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Received November 2001
Revised April 2003

NEW RESEARCH

Economic statistical process control for over-adjusted process mean

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Keywords *Control charts, Markov processes, Statistical process control*

Abstract *An economic adjustment model of a process whose quality can be affected by multiple special causes, resulting in changes of the process mean by incorrect adjustment of the process when it is operating according to its capability. A statistically constrained adjustment model is developed for the economic statistical design of* $\bar{X}$ *control chart to control the process mean affected by multiple special causes. The objective is to determine the design parameters of the* $\bar{X}$ *control chart, which minimize the total quality control cost. A Markov chain approach is used to derive the model. It is demonstrated that the expressions for the expected cycle time and the expected cycle cost with multiple special causes are easier to obtain by the proposed approach than by extending or adopting that in Collani* et al. *Application of the model is demonstrated through a numerical example.*

Introduction

Control charts are important tools of statistical quality control (SQC). These charts are used to decide whether a process has achieved a state of statistical control and to maintain current control of a process. The use of control charts as a process monitoring and control tool has received much attention recently.

Deming (1982) explains that there are two kinds of mistakes the production worker can make on the job. These are to over-adjust a process or to under-adjust a process. He goes on to explain that the control chart provides "a rational and economic guide to minimize loss from both mistakes". Precise methods to design control charts that maximize the profit or minimize the cost of a process have been proposed by a number of authors. These methods yield control chart designs known as economic design. Economic design of a control chart was first proposed by Duncan (1956). The pioneering work of Duncan was later extended by others. A review of the literature is available in Montgomery (1980) and Vance (1983). Economic design optimizes the economic model of a production process by considering the costs of under-adjustment along with other costs, but it assumes that the search for a special cause is perfect.

In reality, a common problem in statistical process control (SPC) is process over-adjustment. Processes may be adjusted since the only information available

Support for this research was provided in part by the National Science Council of the Republic of China, Grant No. NSC-90-2118-M-004-005.

International Journal of Quality & Reliability Management
Vol. 21 No. 4, 2004
pp. 412-424
© Emerald Group Publishing Limited
0265-671X
DOI 10.1108/02656710410530109

about the process state is that due to sampling. Consequently, a control chart signal outside the control limits is associated with process adjustment. If the signal is a false alarm, the process will be adjusted incorrectly. Woodall (1986) noted the effect of this over-adjustment as being an increase in the variability of the process. The increase in variability and the corresponding loss of quality can be quite marked as noted by Collani *et al.* (1994). This problem is common in practice and therefore of importance. Collani *et al.* (1994) first proposed economic adjustment design for $\bar{X}$ control chart to monitor a process with two types of mistakes. They assumed that there exists a single special cause that may cause the shift of a process mean. Their model allows for the determination of the design parameters of the $\bar{X}$ control chart that maximizes the profitability of the process or, equivalently, minimizes the process loss from two types of mistakes of over-adjustment and under-adjustment. However, their calculations for the expected cycle time and expected cycle cost/profitability are complicated, and it is not easy to extend their approach to the case of multiple special causes. In this paper, we consider that the quality of output can be affected by multiple special causes, resulting in shifts in the process mean, due to over-adjustment during operation. The $\bar{X}$ control chart is used to signal any one or any combination of the special causes, which results in a shift of the process mean. A Markovian chain approach is used. The proposed approach would derive the expected cycle time and the expected cycle cost more easily than by extending or adopting that in Collani *et al.* (1994). In the next section, the economic adjustment model is derived by a Markovian chain approach. A direct search optimization technique is used to determine the design parameters of the $\bar{X}$ control chart that minimizes the loss of this process. An example illustrating the proposed method for multiple special causes is given in the third section. A brief summary is provided in the final section of this paper.

Economic adjustment model: a Markov chain approach

A production process may be in control or out of statistical control. If the process is influenced by any special causes then the process is out of control; otherwise the process is in control. Suppose that there exist two special causes, say SC_1 and SC_2, for a production process. We assume that the process mean would be shifted if any one or any combination of the special causes influences the process. In this analysis, we are using the $\bar{X}$ control chart to signal the need for adjustment in the key dimension of the product. The in-control process can be out of control if it is incorrectly adjusted. Specifically, we take a sample of size n units of output every h hours of production time and adjust the process if the sample mean falls outside the control limits of the $\bar{X}$ control chart. Our objective is to derive the economic adjustment model using the Markov chain approach and to find the set of parameters n, h, and k (control coefficient of $\bar{X}$ control chart) such that the average long-term loss of the process is minimized.

Process assumptions and notation

The assumptions of the process are described as follows. Suppose that the product's quality can be represented by one key dimension, say X. When the process is in control, $X \sim N(\mu, \sigma^2)$. There are three situations for the occurred two special causes; that is, either SC_1, SC_2, or both. When any situation of the special causes of poor quality occurs in the process, there is a shift in the distribution of X to $X \sim N(\mu + \delta_j\sigma, \sigma^2)$ with probability w and to $X \sim N(\mu - \delta_j\sigma, \sigma^2)$ with probability $1 - w$, where $\delta_j > 0$ and $0 < w < 1$, where $j = 1, 2, 3$. The time until the occurrence of any special cause is assumed to be independent and exponential with a mean of $1/\lambda_i$, $\lambda_i > 0$, where $i = 1, 2$. We also assume that the process cannot correct itself, and the time to sample and plot $\bar{X}$ is small and hence can be neglected in the model. However, it should be noted that the proposed Markov chain approach could be extended to include cases involving the time to sample and plot $\bar{X}$.

An adjustment to the process is performed if the sample mean falls outside the control limits of the $\bar{X}$ control chart, respectively $LCL_{\bar{X}}$ and $UCL_{\bar{X}}$, where:

$$LCL_{\bar{X}} = \mu - k\sigma/\sqrt{n},$$

$$UCL_{\bar{X}} = \mu + k\sigma/\sqrt{n},$$

where k is the number of standard deviation above or below the center line of the $\bar{X}$ control chart. The adjustment can take one of three forms:

(1) When the shift results in $X \sim N(\mu + \delta_1\sigma, \sigma^2)$, SC_1 is adjusted to let the mean of X be μ; when the shift results in $X \sim N(\mu - \delta_1\sigma, \sigma^2)$, SC_1 is adjusted to let the mean of X be μ.

(2) When the shift results in $X \sim N(\mu + \delta_2\sigma, \sigma^2)$, SC_2 is adjusted to let the mean of X be μ; when the shift results in $X \sim N(\mu - \delta_2\sigma, \sigma^2)$, SC_2 is adjusted let the mean of X be μ.

