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OPTIMAL PROCESS CONTROL FOR MULTIPLE DEPENDENT SUBPROCESSES

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SUMMARY

A renewal theory approach is proposed to derive the cost model for multiple dependent subprocesses. The optimal individual Y control chart and multiple cause-selecting control chart are thus constructed to monitor the overall product quality and specific product quality contributed by the multiple dependent subprocesses. They can be used to maintain the process with minimum cost and effectively distinguish which component of the subprocesses is out of control. The optimal design parameters of the proposed control charts can be determined by minimizing the cost model using a simple grid search method. An example is given to illustrate the application of the optimal individual Y control chart and multiple cause-selecting control chart. ©1998 John Wiley & Sons, Ltd.

KEY WORDS: assignable causes; control charts; multiple dependent process steps; renewal reward processes

INTRODUCTION

Control charts are important tools of statistical quality control. These charts are used to decide whether a process has achieved a state of statistical control and to maintain current control of a process. Today, most products are produced by several different process steps. In multiple-step processes a Shewhart control chart is often used at each individual step. If the steps of the process are independent, then using a Shewhart control chart at each individual step is a meaningful procedure. However, in many processes the steps are not independent and thus the control charts are difficult to interpret. One way to solve this problem is to use a multivariate control chart such as a Hotelling T^2 chart. The disadvantages of using T^2 control charts are that one must assume that the process quality characteristics are multivariate normal random variables and, once an out-of-control signal is given, it is often difficult to determine which component of the process is out of control. An alternative to this approach was proposed by Zhang [1]. He calls his charts 'cause-selecting control charts'. The cause-selecting control chart is constructed for a variable only after the observations have been adjusted for the effect of some other random variables. Zhang's cause-selecting control charts use the concepts of overall quality and specific quality. Zhang defines overall quality as that quality due to the current subprocess and any previous subprocesses. Specific quality is that quality which is due only to the current subprocess. The cause-selecting charts are designed to further distinguish between controllable assignable causes and uncontrollable assignable causes. Controllable assignable causes are those assignable causes that affect the current subprocess but no previous subprocesses. Uncontrollable assignable causes are those assignable causes affecting previous processes that cannot be controlled at the current process level. The advantage of this approach is that once an out-of-control signal is given, it is often easy to determine which component of the subprocesses is out of control. Wade and Woodall [2] review the basic principles of the cause-selecting chart in the simple case of a two-step process and give an example to illustrate its use. They also examine the relationship between the cause-selecting chart and the multivariate T^2 chart. In their opinion the cause-selecting control chart has some advantages over the T^2 chart.

To use any control chart, three design parameters must be specified: the sample size, the sampling interval and the number of standard deviations above or below the centreline of the control chart. The choice of these design parameters influences the costs of sampling and testing, costs of searching and repairing and costs due to the production of non-conforming items. Therefore it is logical to consider the design of control charts from an economic viewpoint.

Duncan [3] first proposed an economic model for the optimal economic design of an $\bar{X}$ control chart. He recommended the use of a concept which he called economic design to obtain the optimal design. The pioneering work of Duncan was later extended by others to include $\bar{X}$ and R charts employed jointly [4–6]. Rahim *et al.* [7] discussed the use of joint $\bar{X}$ and S^2 charts according to economic considerations when sample sizes are moderately large. Collani and Sheil [8] proposed the economic design of an S chart when the assignable cause can only influence the process variance. Yang and Yang [9] first presented the economic design of a simple cause-selecting control chart for a system with a single assignable cause which is assumed to occur in either the current subprocess or the previous subprocess. However, the multiple assignable-cause cost model for multiple dependent subprocesses has not been addressed. In this paper we consider a system with multiple dependent subprocesses, i.e. the current subprocess and the multiple previous sub-

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processes, and each subprocess may be influenced by a single assignable cause. The optimal individual Y control chart proposed to monitor the overall product quality and the multiple cause-selecting control chart proposed to monitor the specific product quality are derived by minimizing a multiple assignable-cause cost model which is obtained by extending the renewal theory approach. Finally, an example is given to illustrate the design procedure and application of the optimal multiple cause-selecting control chart and individual Y control chart.

ECONOMIC DESIGN OF MULTIPLE CAUSE-SELECTING CONTROL CHART

Let $X_1, X_2, \ldots, X_k$ represent the incoming quality measurements of interest for the preceding k steps of the process and let Y (overall quality or outgoing quality) represent the quality measurement of interest for the $k+1$ (final) step. Suppose that a sample with size one is taken at the end of the final process every h hours and observations $(X_{1i}, X_{2i}, \ldots, X_{ki}, Y_i)$ are measured on the same item of production. To control the overall product quality contributed by the current subprocess and all the preceding processes, we have to use the individual Y control chart on the variable. If the outgoing quality to be controlled, Y, depends on k incoming qualities $X_1, X_2, \ldots, X_k$, then this is the cause-selecting case of multiple causes where we need to use the multiple cause-selecting control chart to control the specific quality resulting from the current subprocess itself .

The difference between the simple cause-selecting control chart and the multiple cause-selecting control chart is that the function between the outgoing quality to be controlled and the incoming qualities in the latter case is multiple, not simple. To determine the relationship between the object to be controlled and the multiple incoming qualities, we often use multiple linear regression. The multiple cause-selecting chart is then based on values of the outgoing quality Y that have been adjusted for the values of incoming qualities $X_1, X_2, \ldots, X_k$. The multiple cause-selecting control chart will be used in conjunction with the individual Y control chart to control the overall product quality and specific product quality simultaneously and may effectively distinguish which component of the multiple dependent processes is out of control. An example is given to illustrate the design procedure of the two control charts, and their application on the steps of a manufacturing process is also presented.

Multiple cause-selecting control chart

The procedure for constructing the individual Y control chart and multiple cause-selecting control chart is illustrated as follows.

