The Effects of Imprecise Measurement on the Economic Asymmetric $\overline{X}$ and S Control Charts

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Abstract

The presence of imprecise measurement may seriously affect the efficiency of process control and production cost. A cost model is derived to determine the design parameters of the economic asymmetric $\overline{X}$ and S control charts including measurement errors. The effects of imprecise measurement on the performance of the economic asymmetric $\overline{X}$ and S control charts and production cost are examined for the case where the process mean and process standard deviation may change. Application of the proposed control charts is demonstrated through an example. Numerical examples illustrate the effects of imprecise measurement on the design parameters of the proposed control charts. It shows that the imprecision measurement may seriously affect the ability of the proposed control charts to detect process disturbances quickly, change the sampling frequency, and increase the production cost compared to the control charts excluding measurement errors.

Key words: Asymmetric control charts, measurement error, renewal reward processes.

1. Introduction

Process measurements are used in construction of control charts. The performance of control charts and other statistical process control tools could be seriously affected when the process measurement includes the error due to the measurement instrument.

The effect of measurement error on the operating characteristics of an $\overline{X}$ chart, in cases where only the process mean shifts, is discussed by Bennett (1954), Mizuno (1961), Abraham(1977), Mittag (1993) and Mittag and Stemmann (1993). Kanazuka (1986) and Mittag (1995) investigate the power characteristics of the $\overline{X}$-R control chart where both the process mean and process spread change. Mittag and Stemann (1998) extend the results of Mittag (1995), referring to the $\overline{X}$-S control chart. Rahim (1985) analysis the effects of imprecise measurement devices on the design parameters of the economic $\overline{X}$ control chart. This paper investigates the effect of measurement error on the design parameters of the economic asymmetric $\overline{X}$ and S control charts.

2. Problem Statement

A possible industrial situation is taken to illustrate the application of the proposed control charts. A production process may be in control or out of control. If a process is influenced by any special cause then the process is out of control, otherwise the process is in control. Suppose that the interested product quality characteristics is represented by process variable X, and its process measurement includes the error due to the measurement instrument. In the production system, suppose that there is a machine could be out of control, and thus shifts the process mean upward or downward, and also changes process variance. Suppose that a process engineer would like to develop appropriate control charts to monitor the process mean and variance effectively and economically. The problem here is what is the process control policy under the measurement error? That is, what are the control charts and their design parameters (the sampling interval time, control coefficients of the control charts and the sample size), how is their detecting ability, how the measurement error affects the design parameters of the proposed control charts, and how much is the process cost under the measurement error.

The solutions of these problems are to construct the economic $\overline{X}$ and S control charts to detect the shifts of process mean and variance, calculate and analysis the changes of powers and design parameters of the proposed control charts for different measurement dispersion.

3. Process Description

In this paper, we assume that the process mean and variance may shift when a single special cause occurs in the process. The measurement process is considered, and it has a variance for a measurement device is employed for later measurements. The production process model is described under the following assumptions.

3.1 Assumptions and Notation

(1) It is assumed that the process variable X_T has a normal distribution with mean μ and variance σ^2 when the process is in statistical control. That is, when the process is in control , $X_T \sim N(\mu, \sigma^2)$.

(2) One special cause, say AC, many occur in the process and change the process mean to $\mu+\delta_1\sigma$ and the variance to $\delta_2^2\sigma^2$, with probability ω, where, $\delta_1 \neq 0$, $\delta_2 > 1$, and $\omega \neq 0.5$, or change the process mean

to μ-δ_1*σ and the variance to $\delta_2{}^{*2}\sigma^2$, δ_1*$\neq$0, δ_2*>1, with probability 1-ω. That is, when the process is influenced by AC,

X_T~N(μ+$\delta_1\sigma$, $\delta_2{}^2\sigma^2$) with probability ω, where $\omega\neq0.5$.

X_T~N(μ-δ_1*σ, $\delta_2{}^{*2}\sigma^2$) with probability 1-ω.

(3) The time (T_{AC}) until the AC occurs in the process is assumed exponentially distributed with parameter $\frac{1}{\lambda}$.

(4) It is assumed that the measurement process has a variance $\delta_e{}^2\sigma^2$, $\delta_e\neq0$, for a measurement device is employed for later measurements. That is, the distribution of the measurement error (ε) is illustrated as ε~N(0, $\delta_e{}^2\sigma^2$). Hence, the distribution of the process quality variable (X) with measurement error (ε) is illustrated as follows, when the process is in control.