(3) When the shift results in $X \sim N(\mu + \delta_3\sigma, \sigma^2)$, both SC_1 and SC_2 are adjusted to let the mean of X be μ; when the shift results in $X \sim N(\mu - \delta_3\sigma, \sigma^2)$, both SC_1 and SC_2 are adjusted to let the mean of X be μ.

The decision rule can result in an over-adjustment following false alarms for the process mean. The probabilities to adjust SC_1, SC_2, and SC_3 are g1, g2, and g3 respectively, following an alarm. It is assumed that a transition in the process from in control to out of control during sampling is impossible. The following notation is used. Before deriving the economic adjustment model using Markov chain approach, we define some variables as follows.

$\alpha_{\bar{X}}$ = probability that the process is over-adjusted when the $\bar{X}$ control chart gives a false alarm:

$$\alpha_{\bar{X}} = 1 - P(LCL_{\bar{X}} \leq \bar{X} \leq UCL_{\bar{X}} | x \sim N(\mu, \sigma^2)) = 2\Phi(-k),$$

where $\Phi(.)$ is the cumulative probability of a normal distribution.

$\beta_{\bar{X}_j}$=probability that the process is under-adjusted since it is influenced by either SC_1, SC_2 or both, but $\bar{X}$ control chart gives no true alarm, where:

$$\beta_{\bar{X}_j} = wP(LCL_{\bar{X}} \leq \bar{X} \leq UCL_{\bar{X}} | x \sim N(\mu + \delta_j\sigma, \sigma^2)) + (1 - w)P(LCL_{\bar{X}} \leq \bar{X} \leq UCL_{\bar{X}} | x \sim N(\mu - \delta_j\sigma, \sigma^2)) = \Phi(k - \delta_j\sqrt{n}) - \Phi(-k - \delta_j\sqrt{n}),$$

$j = 1, 2, 3.$

T_f = expected time of over-adjusting either a SC_1, SC_2 or both, following a false alarm.

T_{sci} = time before the special cause SC_i occurs in the process, $T_{sci} \sim \exp(\lambda_i)$, $i = 1, 2$.

T_{sr} = expected time to search and repair SC_1, SC_2 or both.

C_f = expected cost of over-adjusting either SC_1, SC_2 or both.

C_0 = production cost per unit time when the process is in control.

C_1 = production cost per unit time when the process is only affected by SC_1.

C_2 = production cost per unit time when the process is only affected by SC_2.

C_{12} = expected cost per unit time when the process is affected by SC_1 and SC_2.

C_{sr} = expected cost to search and repair SC_1, SC_2 or both.

a = fixed cost per sample and test.

b = cost per unit sampled and tested.

τ_i: expected arrival time of the special cause SC_1, given that it occurred in the first sampling interval, where (see Lorenzen and Vance, 1986):

$$\tau_i = \frac{1 - (1 + \lambda_i h)e^{-\lambda_i h}}{\lambda_i - \lambda_i e^{-\lambda_i h}}, \quad i = 1, 2.$$

$\tau_{(i)}$: expected arrival time of the ith special cause, given that SC_1 and SC_2 occurred within time interval h, $i = 1, 2$, where (for proofs, see Yang, 1997):

$$\begin{aligned}\tau_{(1)} &= E(\min(T_{sc1}, T_{sc2})|T_{sci} < h, \ i = 1, 2) \\ &= [\exp(^{-}\lambda_1^{h-}\lambda_2^{h})(h + 1/\lambda_1^{+}1/\lambda_2^{-}1/(\lambda_1^{+}\lambda_2)) - \exp(^{-}\lambda_1^{h})/\lambda_2 - \exp(^{-}\lambda_2^{h})/\lambda_1 \\ &\quad + 1/(\lambda_1 + \lambda_2)]/[(1 - \exp(^{-}\lambda_1^{h}))(1 - \exp(^{-}\lambda_1^{h}))],\end{aligned}$$

$$\begin{aligned}\tau_{(2)} &= E(\max(T_{sc1}, T_{sc2})|T_{sci} < h, i = 1, 2) \\ &= [\exp(^{-}\lambda_1^{h-}\lambda_2^{h})(h + 1/(\lambda_1^{+}\lambda_2)) - \exp(^{-}\lambda_1^{h})(h + 1/\lambda_1) - \exp(^{-}\lambda_2^{h})(h + 1/\lambda_2) \\ &\quad + 1/\lambda_1 + 1/\lambda_2 - 1/(\lambda_1 + \lambda_2)]/[(1 - \exp(^{-}\lambda_1^{h}))(1 - \exp(^{-}\lambda_2^{h}))].\end{aligned}$$

Description of Markov chain

In order to use the Markov chain approach to derive the expected cycle time (ET) and the expected cycle cost (EC), all possible states at the end of each sampling and testing time must be examined. Depending on the state of the system, the transition probabilities and transition costs can be computed. There are 16 possible states at the end of every sampling and testing time, and these states are defined as follows (Table I).

State	SC_1 occur?	SC_2 occur?	x-bar signal?	Process adjustment?	Process over-adjusted?
1	No	No	No	No	No
2	No	No	Yes	SC_1	Yes
3	No	No	Yes	SC_2	Yes
4	No	No	Yes	$SC_1 + SC_2$	Yes
5	Yes	No	No	No	No
6	Yes	No	Yes	SC_1	No
7	Yes	No	Yes	SC_2	Yes
8	Yes	No	Yes	$SC_1 + SC_2$	Yes
9	No	Yes	No	No	No
10	No	Yes	Yes	SC_1	Yes
11	No	Yes	Yes	SC_2	No
12	No	Yes	Yes	$SC_1 + SC_2$	Yes
13	Yes	Yes	No	No	No
14	Yes	Yes	Yes	SC_1	No
15	Yes	Yes	Yes	SC_2	No
16	Yes	Yes	Yes	$SC_1 + SC_2$	No

Table I.
Definition for each state

These states can be classified into two types of states: transient states and absorbing states. The states 6, 11, and 16 are absorbing states, and the others are transient states. Transition probability from state i to state j in time interval h is described in Appendix 1.

The transition probability matrix is denoted as $P_{11} = [P(i, j)]$, i, j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13,14,15; $P_{12} = [P(i, j)]$, i = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13,14,15, j = 6, 11, 16; zero matrix $0 = [P(i, j)]$, $P_{i,j}=0$ for i = 6, 11, 16, j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13,14,15; identity matrix $I = [P(i, j)]$, $P(i, j) = 1$ for i, j = 6, 11, 16, and matrix P is the combination of sub-matrices P_{11}, P_{12}, I, and 0. That is:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ 0 & I \end{bmatrix}.$$

The cycle time is the time from the start of the process in control until an alarm is detected, repaired, and the process is restarted or equivalently it is the time from transient state 1 to reach any absorbing state. The state variable Y_t ($t = 0, h, 2h, \ldots$) is a Markov chain on the state 1, 2, …, 16 and so the Markov property can be effectively used to find the expected cycle time.