Suppose that there are k incoming qualities $X_1, X_2, \ldots, X_k$; by experiment we have the m sets of observed data

$$(Y_i; X_{1i}, X_{2i}, \ldots, X_{ki}), \quad i = 1, 2, \ldots, m$$

The individual Y chart on the Y variable is constructed to monitor the overall product quality. The Y_i values are assumed independent and $Y_i \sim N(\mu, \sigma^2)$ when all the subprocesses are in control. The centreline (CL), upper control limit (UCL) and lower control limit (LCL) of the individual Y chart are set at μ, $\mu + k_1\sigma$ and $\mu - k_1\sigma$ respectively, where k_1 is the number of standard deviations above or below the centreline of the individual Y chart, μ is the mean of the random variable Y_i, and σ^2 is the variance of the random variable Y_i. Suppose that the overall quality Y is a function of k incoming qualities $X_1, X_2, \ldots, X_k$ and the Y_i values are independent, and the specific quality $Y_i|(X_1, X_2, \ldots, X_k) \sim N(\mu_i, \sigma^{*2})$ when the process is in control, where $\mu_i = f(X_{1i}, X_{2i}, \ldots, X_{ki})$ and σ^{*2} is a constant. Here μ_i is the mean of the random variable $Y_i|(X_1, X_2, \ldots, X_k)$ and σ^{*2} is the variance of the random variable $Y_i|(X_1, X_2, \ldots, X_k)$. Next we have to establish a relationship between $X_1, X_2, \ldots, X_k$ and Y, either empirically or theoretically. If the function $f(x_{1i}, x_{2i}, \ldots, x_{ki})$ was known, the transformation $Z_i = (Y_i - \mu_i)/\sigma^*$ would be used to standardize the Y_i values. The multiple cause-selecting chart is a Shewhart type of control chart for the cause-selecting values Z_i, the values of Y_i adjusted for the effects of $X_{1i}, X_{2i}, \ldots, X_{ki}$. Thus the Z_is are independent $N(0,1)$ random variables. The centreline, upper control limit and lower control limit for the multiple cause-selecting control chart are set at 0, k_2, and $-k_2$ respectively, where k_2 is the number of standard deviations above or below the centreline of the multiple cause-selecting chart. Alternatively, cause-selecting values could also be defined as $Y_i - \mu_i \sim N(0, \sigma^{*2})$. Thus the centreline, upper control limit and lower control limit for the multiple cause-selecting control chart are set at 0, $k_2\sigma^*$ and $-k_2\sigma^*$ respectively.

In practice, the true relationship between $X_1, X_2, \ldots, X_k$ and Y is never known. Hence the mean of $Y|(X_1, X_2, \ldots, X_k)$, $\mathrm{E}(Y|X_1, X_2, \ldots, Xk)$, and the variance of $Y|(X_1, X_2, \ldots, X_k)$, $\mathrm{V}(Y|X_1, X_2, \ldots, X_k)$, have to be estimated from an initial sample of m observations. The estimate for μ_i will be $\hat{Y}_i$, where $\hat{Y}_i$ is the fitted value of $\mathrm{E}(Y_i|(X_1, X_2, \ldots, X_k))$. The estimate for σ^* will be $\sqrt{\mathrm{MSE}}$, where $\sqrt{\mathrm{MSE}}$ is the square root of the mean square error. (For the model fitting methods and diagnosis see e.g. References [10] and [11]). Thus the upper and lower control limits of the optimal multiple cause-selecting chart are set at $k_2\sqrt{\mathrm{MSE}}$ and $-k_2\sqrt{\mathrm{MSE}}$ respectively for residuals e_i, where $e_i = Y_i - \hat{Y}_i$. Zhang [1] estimated σ^* using the average range of the residuals, $\overline{\mathrm{MR}}$, where $\overline{\mathrm{MR}} = \sum_{i=1}^{m-1} \mathrm{MR}_i/(m-1)$ and $\mathrm{MR}_i = |e_{i+1} - e_i|$. In this case $\mathrm{UCL} = k_2\overline{\mathrm{MR}}$, $\mathrm{CL} = 0$ and $\mathrm{LCL} = -k_2\overline{\mathrm{MR}}$.

Derivation of cost model

A current process is out of control when it is influenced by a controllable assignable cause, say A_{k+1}. We also assume that there are k uncontrollable assignable causes, say $A_1, A_2, A_3, \ldots, A_k$, which can only affect the preceding operations $1, 2, \ldots, k$ respectively. A preceding operation j is out of control when it is influenced by an uncontrollable assignable cause, say A_j, $j = 1, 2, \ldots, k$.

Assignable causes $A_1, A_2, A_3, \ldots, A_k$, and A_{k+1} would be allowed to occur in the first step, the second step, ... and the current step of the process simultaneously.

The distributions of the overall quality Y and specific quality $Y|(X_1, X_2, \ldots, X_k)$ would be changed once any assignable causes occurred in the process steps. Before describing the possible distributions of Y and $Y|(X_1, X_2, \ldots, X_k)$, we define some notation as follows:

- W the set of all assignable causes $A_1, A_2, A_3, \ldots, A_k, A_{k+1}$, $W = \{A_1, A_2, A_3, \ldots, A_k, A_{k+1}\}$
- W_1 the set of all uncontrollable assignanble causes $A_1, A_2, A_3, \ldots, A_k$, $W_1 = \{A_1, A_2, A_3, \ldots, A_k\}$
- S a subset of W
- S_1 a non-empty subset of W_1
- S_2 the set of assignable cause A_{k+1}, $S_2 = \{A_{k+1}\}$
- S_{12} the union of S_1 and S_2, $S_{12} = S_1 \cup S_2$

The general distribution of the overall quality, Y, can be expressed as $N(\mu_S, \sigma^2)$, but

(a) when $S = \emptyset, \mu_S = \mu$
(b) when $S = S_1, \mu_S = \mu + \delta_{S1}\sigma$ ($2^k - 1$ cases)
(c) when $S = S_2, \mu_S = \mu + \delta_{S2}\sigma$
(d) when $S = S_{12}, \mu_S = \mu + \delta_{S12}\sigma$ ($2^k - 1$ cases)

The most general distribution of the specific quality $Y|(X_1, X_2, \ldots, X_k)$ can be expressed as $N(\mu_S^*, \sigma^{*2})$, but

(a) when $S = \emptyset, \mu_S^* = \mu_i$
(b) when $S = S_1, \mu_S^* = \mu_i + \delta_{S1}^*\sigma^*$ ($2^k - 1$ cases)
(c) when $S = S_2, \mu_S^* = \mu_i + \delta_{S2}^*\sigma^*$
(d) when $S = S_{12}, \mu_S^* = \mu_i + \delta_{S12}^*\sigma^*$ ($2^k - 1$ cases).