X= X_T +ε ~ N(μ, $\delta_e{}^2$+1)σ^2).

(5) To monitor the process state with minimum cost and effectively, the control limits of the economic $\overline{X}$ and S control charts are determined by the statistical control process. Thus the data used for establishing the limits on the control charts comes form the distribution X~N(μ, ($\delta_e{}^2$+1)σ^2). Since the AC may not shift the process mean to right or left with equal chance, so we consider the asymmetric $\overline{X}$ and S charts. Their control limits are

$$UCL_{\bar{X}} = \mu + k_1\sqrt{\frac{(\delta_e^2+1)\sigma^2}{n}},$$

$$LCL_{\bar{X}} = \mu + k_2\sqrt{\frac{(\delta_e^2+1)\sigma^2}{n}}$$

and $UCL_S = k_3\sqrt{(\delta_e^2+1)\sigma^2}$,

respectively. When the process shifts, the measurement data is assumed to come from an N(μ+$\delta_1\sigma$, ($\delta_e{}^2$+$\delta_2{}^2$)σ^2) population with probability ω, or an N(μ -δ_1*σ, ($\delta_e{}^2$+$\delta_2{}^{*2}$)σ^2) population with probability 1-ω, $\omega\neq0.5$.

(6) We assume that the process is composed of independent and identically distributed cycles (Figure 1).

(7) A cycle is composed of in-control time, out-of-control time and search and repair time until the next starting cycle (Figure 2).

(8) To monitor the process state with minimum cost, a sample of size n is taken and the values of the plotted statistics $\overline{X}$ and S are plotted on the constructed economic $\overline{X}$ and S control charts every h hours.

(9) The time of sampling and testing is small and negligible.

(10) Once at least one of the plotted points falls outside of the control limits of the proposed control charts, the quality engineer has to stop the process to search and remove the AC; otherwise, no action is required and the process continues.

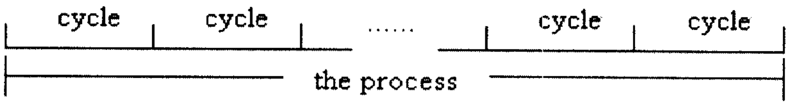

Figure 1. The Process Denoted by Cycles

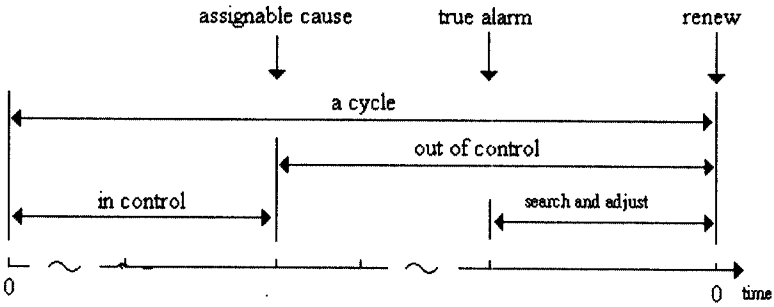

Figure 2. The Complete Cycle

The notation used is described as follows.

$1-\beta_{\bar{x}}$= the probability that the economic $\overline{X}$ chart gives a true alarm.

$1-\beta_s$= the probability that the economic S chart gives a true alarm.

$\alpha_{\bar{X}}$= the probability that the economic $\overline{X}$ chart gives a false alarm.

α_s= the probability that the economic S chart gives a false alarm.

T_{AC}= the time until AC occurs in the process.

T_s= the average time of search a false alarm.

T_{sr}= the average time of search and repair AC.

a= the fixed cost of sampling and testing.

b= the cost per unit sampled and test .

C_o= the average cost per unit production time when the process is in control.

C_1= the average cost per unit production time when the process shifts.

C_s= the average cost of search a false alarm.

C_{sr}= the average cost of search and repair AC.

τ = The average arriving time of AC, given that AC occurs in the fist sampling and testing time h. That is, $\tau = E(T_{AC}|T_{AC} < h) = h(0.5-\lambda h/12)$(see Montgomery 1982).

