think the matrix A is not invertible, explain why.
   (b) (6%) Are the vectors
$$\mathrm{A}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \mathrm{A}_2 = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}, \mathrm{A}_3 = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$$
   linearly independent?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	線性代數	系所別	應用數學系三年級	考試時間	7月5日(三) 第四節

(c) (6%) Write the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ as a linear combination of A_1, A_2, and A_3.

6. (16 %) Find the Jordan canonical form of the matrix

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix}.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。