Other assumptions and the nature of the operation condition are summarized as follows.

1. The time T_{Ai} until the occurrence of assignable cause A_i is assumed exponentially distributed with parameter $\lambda_i, i = 1, 2, \ldots, k+1$. $T_{A1}, T_{A2}, \ldots, T_{A(k+1)}$ are mutually independent.
2. The time of taking a sample, inspection and charting is negligible.
3. The search and repair time is a constant T_{srS} when the process is influenced by the assignable causes in set S. The search and repair time is a constant T_f when there is at least one false alarm for the two charts.
4. The search and repair cost is a constant C_{srS} when the process is influenced by the assignable causes in set S. The search and repair cost is a constant C_f when there is at least one false alarm for the two charts.
5. A quality cycle is defined as the time between the start of successive in-control periods. Then the process is expressed as a series of independent and identical cycles. That is, the process is a renewal process. The accumulated cost per cycle is called the cycle cost. The cycle costs are independent and identically distributed. Such a process is known as a renewal reward process [12].
6. The cost of sampling and testing is a constant b, where $b > 0$.
7. The process is discontinuous. That is, the process ceases during the search state.

The cost model is thus derived using the renewal theory approach. Some notation used is defined as follows:

- $E(T)$ the expected cycle time
- $E(C)$ the expected cycle cost
- α the probability that at least one of the multiple cause-selecting control chart and individual Y control chart has a false alarm.
- β_{S1} the probability that there is no true alarm for the individual Y chart and no false alarm for the multiple cause-selecting chart (a combination of uncontrollable assignable causes $A_1, A_2, A_3, \ldots, A_k$) (i.e. S_1) occurs in the system) (the number of such cases is $2^k - 1$)
- β_{S2} the probability that there are no true alarms for the individual Y chart and multiple cause-selecting chart given that the previous process steps are in control but the current process is out of control
- β_{S12} the probability that there are no alarms for both charts given that some previous process steps and the current process are out of control (the number of such cases is $2^k - 1$)
- T_{srS1} the time of search and repair of the assignable causes in set S_1
- T_{srS2} the time of search and repair of the assignable cause in set S_2
- T_{srS12} the time of search and repair of the assignable causes in set S_{12}
- C_0 the quality cost per hour while production is in control
- C_j the quality cost per hour while the process is only disturbed by any j assignable causes of all assignable causes, $j = 1, 2, \ldots, k+1$
- C_{srS1} the cost of search and repair of the assignable causes in set S_1
- C_{srS2} the cost of search and repair of the assignable cause in set S_2
- C_{srS12} the cost of search and repair of the assignable causes in set S_{12}
- $\tau_{(a',b')}$ the expected arrival time of the a'th arrived assignable cause given that all the assignable causes in set b' occur in the first sampling and testing time interval, i.e. $\tau_{(a',b')} = E(T_{(a')}|T_{(m)} < h)$, where $T_{(m)} = \max(T_{Aj}, A_j \in b')$ and $T_{(a')}$ is the arrival time of the a'th arrived assignable cause.

In order to obtain an expression for the expected cycle time $E(T)$, we decomposed the cycle into the following three components: (1) the in-control period; (2) the time to obtain a true alarm given that the process is out of control; (3) the time to find and repair all the assignable causes and start the process anew. To use the renewal theory approach, we have to study the possible states at the end of the first sampling and testing. There are 2^{k+2} possible states. Depending on the state of the system, one can compute the expected residual cycle length and expected residual cost. These values, together with the associated probabilities, lead us to formulate the renewal equation. The analysis developed below depends on the possible states at the end of the first sampling and testing. These states are defined as follows (Table 1).

Table 1. Definition of each state

State	Previous process steps in control?	Current process in control?	At least one alarm for cause-selecting chart and individual Y chart?
1	Yes	Yes	No
2	Yes	Yes	Yes
3 to 2^k+1	No	Yes	No
2^k+2 to 2^{k+1}	No	Yes	Yes
$2^{k+1}+1$	Yes	No	No
$2^{k+1}+2$	Yes	No	Yes
$2^{k+1}+3$ to 3.2^k+1	No	No	No
3.2^k+2 to 2^{k+2}	No	No	Yes

(a) State 1: the previous process steps and the current process are all in control and there are no false alarms for the individual Y chart and multiple cause-selecting chart.

(b) State 2: the previous process steps and the current process are all in control, but there is at least one false alarm for the charts.

(c) State 3 to state 2^k+1: some previous process steps are out of control because of the occurrence of the uncontrollable assignable causes in set S_1 and the current process is in control, but there are no alarms for the charts. The number of such states is 2^k-1.

(d) State 2^k+2 to state 2^{k+1}: some previous process steps are out of control because of the occurrence of the uncontrollable assignable causes in set S_1 and the current process is in control, and there are at least one alarm for the charts. The number of such states is 2^k-1

(e) State $2^{k+1}+1$: the previous process steps are in control and the current process is out of control because of the occurrence of the controllable assignable cause, but there are no alarms for the charts.

(f) State $2^{k+1}+2$: the previous process steps are in control but the current process is out of control because of the occurrence of the controllable assignable cause, and there is at least one alarm for the charts.

(g) State $2^{k+1}+3$ to state 3.2^{k+1}: some previous process steps are out of control because of the occurrence of the uncontrollable assignable causes in set S_1 and the current process is out of control because of the occurrence of the controllable assignable cause, but there are no true alarms for the charts. The number of such states is 2^k-1.

(h) State 3.2^k+2 to state 2^{k+2}: some previous process steps are out of control because of the occurrence of the uncontrollable assignable causes in set S_1 and the current process is out of control because of the occurrence of the controllable assignable cause, and there is at least one true alarm for the charts. The number of such states is 2^k-1.

