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Exclusion theorem in an efficiency wage model

Chung-cheng Lin[1], **Chao-cheng Mai**[2], **Ching-chong Lai**[3]

[1] Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan
and Department of Public Finance, National Cheng-Chi University,
Taiwan (e-mail: cclin@econ.sinica.edu.tw)
[2] Department of Industrial Economics, Tamkang University, Tamsui, Taipei 251, Taiwan
[3] Sun Yat-Sen Institute for Social Sciences and Philosophy, Academia Sinica,
Taipei 115, Taiwan

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Abstract. This paper uses Shapiro and Stiglitz's (1984) efficiency wage model embodying the firm's choice of location to show the existence of an optimal intermediate location without assuming a transport rate that increases with distance. Based on the viewpoint of Shapiro and Stiglitz, we demonstrate that the more time that the worker spends traveling to the plant, the higher will be the wage that the firm will need to pay to motivate the worker not to shirk. To avoid paying a higher wage, the firm may choose its optimal location at an intermediate rather than a polar location.

JEL classification: J41, R3

1. Introduction

A well-known component of the location theory of the firm, dubbed the "exclusion theorem," has attracted the attention of many economists (for example, Sakashita 1967; Mathur 1979; Higano 1985). The theorem states that when a profit-maximizing or cost-minimizing firm chooses its optimal location between the input and the output markets along a line, there is no possibility of an intermediate location under the reasonable assumption that transport rates are constant or decreasing with distance. Since many firms do not locate at the end-points in the real world, this is a puzzle. Faced with this difficulty, it is not surprising that many studies (e.g., Mai and Shieh 1984, 1991; Hwang and Mai 1990), when exploring the impacts of changes in the output demand (or price) on the firm's optimal location, need to rely on an increasing transport rate assumption to ensure the existence of an intermediate location.[1] A corresponding question is thus how to explain the existence

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[1] One exception is Mai and Hwang (1994). They take into account the conjectural variations of two rival firms and find that an optimal intermediate location is possible.

of such an intermediate location without the analysis being based on the increasing rate assumption. A possible solution will be provided in this paper.

Most existing location literature have made a critical assumption, whereby workers in spatial labor markets are fully employed and paid a wage that is determined competitively.[2] As a matter of fact, involuntary unemployment appears to be a persistent feature of many modern labor markets. A question that naturally arises is whether or not the exclusion theorem will still hold in the presence of involuntary unemployment. To help answer this, this paper sets up an involuntary unemployment model that embodies the firm's optimal choice of location to explore the robustness of the exclusion theorem.

The involuntary unemployment model we use is a shirking model of efficiency wages. The basic tenet of the efficiency wage theory is that worker's effort or productivity is positively related to his or her wage. An increase in the wage will raise the cost (higher wage) as well as the benefit (higher productivity). It may therefore be profitable for firms not to cut wages in the presence of involuntary unemployment. The main justifications that have been provided for the positive relationship between labor productivity and the wage include nutritional concerns (Leibenstein 1957), gift-exchange (Akerlof 1982), adverse selection (Weiss 1980), and the shirking problem (Shapiro and Stiglitz 1984).[3] Of these, the most popular version has been the shirking viewpoint proposed by Shapiro and Stiglitz (1984) and others. Its essential feature is that firms cannot precisely observe the efforts of workers due to incomplete information. This information problem forces the firm to pay a wage that exceeds the worker's opportunity cost to prevent shirking. Equilibrium unemployment is thereby a necessary device for disciplining workers.[4]

In this paper we examine the exclusion theorem along the lines of the shirking models. Unlike the existing literature, this paper provides a possible solution to the puzzle without abandoning the reasonable assumption of a constant (or decreasing) transport rate with distance. By using the classic shirking model of Shapiro and Stiglitz (1984), we show that the longer the workers' traveling time to their workplace is, the higher will be the wage that

[2] The contributions of Zenou and Smith (1995) and Smith and Zenou (1997) are two exceptions.

[3] Excellent surveys of the efficiency wage literature have been provided by Yellen (1984), Akerlof and Yellen (1986), and Katz (1986).