Expected cycle time and cost

Let random variable T_i be the time until absorption from transient state i. Then, using the Markov property and conditioning on the first step:

$$\begin{aligned}
P(T_i = h + T_{sr}) = P(i, j) \quad & \text{where } j = 6, 11, 16, i \neq j, \\
P(T_i = h + T_f + T_j) = P(i, j) \quad & \text{where } i \neq 6, 11, 16, j = 2, 3, 4, 7, 8, 10, \\
P(T_i = h + T_f + T_{sr} + T_j) = P(i, j) \quad & \text{where } i \neq 6, 11, 16, j = 8, 12, \\
P(T_i = h + T_j) = P(i, j) \quad & \text{where } i \neq 6, 11, 16, j = 1, 5, 9, 13, \\
P(T_i = h + T_{sr} + T_j) = P(i, j) \quad & \text{where } i \neq 6, 11, 16, j = 14, 15.
\end{aligned} \tag{1}$$

Equation (1) can be expressed in matrix form:

$$M = h\mathbf{1} + P_{11}M_{sr1} + P_{11}M + P_{12}M_{sr2}SoM$$
$$= h(I - P_{11})^{-1}\mathbf{1} + (I - P_{11})^{-1}P_{11}M_{sr1} + (I - P_{11})^{-1}P_{12}M_{sr2},$$

where: M is a (13×1) vector, with the expected time up to absorption from transient state i, $i \neq 6, 11, 16$; $\mathbf{1}$ is a (13×1) vector, with elements 1; M_{sr1} is a (13×1) vector, $M_{sr1}^T = [0\ T_f\ T_f\ T_f\ 0\ T_f T_f{+}T_{sr}\ 0\ T_f T_f{+}T_{sr}\ 0\ T_{sr} T_{sr}]$; M_{sr2} is a (3×1) vector, $M_{sr2}^T = [T_{sr}\ T_{sr}\ T_{sr}]$, P_{11} is defined as above.

The expected cycle time is the first element of vector M, i.e. M_1 or $E(T_1)$.

Once the expected cycle time is obtained, the expected cycle cost must be calculated, and the economic adjustment model can be derived by taking the ratio of the expected cycle cost to the expected cycle time.

The derivation of the expected cycle cost uses the Markov property in a similar manner to that used for the expected cycle time. Let C(i, j) be the expected cumulative cost that is associated with transition from state i to j in time interval h; i, j = 1, 2, …, 16. The calculation of C(i, j) is illustrated in Appendix 2.

The transition cost matrices are denoted as: $C_{11} = [C(i, j)]$, i, j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15; $C_{12} = [C(i, j)]$, i = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, j = 6, 11, 16; zero matrix $0 = [C(i, j)]$, $C(i, j) = 0$ for i = 6, 11, 16, j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15; $C_{22} = C_{sr}I$, I is identity matrix for i, j = 6, 11, 16, and matrix C is the combination of sub-matrices C_{11}, C_{12}, C_{22}, and 0. That is:

$$C = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix}.$$

The cycle cost is the cumulative cost from the start of the process, in control, until an alarm is detected, the process is repaired and re-started, or equivalently, it is the cost from transient state 1 until it reaches an absorbing state.

Let random variable C_i be the cumulative cost up to absorption from transient state i, i = 1, 2, …, 15. Then using the Markov property and conditioning on the first step:

$$\begin{aligned} P(C_i = C(i,j)) = P(i,j) \qquad & \text{where } j = 6, 11, 16, i \neq j, \\ P(C_i = C(i,j) + Cj) = P(i,j) \qquad & \text{where } i, j \neq 6, 11, 16. \end{aligned} \tag{2}$$

Equation (2) can be expressed in matrix form:

$$U = P_{11} * C_{11}\mathbf{1} + P_{12} * C_{12}\mathbf{1} + P_{11}U,$$

where * denotes the Hadamard product of the two matrices, and U is a (13×1) vector with the expected cost up to absorption from transient state i, $i \neq 6, 11, 16$.

So $U = (I - P_{11})^{-1}[P_{11} * C_{11}\mathbf{1} + P_{12} * C_{12}\mathbf{1}]$, where the first element of the vector, U_1, is the expected cycle cost.

Determination of optimal design parameters

Applying the property of renewal reward processes (Ross, 1993), the objective function (L), the expected cost per unit time is derived by taking the ratio of the expected cycle cost (U_1) to the expected cycle time (M_1); $L = U_1/M_1$. The expected long-term loss is the function of design parameters n, h, k; L(n, k, h). Hence, the optimal design parameters of the economic statistical adjustment design of $\bar{X}$ control chart can be determined by minimization of the statistically constrained objective function, that is:

$$\begin{aligned} &\mathrm{Min} L(n, k, h) \\ \text{s.t.} \quad &\alpha_{\bar{X}} < \alpha_L \\ &\beta_{\bar{X}_j} < \beta_{L_j}, \ \ j = 1, 2, 3 \end{aligned} \tag{3}$$

where α_L is the upper bound of $\alpha_{\bar{X}}$; β_{L_j} is the upper bound of $\beta_{\bar{X}_j}$, j = 1, 2, 3. It may be noted that the proposed approach can also be used to derive the identical economic adjustment model obtained by Collani *et al.* (1994) if the expected time of over-adjustment = 0, the expected time to search and repair a special cause = 0, the expected cost of incorrect adjustment = 0, the expected cost to search and repair a special cause = 0, and profit maximization is used, instead of cost minimization, in a single special-cause economic adjustment model.

A numerical example

A simple case is used to illustrate the proposed method, and the application of the proposed control charts. Suppose that two possible out-of-control machines may influence the process mean, and the process mean may be over-adjusted when a false alarm occurred on the control chart.