The probability for each state is calculated as follows.

$$
\begin{aligned}
P_1 &= \mathrm{P}(T_{\mathrm{A}1} > h, T_{\mathrm{A}2} > h, \ldots, T_{\mathrm{A}(k+1)} > h)(1-\alpha) \\
&= \prod_{j=1}^{k+1} \mathrm{P}(T_{\mathrm{A}j} > h)(1-\alpha) \\
&= \exp\left(-\sum_{j=1}^{k+1} \lambda_j h\right) \times (1-\alpha)
\end{aligned}
$$

$$
\begin{aligned}
P_2 &= \mathrm{P}(T_{\mathrm{A}1} > h, T_{\mathrm{A}2} > h, \ldots, T_{\mathrm{A}(k+1)} > h)\alpha \\
&= \prod_{j=1}^{k+1} \mathrm{P}(T_{\mathrm{A}j} > h)\alpha = \exp\left(-\sum_{j=1}^{k+1} \lambda_j h\right)\alpha
\end{aligned}
$$

For state $i = 3, \ldots, 2^{k+1}$,

$$
\begin{aligned}
\mathrm{P}_i &= \mathrm{P}(T_{\mathrm{S}1} < h, T_{\mathrm{W}1-\mathrm{S}1} > h, T_{\mathrm{A}(k+1)} > h)\beta_{\mathrm{S}1} \\
&= \prod_{j \in S_1} [1-\exp(-\lambda_j h)] \\
&\quad \times \exp\left(-\sum_{j \in (W_1-S_1)\cup S_2} \lambda_j h\right)\beta_{\mathrm{S}1}
\end{aligned}
$$

where $(T_{\mathrm{S}1} < h)$ means that the arrival times of all the assignable causes in set S_1 are all smaller than h, i.e. $(T_{\mathrm{A}j} < h, A_j \in S_1)$, and $(T_{\mathrm{W}1-\mathrm{S}1} > h)$ means that the arrival times of all the assignable causes which are in W_1 but not in S_1 are all greater than h, i.e. $(T_{\mathrm{A}j} > h, A_j \in (W_1 - S_1))$. For state $i = 2^k+2, \ldots, 2^{k+1}$,

$$
\begin{aligned}
P_i &= \mathrm{P}(T_{\mathrm{S}1} < h, T_{\mathrm{W}1-\mathrm{S}1} > h, T_{\mathrm{A}(k+1)} > h)(1-\beta_{\mathrm{S}1}) \\
&= \prod_{j \in S_1} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (W_1-S_1)\cup S_2} \lambda_j h\right) \\
&\quad \times (1-\beta_{\mathrm{S}1})
\end{aligned}
$$

$$
\begin{aligned}
&P(2^{k+1}+1) \\
&= \mathrm{P}(T_{\mathrm{A}1} > h, T_{\mathrm{A}2} > h, \ldots, T_{\mathrm{A}k} > h, T_{\mathrm{A}(k+1)} < h)\beta_{\mathrm{S}2} \\
&= \exp\left(-\sum_{j=1}^{k} \lambda_j h\right)[1-\exp(-\lambda_{k+1} h)]\beta_{\mathrm{S}2}
\end{aligned}
$$

$$
\begin{aligned}
&P(2^{k+1}+2) \\
&= \mathrm{P}(T_{\mathrm{A}1} > h, T_{\mathrm{A}2} > h, \ldots, T_{\mathrm{A}k} > h, T_{\mathrm{A}(k+1)} < h) \\
&\quad \times (1-\beta_{\mathrm{S}2}) \\
&= \exp\left(-\sum_{j=1}^{k} \lambda_j h\right)[1-\exp(-\lambda_{k+1} h)](1-\beta_{\mathrm{S}2})
\end{aligned}
$$

For state $i = 2^{k+1}+3, \ldots, 3.2^k+1$,

$$
\begin{aligned}
P_i &= \mathrm{P}(T_{\mathrm{S}1} < h, T_{\mathrm{W}2-\mathrm{S}1} > h, T_{\mathrm{A}(k+1)} < h)\beta_{\mathrm{S}12} \\
&= \prod_{j \in S_{12}} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (W_1-S_1)} \lambda_j h\right) \\
&\quad \times \beta_{\mathrm{S}12}
\end{aligned}
$$

For state $i = 3.2^k+2, \ldots, 2^{k+2}$,

$$
\begin{aligned}
P_i &= \mathrm{P}(T_{\mathrm{S}1} < h, T_{\mathrm{W}1-\mathrm{S}1} > h, T_{\mathrm{A}(k+1)} < h)(1-\beta_{\mathrm{S}12}) \\
&= \prod_{j \in S_{12}} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (\mathrm{W}_1-\mathrm{S}_1)} \lambda_j h\right) \\
&\quad \times (1-\beta_{\mathrm{S}12})
\end{aligned}
$$

Table 2 displays the possible states of the system, the expected residual cycle times and the associated probabilities. Consequently,

$$
\mathrm{E}(T) = h + P_1 \mathrm{E}(T) + P_2(\mathrm{E}(T) + T_{\mathrm{f}}) + \sum_{i=3}^{2^{k+2}} P_i R_i .
$$