3.2 The Cost Model

The renewal theory approach is used to derive the process cost model of the economic $\overline{X}$ and S control charts. The process cost model is the function of design parameters (n, h, k1, k2, k3) for the two proposed control charts. Since the production process is expressed as renewal reward

Table 1. Definition for 8 possible states on time h

state	AC occurs?	Alarms?		Probability
		$\overline{X}chart$	S chart	
1	No	No	No	$P_1 = e^{-\lambda h}(1-\alpha_{\overline{X}})(1-\alpha_s)$
2	No	Yes	No	$P_2 = e^{-\lambda h}(\alpha_{\overline{X}})(1-\alpha_s)$
3	No	No	Yes	$P_3 = e^{-\lambda h}(1-\alpha_{\overline{X}})(\alpha_s)$
4	No	Yes	Yes	$P_4 = e^{-\lambda h}(\alpha_{\overline{X}})(\alpha_s)$
5	Yes	No	No	$P_5 = (1-e^{-\lambda h})(\beta_{\overline{X}})(\beta_s)$
6	Yes	Yes	No	$P_6 = (1-e^{-\lambda h})(1-\beta_{\overline{X}})(\beta_s)$
7	Yes	No	Yes	$P_7 = (1-e^{-\lambda h})(\beta_{\overline{X}})(1-\beta_s)$
8	Yes	Yes	Yes	$P_8 = (1-e^{-\lambda h})(1-\beta_{\overline{X}})(1-\beta_s)$

Note: Derivations of $\alpha_{\overline{X}}, \alpha_S, 1-\beta_{\overline{X}}$, and $1-\beta_S$ see Appendix I.

processes, applying its property, the cost model is derived by taking the ratio of the expected cycle cost and the expected cycle time (Ross (1993)). The expected cycle time (ET) includes the expected in-control time, the expected out-of-control time, the expected time of search false alarms, and the expected time of search and repair the AC, or equivalently, the expected cycle time is the sum of the first sampling and testing time (h) and the expected residual cycle time. The expected residual cycle time is the time after h before the process is renew. The expected cycle cost (EC) includes the cost of sampling and testing, production cost, the cost of search false alarms, and the cost of search and repair AC. The expected cycle cost is defined as the expected cumulative cost occurred in the cycle time, or equivalently, the expected cycle cost is the sum of cost occurred in time h and the expected residual cost. The expected residual cost is the expected cumulative cost occurred in the residual cycle time.

To derive the expected cycle time and the expected cycle cost, we have to study the possible states at the end of the first sampling and testing time h. Depending on the states of the system, one can compute the expected residual cycle time and the expected residual cost. These values, together with the associated probability of being in each respective state, lead us to formulate the renewal equation. The analysis developed below depends on the possible states at the end of the first sampling and testing time. There are eight possible states on time h, its

Table 2. The expected residual time and expected residual cost for each state

state	expected residual cycle time	expected residual cost
1	$l_1 = ET$	$R_1 = a + bn + c_0 h + EC$
2	$l_2 = ET + T_s$	$R_2 = a + bn + c_0 h + EC + c_s$
3	$l_3 = ET + T_s$	$R_3 = a + bn + c_0 h + EC + c_s$
4	$l_4 = ET + T_s$	$R_4 = a + bn + c_0 h + EC + c_s$
5	$l_5 = \frac{1}{(1-\beta_{\bar{X}}\beta_s)} + T_{sr}$	$R_5 = a + bn + c_0\tau + c_1(h-\tau) + \frac{(a+bn+c_1 h)}{(1-\beta_{\bar{X}}\beta_s)} + c_{sr}$
6	$l_6 = T_{sr}$	$R_6 = a + bn + c_0\tau + c_1(h-\tau) + c_{sr}$
7	$l_7 = T_{sr}$	$R_7 = a + bn + c_0\tau + c_1(h-\tau) + c_{sr}$
8	$l_8 = T_{sr}$	$R_8 = a + bn + c_0\tau + c_1(h-\tau) + c_{sr}$

definition and probability are illustrated in Table 1. The expected residual cycle time and expected residual cost for each state are described in Table 2.