To determine the design parameters of the $\bar{X}$ chart with minimum cost and required statistical properties, the following set of hypothetical process and cost parameters is chosen:

$$\begin{aligned} w_1 &= w_2 = 0.5, \delta_1 = 2, \delta_2 = 2.5, \delta_3 = 3.0, \lambda_1 = 0.05, \lambda_2 = 0.04, a = 0.5, b = 0.1, C_f \\ &= 20, C_{sr} = 35, T_f = 0.3, T_{sr} = 0.6, C_0 = 5, C_1 = 10, C_2 = 15, C_{12} = 35, \mu = 0, \sigma \\ &= 1. \end{aligned}$$

The algorithm used to obtain the approximate optimum values (n*, h*, k*) of the design values (n, h, k), with constraints $0 < k < 6, 1 < n \leq 25$, $0 < h \leq 8$, and $0 < \alpha_{\bar{X}}, \beta_{\bar{X}_j} < 0.1$ is a simple grid search method yielding the following result: $n* = 25$, $h* = 1.0$, $k* = 3.0$.

That is, the upper and lower control limits of the economic statistical $\bar{X}$ chart should be set at 0.6 and -0.6, respectively. To monitor the process states, every one hour a sample of 25 is taken and tested.

There are four possible results for the process. These outcomes with the associated actions are:

(1) The sample mean $\bar{x}$ falls within the control limits of $\bar{X}$ chart – this indicates that the process is in control so the process continues and the next sample is taken after one hour.

(2) The sample mean $\bar{x}$ falls outside of the control limits of $\bar{X}$ chart – this indicates that the process should be stopped and SC_1 is adjusted.

(3) The sample mean $\bar{x}$ falls outside of the control limits of $\bar{X}$ chart – this indicates that the process should be stopped and SC_2 is adjusted.

(4) The sample mean $\bar{x}$ falls outside of the control limits of $\bar{X}$ chart – this indicates that the process should be stopped and SC_1 and SC_2 are adjusted.

Summary

A model of a production process is proposed, whose quality can be affected by the occurrence of two special causes, which result in a different shift in the mean of a process. A shift in either may also result from over-adjustment of the process when the process is in control. Deming (1982) discusses this common situation in practice. The proposed model is an improvement to the economic design, since it has the required statistical properties and considers the effect of process over-adjustment when there are multiple special causes. Using the proposed design, a process may be adjusted with minimum cost and required statistical properties, since the only information about process state available is from sampling.

A Markov chain approach is extended to derive the economic adjustment model used to determine the design parameters of the $\bar{X}$ control charts, which together minimize the long-term cost resulting from process over-adjustment or under-adjustment. It is demonstrated that the expression for the economic adjustment model is easier to obtain through the proposed approach rather than by others. Several important extensions of the developed model can be developed. It is straightforward to extend the proposed model to study other control charts, like EWMA, CUSUM-charts or charts for attributes. The differences between the models lie in the derivation of the probabilities of Type I and Type II errors. One particularly interesting research area for future research involves the economic statistical modeling of multiple dependent production processes.

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Appendix 1

Table AI. Row 1

$P_{1,1} = \exp(-\lambda_1 h)\exp(-\lambda_2 h)\cdot(1 - \alpha_{\bar{X}})$	$P_{1,2} = \exp(-\lambda_1 h)\exp(-\lambda_2 h)\cdot g1(\alpha_{\bar{X}})$	$P_{1,3} = \exp(-\lambda_1 h)\exp(-\lambda_2 h)\cdot g2(\alpha_{\bar{X}})$	$P_{1,4} = \exp(-\lambda_1 h)\exp(-\lambda_2 h)\cdot g3(\alpha_{\bar{X}})$
$P_{1,5} = (1 - \exp(-\lambda_1 h))\exp(-\lambda_2 h)\cdot(\beta_{\bar{X}_1})$	$P_{1,6} = (1 - \exp(-\lambda_1 h))\exp(-\lambda_2 h)\cdot\cdot g1(1 - \beta_{\bar{X}_1})$	$P_{1,7} = (1 - \exp(-\lambda_1 h))\exp(-\lambda_2 h)\cdot g2(1 - \beta_{\bar{X}_1})$	$P_{1,8} = (1 - \exp(-\lambda_1 h))\exp(-\lambda_2 h)\cdot g3(1 - \beta_{\bar{X}_1})$
$P_{1,9} = \exp(-\lambda_1 h)\times(1 - \exp(\lambda_2 h))\cdot(\beta_{\bar{X}_2})$	$P_{1,10} = \exp(-\lambda_1 h)\times(1 - \exp(-\lambda_2 h))\cdot g1(1 - \beta_{\bar{X}_2})$	$P_{1,11} = (\exp(-\lambda_1 h))\times(1 - \exp(-\lambda_2 h))\cdot g2(1 - \beta_{\bar{X}_2})$	$P_{1,12} = (\exp(-\lambda_1 h))\times(1 - \exp(-\lambda_2 h))\cdot g3(1 - \beta_{\bar{X}_2})$
$P_{1,13} = (1 - \exp(-\lambda_1 h))\times(1 - \exp(-\lambda_2 h))\cdot(\beta_{\bar{X}_3})$	$P_{1,14} = (1 - \exp(-\lambda_1 h))\times(1 - \exp(-\lambda_2 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{1,15} = (1 - \exp(-\lambda_1 h))\times(1 - \exp(-\lambda_2 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{1,16} = (1 - \exp(-\lambda_1 h))\times(1 - \exp(-\lambda_2 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AII. Row 2

$P_{2,1} = 0$	$P_{2,2} = 0$	$P_{2,3} = 0$	$P_{2,4} = 0$
$P_{2,5} = \exp(-\lambda_2 h)(\beta_{\bar{X}_1})$	$P_{2,6} = \exp(-\lambda_2 h)\cdot g1(1 - \beta_{\bar{X}_1})$	$P_{2,7} = \exp(-\lambda_2 h)\cdot g2(1 - \beta_{\bar{X}_1})$	$P_{2,8} = \exp(-\lambda_2 h)\cdot g3(1 - \beta_{\bar{X}_1})$
$P_{2,9} = 0$	$P_{2,10} = 0$	$P_{2,11} = 0$	$P_{2,12} = 0$
$P_{2,13} = (1 - \exp(-\lambda_2 h))\cdot(\beta_{\bar{X}_3})$	$P_{2,14} = (1 - \exp(-\lambda_2 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{2,15} = (1 - \exp(-\lambda_2 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{2,16} = (1 - \exp(-\lambda_2 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AIII. Row 3