Table 2. Probability and expected residual cycle time for each state

State	Probability	Expected residual cycle time
1	$P_1 = \exp\left(-\sum_{j=1}^{k+1} \lambda_j h\right)(1-\alpha)$	$R_1 = \mathrm{E}(T)$
2	$P_2 = \exp\left(-\sum_{j=1}^{k+1} \lambda_j h\right)\alpha$	$R_2 = T_f + \mathrm{E}(T)$
3 to 2^k+1	$P_i = \prod_{j \in S_1} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (W_1 - S_1) \cup S_2} \lambda_j h\right) \beta_{S1}$	$R_i = h/(1-\beta_{S1}) + T_{srS1}$
2^k+2 to 2^{k+1}	$P_i = \prod_{j \in S_1} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (W_1 - S_1) \cup S_2} \lambda_j h\right)(1-\beta_{S1})$	$R_i = T_{srS1}$
$2^{k+1}+1$	$P_i = \exp\left(-\sum_{j=1}^{k} \lambda_j h\right)[1-\exp(-\lambda_{k+1} h)]\beta_{S2}$	$R_i = h/(1-\beta_{S2}) + T_{srS2}$
$2^{k+1}+2$	$P_i = \exp\left(-\sum_{j=1}^{k} \lambda_j h\right)[1-\exp(-\lambda_{k+1} h)](1-\beta_{S2})$	$R_i = T_{srS2}$
$2^{k+1}+3$ to 3.2^k+1	$P_i = \prod_{j \in S_{12}} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (W_1 - S_1)} \lambda_j h\right)(\beta_{S12})$	$R_i = h/(1-\beta_{S12}) + T_{srS12}$
3.2^k+2 to 2^{k+2}	$P_i = \prod_{j \in S_{12}} [1-\exp(-\lambda_j h)] \exp\left(-\sum_{j \in (W_1 - S_1)} \lambda_j h\right)(1-\beta_{S12})$	$R_i = T_{srS12}$

Simplifying this, we get

$$\mathrm{E}(T) = \frac{h + P_2 T_f}{1 - P_1 - P_2} + \frac{\sum_{i=3}^{2^{k+2}} P_i R_i}{1 - P_1 - P_2} \qquad (1)$$

In order to obtain an expression for the expected cycle cost ($\mathrm{E}(C)$), we decomposed the cycle cost into the following two components: (1) the cost incurred in the first sampling and testing; (2) the expected residual cost, which is the cost incurred from the time that the process is influenced by any one assignable cause until all occurred assignable causes are repaired. We present the possible states of the system, the costs incurred in the first sampling and testing and the expected residual costs in Table 3. Consequently,

$$\begin{aligned}\mathrm{E}(C) \quad &= \quad P_1[(b + C_0 h) + \mathrm{E}(C)] \\ &\quad + P_2[(b + C_0 h) + \mathrm{E}(C) + C_f] + \sum_{i=3}^{2^{k+2}} P_i R_i'\end{aligned}$$

Simplifying this, we get

$$\mathrm{E}(C) = \frac{(P_1 + P_2)(b + C_0 h) + P_2 C_f}{1 - P_1 - P_2} + \frac{\sum_{i=3}^{2^{k+2}} P_i R_i'}{1 - P_1 - P_2} \quad (2)$$

Applying the property of the renewal reward process [12], the objective function (the expected cost per unit time) $\mathrm{E}(V_\infty)$ is derived by taking the ratio of the expected cycle cost $\mathrm{E}(C)$ and the expected cycle time $\mathrm{E}(T)$: $\mathrm{E}(V_\infty) = \mathrm{E}(C)/\mathrm{E}(T)$. The objective function is a function of the design parameters h, k_1 and k_2. Hence the optimal design parameters of the proposed control charts can be determined by minimizing the objective function.

Application of Optimal Multiple Cause-Selecting Control Chart and Individual Y Control Chart

We illustrate the application of the optimal multiple cause-selecting control chart and individual Y control chart in this subsection. Suppose that the approximate optimal values h^*, k_1^* and k_2^* have been obtained using an optimization technique. That is, the upper and lower control limits of the optimal individual Y chart are set at $\mu + k_1^*\sigma$ and $\mu - k_1^*\sigma$ (if μ and σ are unknown, we use $\bar{Y}$ (sample mean) and S (sample standard deviation) to estimate them) respectively for the plotted statistic Y_i. The upper and lower control limits of the optimal multiple cause-selecting chart are set at k_2^* and $-k_2^*$ respectively for the plotted statistic Z_i, or set at $k_2^*\sqrt{\mathrm{MSE}}$ and $-k_2^*\sqrt{\mathrm{MSE}}$ respectively for residuals e_i. To monitor the process states, every h^* hours a sample with size one $(X_{1i}, X_{2i}, X_{3i}, \ldots, Y_i)$ is taken and tested. There are three possible test results for the multiple process steps. These outcomes and the associated various actions are given in Table 4. Combination 1 means that all the previous and current process steps are in control, so the process continues and the next sample is taken after h^* hours. Combination 2 means that some previous process steps are out of control but the current process is in control, hence the process has to be stopped and the preceding process steps need to be check and repaired. Combination 3 may mean that the previous process steps are in control but the current process is out of control, hence the process has to be stopped and the current process needs to be check and repaired; or some previous process steps and the current process are out of control, hence the process

Table 3. Cost for each state

State	Cost in first sampling and testing + [expected residual cost]
1	$R'_1 = b + C_0 h + [\mathrm{E}(C)]$
2	$R'_2 = b + C_0 h + [C_\mathrm{f} + \mathrm{E}(C)]$
3 to 2^k+1	$R'_i = b + C_0\tau_{(1,\mathrm{S1})} + \sum_{j=2}^{m} C_{j-1}(\tau_{(j,\mathrm{S1})} - \tau_{(j-1,\mathrm{S1})}) + C_m(h - \tau_{(m,\mathrm{S1})}) + [hC_m/(1-\beta_{\mathrm{S1}}) + C_{\mathrm{srS1}}]$
2^k+2 to 2^{k+1}	$R'_i = b + C_0\tau_{(1,\mathrm{S1})} + \sum_{j=2}^{m} C_{j-1}(\tau_{(j,\mathrm{S1})} - \tau_{(j-1,\mathrm{S1})}) + C_m(h - \tau_{(m,\mathrm{S1})}) + [C_{\mathrm{srS1}}]$
$2^{k+1}+1$	$R'_i = b + C_0\tau_{(1,\mathrm{S2})} + C_1(h - \tau_{(1,\mathrm{S2})}) + [hC_1/(1-\beta_{\mathrm{S2}}) + C_{\mathrm{srS2}}]$
$2^{k+1}+2$	$R'_i = b + C_0\tau_{(1,\mathrm{S2})} + C_1(h - \tau_{(1,\mathrm{S2})}) + [C_{\mathrm{srS2}}]$
$2^{k+1}+3$ to 3.2^k+1	$R'_i = b + C_0\tau_{(1,\mathrm{S12})} + \sum_{j=2}^{m+1} C_{j-1}(\tau_{(j,\mathrm{S12})} - \tau_{(j-1,\mathrm{S12})}) + C_{m+1}(h - \tau_{(m+1,\mathrm{S12})}) + [hC_{m+1}/(1-\beta_{\mathrm{S12}}) + C_{\mathrm{srS12}}]$
3.2^k+2 to 2^{k+2}	$R'_i = b + C_0\tau_{(1,\mathrm{S12})} + \sum_{j=2}^{m+1} C_{j-1}(\tau_{(j,\mathrm{S12})} - \tau_{(j-1,\mathrm{S12})}) + C_{m+1}(h - \tau_{(m+1,\mathrm{S12})}) + [C_{\mathrm{srS12}}]$