The renewal equation for ET is

$$\mathrm{ET} = h + \sum_{i=1}^{8} l_i P_i = h + P_1 ET + P_2 ET + P_3 ET + P_4 ET + (P_2 + P_3 + P_4)T_s + \sum_{i=5}^{8} l_i P_i$$

$$= h + E(T)(P_1 + P_2 + P_3 + P_4) + (P_2 + P_3 + P_4)T_s + \sum_{i=5}^{8} l_i P_i$$

$$= h + (P_2 + P_3 + P_4)T_s + \sum_{i=5}^{8} l_i P_i + E(T)(P_1 + P_2 + P_3 + P_4) \tag{1}$$

Consequently, the expected cycle time is

$$ET = \frac{h + (P_2 + P_3 + P_4)T_s + \sum_{i=5}^{8} l_i P_i}{(1 - P_1 - P_2 - P_3 - P_4)} \tag{2}$$

The renewal equation for EC is

$$EC = \sum_{i=1}^{8} P_i R_i = (P_1 + P_2 + P_3 + P_4)(a + bn + c_0 h + EC) + (P_2 + P_3 + P_4)C_s + \sum_{i=5}^{8} P_i R_i \tag{3}$$

Consequently, the expected cycle cost is

$$(1 - P_1 - P_2 - P_3 - P_4)EC = (P_2 + P_3 + P_4)C_s + (P_1 + P_2 + P_3 + P_4)(a + bn + C_0 h) + \sum_{i=5}^{8} P_i R_i$$

$$EC = \frac{(P_2 + P_3 + P_4)C_s + (P_1 + P_2 + P_3 + P_4)(a + bn + C_0 h) + \sum_{i=5}^{8} P_i R_i}{(1 - P_1 - P_2 - P_3 - P_4)} \tag{4}$$

The cost model (EV) is the ratio of the expected cycle cost and the expected cycle time; EV=EC/ET, which is the function of the design parameters (n, h, k1, k2, k3) of the economic $\overline{X}$ and S control charts. Minimize the cost model using optimization technique, the optimum design parameters of the $\overline{X}$ and S control charts can be determined. Consequently, the $\overline{X}$ and S control charts are constructed to monitor the measurement process with minimum cost.

3.3 Numerical Example

To know the effect of measurement error on the design parameters of the economic asymmetric $\overline{X}$ and S control charts. We assume that the process and cost parameters estimated from process quality department and finance department are $\delta_1 = 1.5, \delta_1^* = 2.0, \delta_2 = 1.5, \delta_2^* = 2.5, \lambda = 0.01$, $C_1 = 20$, $\mu = 0, \sigma = 1.0$, $a = 1$ (dollars), $b = 0.1$, $w = 0.3$, $T_s = 0.1$, $T_{sr} = 0.4$, $\delta_e^2 = 1$, $C_s = 20$, $C_{sr} = 40, C_0 = 10$. The estimated variance of measurement process is employed before the control charts are set up. Put these values in the derived cost model and minimum the cost model using Fortran program with IMSL BDCONF subroutine, the optimal design parameters and minimum cost are obtained as follows; n=12, h=6.75, k1=2.52, k2=2.77, k3=1.61, and EV=11.08 (dollars). That is, the control limits and central limit of the proposed $\overline{X}$ and S control charts are $(UCL_{\overline{X}} = 0.73, CL_{\overline{X}} = 0, LCL_{\overline{X}} = -0.8)$, and $UCL_S = 1.61$, respectively. To monitor the process mean and variance with minimal cost, a sample with size 12 is taken from the process QC station every 6.75 hours and plotted statistics $(\overline{X}, S)$ are calculated and plotted on the proposed $\overline{X}$ and S charts. When at least one of the sample $\overline{X}$ or S falls outside of the proposed control charts, the process engineer has to check and remove the special cause; otherwise, the process continues and the next sample is taken after 6.75 hours. The in-control average run length (ARL_I) of the proposed control charts is 11.11, and the out-of-control average run length (ARL_O) of the proposed control charts is 1.00. The detection ability of the proposed control charts is good when the process is out of control, and it only takes 11.08 dollars to monitor the production process.

If we consider that the measurement error occurs after the proposed control charts are set up, then the control limits of the $\overline{X}$ and S charts are set at $\text{UCL}_{\overline{X}} = \mu + k_1\sqrt{\frac{\delta^2}{n}}$, $\text{LCL}_{\overline{X}} = \mu - k_2\sqrt{\frac{\delta^2}{n}}$ and $\text{UCLs} = k_3\sqrt{\delta^2}$, respectively. All other situation is the same as described before, then the optimal design parameters are n=12, h=6.74, k1=3.62, k2=3.78, k3=3.28, and EV=11.08 (dollars). Its ARL_I is 86.58, and ARL_O is 1.04.

Compare their results, we find that their EV, n and h are almost same, but k1, k2, and k3 for the economic $\overline{X}$ and S control charts including measurement error are much smaller than those of the economic $\overline{X}$ and S control charts excluding measurement error. It indicates that the detection ability for the economic $\overline{X}$ and S control charts excluding measurement error is much better.