$P_{3,1} = 0$	$P_{3,2} = 0$	$P_{3,3} = 0$	$P_{3,4} = 0$
$P_{3,5} = 0$	$P_{3,6} = 0$	$P_{3,7} = 0$	$P_{3,8} = 0$
$P_{3,9} = \exp(-\lambda_1 h)(\beta_{\bar{X}_2})$	$P_{3,10} = \exp(-\lambda_1 h)\cdot g1(1 - \beta_{\bar{X}_2})$	$P_{3,11} = \exp(-\lambda_1 h)\cdot g2(1 - \beta_{\bar{X}_2})$	$P_{3,12} = \exp(-\lambda_1 h)\cdot g3(1 - \beta_{\bar{X}_2})$
$P_{3,13} = (1 - \exp(-\lambda_1 h))\cdot(\beta_{\bar{X}_3})$	$P_{3,14} = (1 - \exp(-\lambda_1 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{3,15} = (1 - \exp(-\lambda_1 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{3,16} = (1 - \exp(-\lambda_1 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AIV. Row 4

$P_{4,1} = 0$	$P_{4,2} = 0$	$P_{4,3} = 0$	$P_{4,4} = 0$
$P_{4,5} = 0$	$P_{4,6} = 0$	$P_{4,7} = 0$	$P_{4,8} = 0$
$P_{4,9} = 0$	$P_{4,10} = 0$	$P_{4,11} = 0$	$P_{4,12} = 0$
$P_{4,13} = \beta_{\bar{X}_3}$	$P_{4,14} = g1(1 - \beta_{\bar{X}_3})$	$P_{4,15} = g2(1 - \beta_{\bar{X}_3})$	$P_{4,16} = g3(1 - \beta_{\bar{X}_3})$

Table AV. Row 5

$P_{5,1} = 0$	$P_{5,2} = 0$	$P_{5,3} = 0$	$P_{5,4} = 0$
$P_{5,5} = \exp(-\lambda_2 h)(\beta_{\bar{X}_1})$	$P_{5,6} = \exp(-\lambda_2 h)\cdot g1(1 - \beta_{\bar{X}_1})$	$P_{5,7} = \exp(-\lambda_2 h)\cdot g2(1 - \beta_{\bar{X}_1})$	$P_{5,8} = \exp(-\lambda_2 h)\cdot g3(1 - \beta_{\bar{X}_1})$
$P_{5,9} = 0$	$P_{5,10} = 0$	$P_{5,11} = 0$	$P_{5,12} = 0$
$P_{5,13} = (1 - \exp(-\lambda_2 h))\cdot(\beta_{\bar{X}_3})$	$P_{5,14} = (1 - \exp(-\lambda_2 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{5,15} = (1 - \exp(-\lambda_2 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{5,16} = (1 - \exp(-\lambda_2 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AVI. Row 6

$P_{6,1} = 0$	$P_{6,2} = 0$	$P_{6,3} = 0$	$P_{6,4} = 0$
$P_{6,5} = 0$	$P_{6,6} = 1$	$P_{6,7} = 0$	$P_{6,8} = 0$
$P_{6,9} = 0$	$P_{6,10} = 0$	$P_{6,11} = 0$	$P_{6,12} = 0$
$P_{6,13} = 0$	$P_{6,14} = 0$	$P_{6,15} = 0$	$P_{6,16} = 0$

Table AVII. Row 7

$P_{7,1} = 0$	$P_{7,2} = 0$	$P_{7,3} = 0$	$P_{7,4} = 0$
$P_{7,5} = 0$	$P_{7,6} = 0$	$P_{7,7} = 0$	$P_{7,8} = 0$
$P_{7,9} = 0$	$P_{7,10} = 0$	$P_{7,11} = 0$	$P_{7,12} = 0$
$P_{7,13} = \beta_{\bar{X}_3}$	$P_{7,14} = g1(1 - \beta_{\bar{X}_3})$	$P_{7,15} = g2(1 - \beta_{\bar{X}_3})$	$P_{7,16} = g3(1 - \beta_{\bar{X}_3})$

$P_{8,1} = 0$	$P_{8,2} = 0$	$P_{8,3} = 0$	$P_{8,4} = 0$
$P_{8,5} = 0$	$P_{8,6} = 0$	$P_{8,7} = 0$	$P_{8,8} = \exp(-\lambda_1 h)(\beta)(\beta_{s^2})$
$P_{8,9} = \exp(-\lambda_1 h)\cdot(\beta_{\bar{X}_2})$	$P_{8,10} = \exp(-\lambda_1 h)\cdot g1(1 - \beta_{\bar{X}_2})$	$P_{8,11} = \exp(-\lambda_1 h)\cdot g2(1 - \beta_{\bar{X}_2})$	$P_{8,12} = \exp(-\lambda_1 h)\cdot g3(1 - \beta_{\bar{X}_2})$
$P_{8,13} = (1 - \exp(-\lambda_1 h))\cdot(\beta_{\bar{X}_3})$	$P_{8,14} = (1 - \exp(-\lambda_1 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{8,15} = (1 - \exp(-\lambda_1 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{8,16} = (1 - \exp(-\lambda_1 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AVIII. Row 8

$P_{9,1} = 0$	$P_{9,2} = 0$	$P_{9,3} = 0$	$P_{9,4} = 0$
$P_{9,5} = 0$	$P_{9,6} = 0$	$P_{9,7} = 0$	$P_{9,8} = 0$
$P_{9,9} = \exp(-\lambda_1 h)\cdot(\beta_{\bar{X}_2})$	$P_{9,10} = \exp(-\lambda_1 h)\cdot g1(1 - \beta_{\bar{X}_2})$	$P_{9,11} = \exp(-\lambda_1 h)\cdot g2(1 - \beta_{\bar{X}_2})$	$P_{9,12} = \exp(-\lambda_1 h)\cdot g3(1 - \beta_{\bar{X}_2})$
$P_{9,13} = (1 - \exp(-\lambda_1 h))\cdot(\beta_{\bar{X}_3})$	$P_{9,14} = (1 - \exp(-\lambda_1 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{9,15} = (1 - \exp(-\lambda_1 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{9,16} = (1 - \exp(-\lambda_1 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AIX. Row 9

$P_{10,1} = 0$	$P_{10,2} = 0$	$P_{10,3} = 0$	$P_{10,4} = 0$
$P_{10,5} = 0$	$P_{10,6} = 0$	$P_{10,7} = 0$	$P_{10,8} = 0$
$P_{10,9} = 0$	$P_{10,10} = 0$	$P_{10,11} = 0$	$P_{10,12} = 0$
$P_{10,13} = \beta_{\bar{X}_3}$	$P_{10,14} = g1(1 - \beta_{\bar{X}_3})$	$P_{10,15} = g2(1 - \beta_{\bar{X}_3})$	$P_{10,16} = g3(1 - \beta_{\bar{X}_3})$