Note: m is the number of assignable causes in S_1 and $m+1$ is the number of assignable causes in S_{12}.

Table 4. Decision rules

Combination	Individual Y chart signal?	Cause-selecting chart signal?	Action process stops?
1	No	No	No
2	Yes	No	Yes search and repair S_1
3	Yes	Yes	Yes search and repair A_{k+1} or search and repair S_1 and A_{k+1}

has to be stopped and the previous process steps and the current process need to be checked and repaired.

The multiple cause-selecting chart is used in conjunction with the individual Y chart for the multiple dependent process steps. We find that they can distinguish the uncontrollable assignable causes and controllable assignable cause effectively.

AN EXAMPLE

A two-dependent-step process is performed to illustrate the approach proposed. Let X represent the incoming quality measurements of interest for the preceding step of the process and let Y (overall quality) represent the quality measurement of interest for the current (final) step. Suppose that a sample with size one is taken at the end of the final process every h hours and observations (X_i, Y_i) are measured on the same item of production. A process is out of control when it is influenced by assignable causes. We assume that there are an uncontrollable assignable cause and a controllable assignable cause, say A_1 and A_2. A_1 can only affect the previous process and cannot be controlled at the current process.

Table 5. Data and residual values for example

No.	X_i	Y_i	$\hat{Y}_i$	$e_i = Y_i - \hat{Y}_i$
1	85	99	91.772	7.22803
2	82	93	91.552	1.44847
3	75	99	95.289	3.71065
4	74	97	95.625	1.37534
5	76	90	94.811	−4.81102
6	74	96	95.625	0.37534
7	73	93	95.77	−2.77026
8	93	109	110.029	−1.02874
9	70	88	94.602	−6.60206
10	82	89	91.552	−2.55153
11	80	93	92.401	0.59898
12	77	94	94.236	−0.23634
13	82	86	91.552	−5.55153
14	76	91	94.811	−3.81102
15	74	100	95.625	4.37534
16	71	98	95.306	2.69436
17	70	101	94.602	6.39794
18	64	80	80.842	−0.84194

A_2 can only affect the current process but no previous subprocess. We adopt 18 sets of observed data from Reference [13], p. 835 (see Table 5).

The variables measured were X = fibre length (10^{-2} inch) for the preceding step and Y = skein length (lb) for the final step. The 18 data points are used to establish the relationship between the two variables and to calculate the control limits, since these points are obtained when the process is in control.

The relationship between X and Y is found using least squares regression with Y as the dependent variable and X as the independent variable. The least squares fit obtained is

$$\hat{Y}_i = -3558 + 142X - 1.82X^2 + 0.00778X^3$$

The next step is to calculate the residuals for the 18 observations. The residuals are given in Table 5.

Next the control limits of the cause-selecting chart for the residual e_i, and the individual Y chart are calculated.

The centreline and control limits of the simple cause-selecting chart are

$$\begin{aligned} \text{UCL} &= k_2\sqrt{\text{MSE}} = 4.36k_2 \\ \text{CL} &= 0 \\ \text{LCL} &= -k_2\sqrt{\text{MSE}} = -4.36k_2 \end{aligned}$$

The centreline and control limits of the individual Y chart are

$$\begin{aligned} \text{UCL} &= \bar{Y} + k_1 S = 94.22 + 6.58k_1 \\ \text{CL} &= \bar{Y} \\ \text{LCL} &= \bar{Y} - k_1 S = 94.33 - 6.58k_1, \end{aligned}$$

where $\bar{Y}$ is the average of the 18 Y_is and S is the standard deviation of the 18 observations.

Let $\beta_{S2} = \beta_{01}$, $\beta_{S1} = \beta_{10}$, $\beta_{S12} = \beta_{11}$, $T_{srS1} = T_{srS2} = T_{sr}$, $C_{srS1} = C_{srS2} = C_{sr}$, $\tau_{(1,\{A_1\})} = \tau_1$, $\tau_{(2,\{A_2\})} = \tau_2$, $\tau_{(1,\{A_1,A_2\})} = \tau_{(1)}$, $\tau_{(2,\{A_1,A_2\})} = \tau_{(2)}$, $\delta_{S1} = \delta^*_{S1} = \delta_1$, $\delta_{S2} = \delta^*_{S2} = \delta_2$ and $\delta_{S12} = \delta^*_{S12} = \delta_{12}$, where:

β_{01} the probability that there are no alarms for the charts given that previous process is in control and the current process is out of control

β_{10} the probability that there are no alarms for the charts given that the previous process is out of control and the current process is in control

β_{11} the probability that there are no alarms for the charts given that the previous process and the current process are both out of control

T_{sr} the time of search and repair of any assignable causes when there is at least one true alarm for the individual Y chart and cause-selecting chart

C_{sr} the cost of search and repair of any assignable causes when there is at least one true alarm for the two charts

τ_j the expected arrival time of assignable cause A_j given that it occurred in the first sampling and testing interval, $j = 1, 2$, i.e.

$$\tau_j = \text{E}(T_{Aj} | T_{Aj} < h) = \frac{1 - e^{-\lambda_j h} - \lambda_j h\, e^{-\lambda_j h}}{\lambda_j(1 - e^{-\lambda_j h})}$$