4. Results Comparison for Control Charts Constructed without Imprecise Measurement and with Imprecise Measurement

To understand the effect of measurement error on the design parameters of the proposed control charts, we change the estimated variance (δ_e^2) of measurement error from 0.5 to 2.5. For the $\overline{X}$ and S control charts constructed without imprecise measurement, the optimal solutions, optimal values and average run lengths are illustrated in Table 3. We may find that k1, k2, k3, h, EV, and ARL_O all increase and ARL_I decreases when δ_e^2 increases. It means the widths of control limits of the proposed control charts would be larger when the variation of measurement error becomes larger. It leads to poor detection ability for the process control. It also shows that the measurement error has significant effect on the design parameters of the proposed control charts and ARL_I. Consequently, the detection ability of the control charts is significantly influenced by the measurement error of the measurement device.

When the proposed $\overline{X}$ and S control charts are set up with imprecise measurement error, from Table 4, we find that n, k1, k2 and decrease, but h, k3, EV, and ARL_O increase, when δ_e^2 increases. It means the widths of the control limits of the proposed $\overline{X}$ control charts would be smaller, but the widths of the control limits the proposed S control chart would be wider when the variation of measurement error becomes larger. It leads to poor detection ability of the proposed control charts. The measurement error has significant effect on the design parameters of the proposed $\overline{X}$ and S control charts and in-control average run length.

Comparing the results between the proposed $\overline{X}$ and S control charts including imprecise and excluding imprecise measurement error, we find that n, k1, k2 and ARL_I in the former are much smaller than those

Table 3. The optimum solutions and values for different values of δ_e^2 (The proposed $\overline{X}$ and S control charts excluding measurement error)

δ_e^2	n	h	k1	k2	k3	EV	ARL_1	ARL_0
0.5	10	6.40	3.20	3.39	2.08	11.04	106.56	1.04
1.5	14	7.04	3.92	4.09	2.45	11.12	68.49	1.04
2.5	18	7.59	4.49	4.65	2.73	11.19	52.36	1.05

Table 4. The optimum solutions and values for different values of δ_e^2
(The proposed $\overline{X}$ and S control charts including measurement error)

δ_e^2	n	h	k1	k2	k3	EV	ARL_1	ARL_0
0.5	10	6.38	2.65	2.76	1.7	11.04	111.86	1.04
1.5	8	8	2.12	0.10	3.43	12.13	2.09	1.10
2.5	8	8	1.9	0.10	3.49	12.19	2.05	1.12

in the latter, but h and k3 are much larger. There is no obvious difference for their EV and ARL_O. It indicates that the operator takes longer time to take a smaller sample of size and easier to have incorrect detection results on the process, when the proposed $\overline{X}$ and S control charts including imprecise measurement error.

5. Summary

Measurement errors may occur in the measurement device. The effects of the measurement error on the design parameters of the proposed economic asymmetric $\overline{X}$ and S control charts are discussed. Numerical examples illustrate that the larger process measurement spread results in much smaller sampling size, the widths of control limits, in-control average run length for the proposed control charts including measurement errors. However, the larger measurement spread results in much larger sampling size, the sampling interval, the widths of control limits, but smaller in-control average run length for the proposed control charts excluding measurement errors. Compare the detection ability and cost between the proposed control charts including measurement error and excluding measurement error, the former has much smaller ARL_I and larger cost than the latter. It indicates that the proposed control charts including measurement errors is easier to have incorrect detection results on the production process.

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Appendix

Derivation of $\alpha_{\overline{X}}$, α_S, $1-\beta_{\overline{X}}$, and $1-\beta_S$

The distribution of the plotted statistic with error when the process is in control

$\overline{X} \sim N(\mu, (1+\delta_e^2)\sigma^2)$

$$\alpha_{\overline{X}} = P\left(\overline{X} > \mu + K_1\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}}\right) + P\left(\overline{X} < \mu - K_2\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}}\right)$$

$$= P\left(\frac{\overline{X}-\mu}{\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}}} > K_1\right) + P(Z_2 < -K_2)$$

$$= P(Z_2 > K_1) + P(Z_2 < -K_2)$$

$$\alpha_s = P\left(S > K_3\sqrt{(1+\delta_e^2)\sigma^2}\right)$$

$$= P\left(\frac{(n-1)s^2}{(1+\delta_e^2)\sigma^2} > (n-1)K_3^2\right)$$

$$= P\left(\chi_2^2 > (n-1)K_3^2\right)$$

The distribution of the plotted statistic with error when the process shifts

$X \sim N(\mu+\delta_1\sigma, (\delta_e^2+\delta_2^2)\sigma^2)$ w.p ω,

$X \sim N(\mu-\delta_1^*\sigma, (\delta_e^2+\delta_2^{*2})\sigma^2)$ w.p. $1-\omega$.