Table AX. Row 10

$P_{11,1} = 0$	$P_{11,2} = 0$	$P_{11,3} = 0$	$P_{11,4} = 0$
$P_{11,5} = 0$	$P_{11,6} = 0$	$P_{11,7} = 0$	$P_{11,8} = 0$
$P_{11,9} = 0$	$P_{11,10} = 0$	$P_{11,11} = 1$	$P_{11,12} = 0$
$P_{11,13} = 0$	$P_{11,14} = 0$	$P_{11,15} = 0$	$P_{11,16} = 0$

Table AXI. Row 11

$P_{12,1} = 0$	$P_{12,2} = 0$	$P_{12,3} = 0$	$P_{12,4} = 0$
$P_{12,5} = \exp(-\lambda_2 h)(\beta_{\bar{X}_1})$	$P_{12,6} = \exp(-\lambda_2 h)\cdot g1(1 - \beta_{\bar{X}_1})$	$P_{12,7} = \exp(-\lambda_2 h)\cdot g2(1 - \beta_{\bar{X}_1})$	$P_{12,8} = \exp(-\lambda_2 h)\cdot g3(1 - \beta_{\bar{X}_1})$
$P_{12,9} = 0$	$P_{12,10} = 0$	$P_{12,11} = 0$	$P_{12,12} = 0$
$P_{12,13} = (1 - \exp(-\lambda_2 h))\cdot(\beta_{\bar{X}_3})$	$P_{12,14} = (1 - \exp(-\lambda_2 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{12,15} = (1 - \exp(-\lambda_2 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{12,16} = (1 - \exp(-\lambda_2 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AXII. Row 12

$P_{13,1} = 0$	$P_{13,2} = 0$	$P_{13,3} = 0$	$P_{13,4} = 0$
$P_{13,5} = 0$	$P_{13,6} = 0$	$P_{13,7} = 0$	$P_{13,8} = 0$
$P_{13,9} = 0$	$P_{13,10} = 0$	$P_{13,11} = 0$	$P_{13,12} = 0$
$P_{13,13} = (1 - \exp(-\lambda_1 h))\cdot(\beta_{\bar{X}_2})$	$P_{13,14} = g1(1 - \beta_{\bar{X}_3})$	$P_{13,15} = g2(1 - \beta_{\bar{X}_3})$	$P_{13,16} = g3(1 - \beta_{\bar{X}_3})$

Table AXIII. Row 13

$P_{14,1} = 0$	$P_{14,2} = 0$	$P_{14,3} = 0$	$P_{14,4} = 0$
$P_{14,5} = 0$	$P_{14,6} = 0$	$P_{14,7} = 0$	$P_{14,8} = 0$
$P_{14,9} = \exp(-\lambda_1 h)\cdot(\beta_{\bar{X}_2})$	$P_{14,10} = \exp(-\lambda_1 h)\cdot g1(1 - \beta_{\bar{X}_2})$	$P_{14,11} = \exp(-\lambda_1 h)\cdot g2(1 - \beta_{\bar{X}_2})$	$P_{14,12} = \exp(-\lambda_1 h)\cdot g3(1 - \beta_{\bar{X}_2})$
$P_{14,13} = (1 - \exp(-\lambda_1 h))\cdot(\beta_{\bar{X}_3})$	$P_{14,14} = (1 - \exp(-\lambda_1 h))\cdot g1(1 - \beta_{\bar{X}_3})$	$P_{14,15} = (1 - \exp(-\lambda_1 h))\cdot g2(1 - \beta_{\bar{X}_3})$	$P_{14,16} = (1 - \exp(-\lambda_1 h))\cdot g3(1 - \beta_{\bar{X}_3})$

Table AXIV. Row 14

Table AXV. Row 15

$P_{15,1} = 0$	$P_{15,2} = 0$	$P_{15,3} = 0$	$P_{15,4} = 0$
$P_{15,5} = \exp(-\lambda_2 h)(\beta_{\bar{X}_1})$	$P_{15,6} = \exp(-\lambda_2 h)\cdot g1(1-\beta_{\bar{X}_1})$	$P_{15,7} = \exp(-\lambda_2 h)\cdot g2(1-\beta_{\bar{X}_1})$	$P_{15,8} = \exp(-\lambda_2 h)\cdot g3(1-\beta_{\bar{X}_1})$
$P_{15,9} = 0$	$P_{15,10} = 0$	$P_{15,11} = 0$	$P_{15,12} = 0$
$P_{15,13} = (1 - \exp(-\lambda_2 h))\cdot(\beta_{\bar{X}_3})$	$P_{15,14} = (1 - \exp(-\lambda_2 h))\cdot g1(1-\beta_{\bar{X}_3})$	$P_{15,15} = (1 - \exp(-\lambda_2 h))\cdot g2(1-\beta_{\bar{X}_3})$	$P_{15,16} = (1 - \exp(-\lambda_2 h))\cdot g3(1-\beta_{\bar{X}_3})$

Table AXVI. Row 16

$P_{16,1} = 0$	$P_{16,2} = 0$	$P_{16,3} = 0$	$P_{16,4} = 0$
$P_{16,5} = 0$	$P_{16,6} = 0$	$P_{16,7} = 0$	$P_{16,8} = 0$
$P_{16,9} = 0$	$P_{16,10} = 0$	$P_{16,11} = 0$	$P_{16,12} = 0$
$P_{16,13} = 0$	$P_{16,14} = 0$	$P_{16,15} = 0$	$P_{16,16} = 1$