$\tau_{(j)}$ the expected arrival time of the jth arrived assignable cause given that A_1 and A_2 occurred in the first sampling and testing interval, $j = 1, 2$, i.e.

$$\begin{aligned} \tau_{(1)} = {} & \{e^{-(\lambda_1+\lambda_2)h}[h + 1/\lambda_1 + 1/\lambda_2 - 1/(\lambda_1+\lambda_2)] \\ & -e^{-\lambda_1 h}/\lambda_2 - e^{-\lambda_2 h}/\lambda_1 + 1/(\lambda_1+\lambda_2)\} \\ & /[(1-e^{-\lambda_1 h})(1-e^{-\lambda_2 h})] \\ \tau_{(2)} = {} & \{e^{-(\lambda_1+\lambda_2)h}[h + 1/(\lambda_1+\lambda_2)] - e^{-\lambda_1 h}/(h+1/\lambda_1) \\ & -e^{-\lambda_2 h}(h+1/\lambda_2) + 1/\lambda_1 + 1/\lambda_2 - 1/(\lambda_1+\lambda_2)\} \\ & /[(1-e^{-\lambda_1 h})(1-e^{-\lambda_2 h})] \end{aligned}$$

(for proofs see Reference [14])

δ_1 the mean shift size of δ_1 standard deviation when the preceding process is influenced by assignable cause A_1

δ_2 the mean shift size of δ_2 standard deviation when the current process is influenced by assignable cause A_2

Table 6. Definition of each state

State	Previous process steps in control?	Current process in control?	At least one alarm for cause-selecting chart and individual X chart?
1	Yes	Yes	No
2	Yes	Yes	Yes
3	No	Yes	No
4	No	Yes	Yes
5	Yes	No	No
6	Yes	No	Yes
7	No	No	No
8	No	No	Yes

δ_{12} the mean shift size of δ_{12} standard deviation when the current process is influenced by assignable cause A_2 and the preceding process is influenced by assignable cause A_1.

It is noted that the numerical calculations for α, β_{01}, β_{10} and β_{11} are not easy because the individual Y control chart and multiple cause-selecting control chart are not independent. We do know that $\alpha > \alpha_1 + \alpha^*_1 - \alpha_1\alpha^*_1$, $\beta_{10} < \beta_1(1-\alpha^*_1)$, $\beta_{01} < \beta_2\beta^*_2$ and $\beta_{11} < \beta_{12}\beta^*_2$, where α_1 is the probability that the individual Y chart has a false alarm, α^*_1 is the probability that the multiple cause-selecting chart has a false alarm, β_1 is the probability that the individual Y chart has no true alarm given that the overall quality is influenced by A_1, β^*_2 is the probability that the multiple cause-selecting chart has no true alarm given that the specific quality is influenced by A_2, β_2 is the probability that the individual Y chart has no true alarm given that the overall quality is influenced by A_2, and β_{12} is the probability that the individual Y chart has no true alarm given that the overall quality is influenced by A_1 and A_2. To simplify the calculation, we let $\alpha = \alpha_1 + \alpha^*_1 - \alpha_1\alpha^*_1$, $\beta_{10} = \beta_1(1-\alpha^*_1)$, $\beta_{01} = \beta_2\beta^*_2$ and $\beta_{11} = \beta_{12}\beta^*_2$. Without losing generality, the optimal design parameters for the proposed control charts with required powers will be obtained.

There are eight possible states at the end of the first sampling and testing time. Table 6 gives the definitions of the eight states. The probability and expected residual cycle time for each state and the cost for each state are presented in Tables 7 and 8 respectively.

Before determining the optimal values of k_1, k_2 and sampling interval h, we apply equations (1) and (2) to obtain the expected cost per hour for the simple case of a two-step process. Hence the expected cycle time is

$$\begin{aligned} \text{E}(T) = {} & [h + e^{-(\lambda_1+\lambda_2)h}\alpha T_f + (1 - e^{-(\lambda_1+\lambda_2)h})T_{sr} \\ & +(1-e^{-\lambda_1 h})\, e^{-\lambda_2 h} h\beta_{10}/(1-\beta_{10}) \\ & +e^{-\lambda_1 h}(1-e^{-\lambda_2 h})h\beta_{01}/(1-\beta_{01}) \\ & +(1-e^{-\lambda_1 h})(1-e^{-\lambda_2 h})h\beta_{11}/(1-\beta_{11})] \\ & /[1-e^{-(\lambda_1+\lambda_2)h}] \end{aligned} \quad (3)$$

The expected cycle cost is

$$\begin{aligned} \text{E}(C) = {} & \{b + C_0 h\, e^{-(\lambda_1+\lambda_2)h} + e^{-(\lambda_1+\lambda_2)h}\alpha C_f \\ & +(1-e^{-(\lambda_1+\lambda_2)h})C_{sr} + [C_0\tau_1 + C_1(h-\tau_1)] \end{aligned}$$

Table 7. Probability and expected residual cycle time for each state

State	Probability	Expected residual cycle time
1	$P_1 = e^{-\lambda_1 h} e^{-\lambda_2 h}(1-\alpha)$	$R_1 = E(T)$
2	$P_2 = e^{-\lambda_1 h} e^{-\lambda_2 h}\alpha$	$R_2 = T_f + E(T)$
3	$P_3 = (1-e^{-\lambda_1 h}) e^{-\lambda_2 h}\beta_{10}$	$R_3 = h/(1-\beta_{10}) + T_{sr}$
4	$P_4 = (1-e^{-\lambda_1 h}) e^{-\lambda_2 h}(1-\beta_{10})$	$R_4 = T_{sr}$
5	$P_5 = e^{-\lambda_1 h}(1-e^{-\lambda_2 h})\beta_{01}$	$R_5 = h/(1-\beta_{01}) + T_{sr}$
6	$P_6 = e^{-\lambda_1 h}(1-e^{-\lambda_2 h})(1-\beta_{10})$	$R_6 = T_{sr}$
7	$P_7 = (1-e^{-\lambda_1 h})(1-e^{-\lambda_2 h})\beta_{11}$	$R_7 = h/(1-\beta_{11}) + T_{sr}$
8	$P_8 = (1-e^{-\lambda_1 h})(1-e^{-\lambda_2 h})(1-\beta_{11})$	$R_8 = T_{sr}$