$$1-\beta_{\bar{x}} = WP\left(\overline{X} > \mu + K_1\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}}, \overline{X} < \mu - K_2\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}} \mid X \sim N(\mu+\delta_1\sigma, (\delta_e^2+\delta_2^2)\sigma^2)\right) +$$

$$(1-W)P\left(\overline{X} > \mu + K_1\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}}, \overline{X} < \mu - K_2\sqrt{\frac{(1+\delta_e^2)\sigma^2}{n}} \mid X \sim N(\mu-\delta_1^*\sigma, (\delta_e^2+\delta_2^2)\sigma^2)\right)$$

$$= W\left(P\left(\frac{\overline{X}-(\mu+\delta_1\sigma)}{\sqrt{\frac{(\delta_e^2+\delta_2^2)\sigma^2}{n}}} > \frac{-\delta_1\sqrt{n}+K_1\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right) + P\left(\frac{\overline{X}-(\mu+\delta_1\sigma)}{\sqrt{\frac{(\delta_e^2+\delta_2^2)\sigma^2}{n}}} < \frac{-\delta_1\sqrt{n}-K_2\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right)\right)$$

$$+(1-W)\left(P\left(\frac{\overline{X}-(\mu-\delta_1^*\sigma)}{\sqrt{\frac{(\delta_e^2+\delta_2^2)\sigma^2}{n}}} > \frac{\delta_1^*\sigma+K_1\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right) + P\left(\frac{\overline{X}-(\mu-\delta_1^*\sigma)}{\sqrt{\frac{(\delta_e^2+\delta_2^2)\sigma^2}{n}}} < \frac{\delta_1^*\sqrt{n}-K_2\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right)\right)$$

$$= W\left(P\left(Z_3 > \frac{-\delta_1\sqrt{n}+K_1\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right) + P\left(Z_3 < \frac{-\delta_1\sqrt{n}-K_2\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right)\right) +$$

$$(1-W)\left(P\left(Z_3^* > \frac{\delta_1^*\sqrt{n}+K_1\sqrt{(1+\delta_e^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right) + P\left(Z_3^* < \frac{\delta_1^*\sqrt{n}-K_2\sqrt{(1+\delta_2^2)}}{\sqrt{(\delta_e^2+\delta_2^2)}}\right)\right)$$

$$
\begin{aligned}
1-\beta_s &= WP\left(S > K_3\sqrt{\left(1+\delta_e^2\right)\sigma^2} \mid X \sim N\left(\mu+\delta_1\sigma,\left(\delta_e^2+\delta_2^2\right)\sigma^2\right)\right)+ \\
&\quad (1-W)P\left(S > K_3\sqrt{\left(1+\delta_e^2\right)\sigma^2} \mid N\left(\mu-\delta_1\sigma,\left(\delta_e^2+\delta_2^{*2}\right)\sigma^2\right)\right) \\
&= WP\left(\frac{(n-1)s^2}{\left(\delta_e^2+\delta_2^2\right)\sigma^2} > \frac{(n-1)K_3^2\left(1+\delta_e^2\right)}{\left(\delta_e^2+\delta_2^2\right)}\right)+(1-W)P\left(\frac{(n-1)s^2}{\left(\delta_e^2+\delta_2^{*2}\right)\sigma^2} > \frac{(n-1)K_3^2\left(1+\delta_e^2\right)\sigma^2}{\left(\delta_e^2+\delta_2^{*2}\right)\sigma^2}\right) \\
&= WP\left(X_3^2 > \frac{(n-1)K_3^2\left(1+\delta_e^2\right)}{\left(\delta_e^2+\delta_2^2\right)}\right)+(1-W)P\left(X_3^{*2} > \frac{(n-1)K_3^2\left(1+\delta_e^2\right)}{\left(\delta_e^2+\delta_2^{*2}\right)}\right)
\end{aligned}
$$

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