Appendix 2

Table AXVII. Row 1

$C_{1,1} = (C_0 h) + (a+bn)$	$C_{1,2} = (C_0 h) + (a+bn) + C_f$	$C_{1,3} = (C_0 h) + (a+bn) + C_f$	$C_{1,4} = (C_0 h) + (a+bn) + C_f$
$C_{1,5} = (C_0\tau_1 + (C_1(h-\tau_1))) + (a+bn)$	$C_{1,6} = (C_0\tau_1 + (C_1(h-\tau_1))) + (a+bn) + C_{sr}$	$C_{1,7} = (C_0\tau_1 + (C_1(h-\tau_1))) + (a+bn) + C_f$	$C_{1,8} = (C_0\tau_1 + (C_1(h-\tau_1))) + (a+bn) + C_{sr} + C_f$
$C_{1,9} = C_0\tau_2 + C_1(h-\tau_2) + (a+bn)$	$C_{1,10} = C_0\tau_2 + C_1(h-\tau_2)+(a+bn)+C_f$	$C_{1,11} = C_0\tau_2 + (C_1(h-\tau_2) + (a+bn) + C_{sr}$	$C_{1,12} = C_0\tau_2 + (C_1(h-\tau_2) + (a+bn) + C_{sr} + C_f$
$C_{1,13} = C_0\tau_{(1)} + (\frac{\lambda_1 C_1+\lambda_2 C_2}{\lambda_1+\lambda_2})\cdot(\tau_{(2)} - \tau_{(1)}) + C_{12}(h - \tau_{(2)}) + (a+bn)$	$C_{1,14} = C_0\tau_{(1)} + (\frac{\lambda_1 C_1+\lambda_2 C_2}{\lambda_1+\lambda_2})\cdot(\tau_{(2)} - \tau_{(1)}) + C_{12}(h - \tau_{(2)}) + (a+bn) + C_{sr}$	$C_{1,15} = C_0\tau_{(1)} + (\frac{\lambda_1 C_1+\lambda_2 C_2}{\lambda_1+\lambda_2})\cdot(\tau_{(2)} - \tau_{(1)}) + C_{12}(h - \tau_{(2)}) + (a+bn) + C_{sr}$	$C_{1,16} = C_0\tau_{(1)} + (\frac{\lambda_1 C_1+\lambda_2 C_2}{\lambda_1+\lambda_2})\cdot(\tau_{(2)} - \tau_{(1)}) + C_{12}(h - \tau_{(2)}) + (a+bn) + C_{sr}$

Table AXVIII. Row 2

$C_{2,1} = 0$	$C_{2,2} = 0$	$C_{2,3} = 0$	$C_{2,4} = 0$
$C_{2,5} = (C_1 h) + (a+bn)$	$C_{2,6} = (C_1 h) + (a+bn) + C_{sr}$	$C_{2,7} = (C_1 h) + (a+bn) + C_f$	$C_{2,8} = (C_1 h)+(a+bn)+C_{sr} + C_f$
$C_{2,9} = 0$	$C_{2,10} = 0$	$C_{2,11} = 0$	$C_{2,12} = 0$
$C_{2,13} = C_1\tau_2 > + C_{12}(h-\tau_2) + (a+bn)$	$C_{2,14} = C_1\tau_2 + C_{12}(h-\tau_2) + (a+bn)+C_{sr}$	$C_{2,15} = C_1\tau_2 + C_{12}(h-\tau_2) + (a+bn)+C_{sr}$	$C_{2,16} = C_1\tau_2 + C_{12}(h-\tau_2) + (a+bn)+C_{sr}$

Table AXIX. Row 3

$C_{3,1} = 0$	$C_{3,2} = 0$	$C_{3,3} = 0$	$C_{3,4} = 0$
$C_{3,5} = 0$	$C_{3,6} = 0$	$C_{3,7} = 0$	$C_{3,8} = 0$
$C_{3,9} = (C_2 h) + (a+bn)$	$C_{3,10} = (C_2 h) + (a+bn) + C_f$	$C_{3,11} = (C_2 h) + (a+bn) + C_{sr}$	$C_{3,12} = (C_2 h)+(a+bn)+C_{sr} + C_f$
$C_{3,13} = C_2\tau_1 + C_{12}(h-\tau_1) + (a+bn)$	$C_{3,14} = C_2\tau_1 + C_{12}(h-\tau_1) + (a+bn) + C_{sr}$	$C_{3,15} = C_2\tau_1 + C_{12}(h-\tau_1) + (a+bn) + C_{sr}$	$C_{3,16} = C_2\tau_1 + C_{12}(h-\tau_1) + (a+bn) + C_{sr}$

Table AXX. Row 4

$C_{4,1} = 0$	$C_{4,2} = 0$	$C_{4,3} = 0$	$C_{4,4} = 0$
$C_{4,5} = 0$	$C_{4,6} = 0$	$C_{4,7} = 0$	$C_{4,8} = 0$
$C_{4,9} = 0$	$C_{4,10} = 0$	$C_{4,11} = 0$	$C_{4,12} = 0$
$C_{4,13} = (C_{12} h) + (a+bn)$	$C_{4,14} = (C_{12} h)+(a+bn)+C_{sr}$	$C_{4,15} = (C_{12} h)+(a+bn)+C_{sr}$	$C_{4,16} = (C_{12} h)+(a+bn)+C_{sr}$

$C_{5,1} = 0$	$C_{5,2} = 0$	$C_{5,3} = 0$	$C_{5,4} = 0$
$C_{5,5} = (C_1h) + (a + bn)$	$C_{5,6} = (C_1h) + (a + bn) + C_{sr}$	$C_{5,7} = (C_1h) + (a + bn) + C_f$	$C_{5,8} = (C_1h)+(a+bn)+C_f + C_{sr}$
$C_{5,9} = 0$	$C_{5,10} = 0$	$C_{5,11} = 0$	$C_{5,12} = 0$
$C_{5,13} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn)$	$C_{2,14} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn) + C_{sr}$	$C_{5,15} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn) + C_{sr}$	$C_{5,16} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn) + C_{sr}$

Table AXXI. Row 5

$C_{6,1} = 0$	$C_{6,2} = 0$	$C_{6,3} = 0$	$C_{6,4} = 0$
$C_{6,5} = 0$	$C_{6,6} = C_{sr}$	$C_{6,7} = 0$	$C_{6,8} = 0$
$C_{6,9} = 0$	$C_{6,10} = 0$	$C_{6,11} = 0$	$C_{6,12} = 0$
$C_{6,13} = 0$	$C_{6,14} = 0$	$C_{6,15} = 0$	$C_{6,16} = 0$

Table AXXII. Row 6

$C_{7,1} = 0$	$C_{7,2} = 0$	$C_{7,3} = 0$	$C_{7,4} = 0$
$C_{7,5} = 0$	$C_{7,6} = 0$	$C_{7,7} = 0$	$C_{7,8} = (C_2h) + (a + bn)$
$C_{7,9} = 0$	$C_{7,10} = 0$	$C_{7,11} = 0$	$C_{7,12} = 0$
$C_{7,13} = C_{12}h + (a + bn)$	$C_{7,14} = C_{12}h + (a + bn) + C_{sr}$	$C_{7,15} = C_{12}h + (a + bn) + C_{sr}$	$C_{7,16} = C_{12}h + (a + bn) + C_{sr}$

Table AXXIII. Row 7

$C_{8,1} = 0$	$C_{8,2} = 0$	$C_{8,3} = 0$	$C_{8,4} = 0$
$C_{8,5} = 0$	$C_{8,6} = 0$	$C_{8,7} = 0$	$C_{8,8} = 0$
			$C_{8,12} = (C_2h)+(a+bn) + C_{sr}+ C_f$
$C_{8,9} = (C_2h) + (a + bn)$	$C_{8,10} = (C_2h) + (a + bn) + C_f$	$C_{8,11} = (C_2h) + (a + bn) + C_{sr}$	
$C_{8,13} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn)$	$C_{8,14} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$	$C_{8,15} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$	$C_{8,16} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$