Table 8. Cost for each state

State	Cost in first sampling and testing + [expected residual cost]
1	$R'_1 = b + C_0 h + [E(C)]$
2	$R'_2 = b + C_0 h + [C_f + E(C)]$
3	$R'_3 = b + C_0\tau_1 + C_1(h-\tau_1) + [hC_1/(1-\beta_{10}) + C_{sr}]$
4	$R'_4 = b + C_0\tau_1 + C_1(h-\tau_1) + [C_{sr}]$
5	$R'_5 = b + C_0\tau_2 + C_1(h-\tau_2) + [hC_1/(1-\beta_{01}) + C_{sr}]$
6	$R'_6 = b + C_0\tau_2 + C_1(h-\tau_2) + [C_{sr}]$
7	$R'_7 = b + C_0\tau_{(1)} + (\tau_{(2)} - \tau_{(1)})(C_1\lambda_1 + C_1\lambda_2)/(\lambda_1+\lambda_2) + C_2(h-\tau_{(2)}) + [hC_2/(1-\beta_{11}) + C_{sr}]$
8	$R'_8 = b + C_0\tau_{(1)} + (\tau_{(2)} - \tau_{(1)})(C_1\lambda_1 + C_1\lambda_2)/(\lambda_1+\lambda_2) + C_2(h-\tau_{(2)}) + [C_{sr}]$

$$\begin{aligned}&\times(1-e^{-\lambda_1 h})e^{-\lambda_2 h}\\ &+(1-e^{-\lambda_1 h})e^{-\lambda_2 h} h\beta_{10}C_1/(1-\beta_{10})\\ &+[C_0\tau_2 + C_1(h-\tau_2)](1-e^{-\lambda_2 h})e^{-\lambda_1 h}\\ &+(1-e^{-\lambda_2 h})e^{-\lambda_1 h} h\beta_{01}C_1/(1-\beta_{01})\\ &+[C_0\tau_{(1)} + (\tau_{(2)}-\tau_{(1)})C_1 + C_2(h-\tau_{(2)})]\\ &\times(1-e^{-\lambda_2 h})(1-e^{-\lambda_1 h})\\ &+(1-e^{-\lambda_1 h})(1-e^{-\lambda_2 h})h\beta_{11}C_2/(1-\beta_{11})\}\\ &/(1-e^{-(\lambda_1+\lambda_2)h}) \qquad (4)\end{aligned}$$

Consequently, the expected cost per hour is the ratio of the expected cycle cost and the expected cycle time: $E(V_\infty) = E(C)/E(T)$.

Suppose that the combination of artifical cost and process parameters is ($\delta_1 = 3.1$, $\delta_2 = 2.8$, $\delta_{12} = 3.5$, $\lambda_1 = 0.01$, $\lambda_2 = 0.05$, $b = 5$, $T_{sr} = 0.8$, $T_f = 0.2$, $C_0 = 5$, $C_1 = 10$, $C_2 = 25$, $C_f = 30$, $C_{sr} = 50$). In the process of obtaining the approximate optimal values h^*, k_1^* and k_2^*, we treat h, k_1 and k_2 as discrete variables and assume that the values of h, k_1 and k_2 are within the ranges 0.0–8.0 ($0 < h \le 8.0$ and the unit length of h is 0.1), 0.0–4.0 ($0 < k_1 \le 4.0$, $0 < k_2 \le 4.0$ and the unit lengths of k_1 and k_2 are 0.1) respectively. We also add constraints ($\alpha \le 0.1$, $\beta_{10} < 0.2$, $\beta_{01} < 0.2$, $\beta_{11} < 0.2$) to the model, because in many economic designs the probability of type I error of control charts is much higher than that in a statistical design, which will result in more false alarms than expected [15], and we also hope that the powers of control charts are as required. The algorithm used to obtain the approximate values h^*, k_1^* and k_2^* of the design variables h, k_1 and k_2 is the simple grid search method. Consequently, $h^* = 8.0$, $k_1^* = 2.2$, $k_2^* = 1.8$, $E(V_\infty) = 5.697$, $\alpha^* = 0.0977$, $\beta_{10}^* = 0.1708$, $\beta_{01}^* = 0.1542$ and $\beta_{11}^* = 0.0292$. That is, the upper and lower control limits of the optimal individual Y chart should be set at 109.70 and 79.74 respectively. The upper and lower control limits of the optimal cause-selecting chart should be set at 7.85 and −7.85 respectively. To monitor the process states, every 8 h a sample with size one (X_i, Y_i) is taken and tested. There are three possible test results for the two subprocesses. These outcomes and the associated various actions are given in Table 9. Combination 1 means that Y_i falls inside the control limits of the individual Y chart and the cause-selecting value e_i also falls inside the cause-selecting chart, so the process continues and the next sample is taken after 8 h. Interpretations for combinations 2 and 3 are similar to that for combination 1.

Table 9. Decision rules

Combination	Individual Y chart signal?	Cause-selecting chart signal?	Action process stops?
1	No	No	No
2	Yes	No	Yes search and repair A_1
3	Yes	Yes	Yes search and repair A_2 or search and repair A_1 and A_2

CONCLUSIONS

The multiple cause-selecting chart can be used in conjunction with the individual Y chart for multiple dependent process steps. They can effectively distinguish between the uncontrollable assignable causes and the controllable assignable cause. A method of designing the optimal multiple cause-selecting chart and individual Y chart simultaneously has been proposed.

Constraints on powers and type I error probability are allowed in the economic design of control charts. This can be viewed as an improvement in economic design while achieving desirable statistical properties.

In practice, if engineers wish to maintain processes with minimum cost and desired statistical properties and to determine effectively which component of the subprocesses is out of control, then use of the optimal cause-selecting control chart and individual Y control chart is preferable.

The method proposed can be extended to the case of multiple assignable causes occurring in both the current process and previous process steps.

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