Table AXXIV. Row 8

$C_{9,1} = 0$	$C_{9,2} = 0$	$C_{9,3} = 0$	$C_{9,4} = 0$
$C_{9,5} = 0$	$C_{9,6} = 0$	$C_{9,7} = 0$	$C_{9,8} = 0$
			$C_{9,12} = (C_2h)+(a+bn)+C_{sr} + C_f$
$C_{9,9} = (C_2h) + (a + bn)$	$C_{9,10} = (C_2h) + (a + bn) + C_f$	$C_{9,11} = (C_2h) + (a + bn) + C_{sr}$	
$C_{9,13} = (C_2\tau_1 + (C_{12}(h - \tau_1)) + (a + bn)$	$C_{9,14} = C_2\tau_1 + C_{12}(h - \tau_1)+ (a+bn)+C_{sr}$	$C_{9,15} = C_2\tau_1 + C_{12}(h - \tau_1)+ (a+bn)+C_{sr}$	$C_{9,16} = C_2\tau_1 + C_{12}(h - \tau_1)+ (a+bn)+C_{sr}$

Table AXXV. Row 9

$C_{10,1} = 0$	$C_{10,2} = 0$	$C_{10,3} = 0$	$C_{10,4} = 0$
$C_{10,5} = 0$	$C_{10,6} = 0$	$C_{10,7} = 0$	$C_{10,8} = 0$
$C_{10,9} = 0$	$C_{10,10} = 0$	$C_{10,11} = 0$	$C_{10,12} = 0$
$C_{10,13} = C_{12}h + (a + bn)$	$C_{10,14} = C_{12}h + (a + bn) + C_{sr}$	$C_{10,15} = C_{12}h + (a + bn) + C_{sr}$	$C_{10,16} = C_{12}h + (a + bn) + C_{sr}$

Table AXXVI. Row 10

$C_{11,1} = 0$	$C_{11,2} = 0$	$C_{11,3} = 0$	$C_{11,4} = 0$
$C_{11,5} = 0$	$C_{11,6} = 0$	$C_{11,7} = 0$	$C_{11,8} = 0$
$C_{11,9} = 0$	$C_{11,10} = 0$	$C_{11,11} = Csr$	$C_{11,12} = 0$
$C_{11,13} = 0$	$C_{11,14} = 0$	$C_{11,15} = 0$	$C_{11,16} = 0$

Table AXXVII. Row 11

Table AXXVIII. Row 12

$C_{12,1} = 0$	$C_{12,2} = 0$	$C_{12,3} = 0$	$C_{12,4} = 0$
$C_{12,5} = (C_1h) + (a + bn)$	$C_{12,6} = (C_1h) + (a + bn) + C_{sr}$	$C_{12,7} = (C_1h) + (a + bn) + C_f$	$C_{12,8} = (C_1h)+(a+bn)+C_f + C_{sr}$
$C_{12,9} = 0$	$C_{12,10} = 0$	$C_{12,11} = 0$	$C_{12,12} = 0$
$C_{12,13} = (C_1\tau_2 + (C_{12}(h - \tau_2)) + (a + bn)$	$C_{12,14} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$	$C_{12,15} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn) + C_{sr}$	$C_{12,16} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn) + C_{sr}$

Table AXXIX. Row 13

$C_{13,1} = 0$	$C_{13,2} = 0$	$C_{13,3} = 0$	$C_{13,4} = 0$
$C_{13,5} = 0$	$C_{13,6} = 0$	$C_{13,7} = 0$	$C_{13,8} = 0$
$C_{13,9} = 0$	$C_{13,10} = 0$	$C_{13,11} = 0$	$C_{13,12} = 0$
$C_{13,13}=C_{12}h+(a+bn)$	$C_{13,14} = C_{12}h + (a + bn) + C_{sr}$	$C_{13,15} = C_{12}h + (a + bn) + C_{sr}$	$C_{13,16} = C_{12}h + (a + bn) + C_{sr}$

Table AXXX. Row 14

$C_{14,1} = 0$	$C_{14,2} = 0$	$C_{14,3} = 0$	$C_{14,4} = 0$
$C_{14,5} = 0$	$C_{14,6} = 0$	$C_{14,7} = 0$	$C_{14,8} = 0$
$C_{14,9} = (C_2h) + (a + bn)$	$C_{14,10} = (C_2h) + (a + bn) + C_f$	$C_{14,11} = (C_2h) + (a + bn) + C_{sr}$	$C_{14,12} = (C_2h)+(a+bn)+C_{sr} + C_f$
$C_{14,13} = (C_2\tau_1 + (C_{12}(h - \tau_1)) + (a + bn)$	$C_{14,14} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$	$C_{14,15} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$	$C_{14,16} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$

Table AXXXI. Row 15

$C_{15,1} = 0$	$C_{15,2} = 0$	$C_{15,3} = 0$	$C_{15,4} = 0$
$C_{15,5} = (C_1h) + (a + bn)$	$C_{15,6} = (C_1h) + (a + bn) + C_{sr}$	$C_{15,7} = (C_1h) + (a + bn) + C_f$	$C_{15,8} = (C_1h)+(a+bn)+C_f + C_{sr}$
$C_{15,9} = 0$	$C_{15,10} = 0$	$C_{15,11} = 0$	$C_{15,12} = 0$
$C_{15,13} = C_1\tau_2 + (C_{12}(h - \tau_2) + (a + bn)$	$C_{15,14} = C_2\tau_1 + C_{12}(h - \tau_1) + (a + bn) + C_{sr}$	$C_{15,15} = C_1\tau_2 + C_{12}(h - \tau_2) + (a + bn) + C_{sr}$	$C_{15,16} = C_1\tau_2 + (C_{12}(h - \tau_2) + (a + bn) + C_{sr}$

Table AXXXII. Row 16

$C_{16,1} = 0$	$C_{16,2} = 0$	$C_{16,3} = 0$	$C_{16,4} = 0$
$C_{16,5} = 0$	$C_{16,6} = 0$	$C_{16,7} = 0$	$C_{16,8} = 0$
$C_{16,9} = 0$	$C_{16,10} = 0$	$C_{16,11} = 0$	$C_{16,12} = 0$
$C_{16,13} = 0$	$C_{16,14} = 0$	$C_{16,15} = 0$	$C_{16,16} = C_{sr}$