[4] However, the shirking viewpoint is also the most controversial version of the efficiency wage theory. As pointed out by Akerlof and Yellen (1986, pp. 3–4), shirking models suffer from a serious theoretical criticism since employment contracts (e.g., bonding, deferred wage payments, and tournaments) that are more ingenious than the simple wage schemes considered can reduce or eliminate involuntary unemployment. In response to the critics, Dickens (1989, 1990) argues that by requiring workers to post large bonds or submit to other forms of punishment, firms could virtually eliminate monitoring expenditures. Yet, the prediction that firms should not monitor workers fails dramatically as an empirical proposition. The empirical evidence shows that a sizeable fraction of the monitoring of employees by employers is directed at deterring worker malfeasance. Pervasive monitoring at the workplace implies that a more ingenious employment contract cannot completely eliminate the need to pay a premium wage to detect shirking. See Carmichael (1985) and Shapiro and Stiglitz (1985) for a debate on the possibility of bonding schemes; see also Carmichael (1990) and Lang and Kahn (1990) for two contrasting views on the issue. For further discussion, please see Eaton and White (1982), Ritter and Taylor (1994), and Lin and Yang (2001).

the firm needs to pay to discipline its workers. To avoid paying an increasingly high wage, a firm may hence locate its plant at an intermediate point, rather than at an end-point.

The rest of the paper is organized as follows. Section 2 presents our main result based on the basic shirking model of Shapiro and Stiglitz (1984). Some extensions are discussed in Sect. 3. Concluding remarks are provided in the final section.

2. A basic model

The virtue of this model lies in the extreme simplicity with which it captures the effect of the asymmetric information problem regarding the worker's effort (the moral hazard problem) on the firm's location decisions. Consider a very simple location model where a representative firm hires a number of identical workers to produce a single commodity.[5] All workers together reside in one city and the output can only be sold in the other city. To enter the market, the firm has to choose its plant location between the two cities.[6] Since workers prefer shirking to working and the firm cannot accurately observe work effort, there must be some incentive devices to prevent shirking. The shirking-type efficiency wage models emphasize that firms will pay premium wages and fire those who are caught shirking.

In what follows, the shirking model of Shapiro and Stiglitz (1984), which embodies the firm's choice of location, is developed to illustrate the possibility of an intermediate optimal location under the assumption of a constant transport rate.

2.1. The worker

The worker enjoys consuming goods by spending income and dislikes putting forth effort and traveling between home and the workplace. For simplicity, the utility function of the worker is assumed to be $u(w, e; x, \lambda) = w^{1/2} - e - x$, where w is the wage, or equivalently consumption, e is effort, and x is the time spent on traveling.[7] In line with Shapiro and Stiglitz (1984), the levels of effort can take only two values: $e = 0$ (shirking) or $e = 1$ (no-shirking). Let the

[5] The commodity market is supposed to be competitive and firms are price-takers and have no strategic behavior.

[6] One may wonder why the workers do not inhabit the city where the output is sold since; after all, workers are consumers. In this most simple and imaginary economy, let us assume that the output market is located next to a harbor. Firms deliver their products to the buyers at the harbor who pay the delivery costs. This is a very restricted example in the real world. Fortunately, our insights would hold in a more complex model where i) two or more inputs are considered, and ii) the workers and the output market are located in the same city and other inputs are available in the other city.

[7] Our purpose is to show the possibility of an optimal intermediate location without assuming that the transport rate increases with distance. Therefore, the utility function is specified to be as simple as possible to ensure that the first-and second-order conditions in the basic model in this section and in the extended models in the next section are all satisfied. For further discussions, please see Footnote 13.

traveling time per unit of distance be normalized to 1, and thus x also stands for the distance from the worker's house to the plant. Given the fact that more traveling time implies less leisure time, the worker's utility function is specified to be negatively related to the traveling time x.

There is an instantaneous probability b that a worker will quit his or her job for exogenous reasons. If the worker shirks and is caught, he or she will be fired. The instantaneous probability of a shirking worker being caught and fired is q. A no-shirking worker can certainly keep his or her job, i.e. no-shirking workers will never be caught shirking by error. Workers who are caught shirking will be fired and enter the unemployment pool. Before they are rehired by another firm, the original firm will pay them $\bar{w}$ as unemployment benefits. Workers who are not caught shirking will be paid w, the same as other non-shirkers.

A worker who is employed will choose either to shirk or not. Let subscript E represent the status of being employed, and superscripts S and N represent the status of shirking and no-shirking, respectively. The expected utility of an employed agent who chooses to shirk (not to shirk) is denoted by V_E^S (V_E^N). As shown in Shapiro and Stiglitz (p. 436) and Romer (1996, pp. 452–454), regular dynamic optimization should sustain the following two asset equations:[8]

$$rV_E^S = w^{1/2} - x + (b+q)(V_u - V_E^S), \tag{1}$$

$$rV_E^N = w^{1/2} - e - x + b(V_u - V_E^N), \tag{2}$$

where r is the discount rate and V_u is the indirect utility of an unemployed agent. A worker who is at work and contemplating whether or not to shirk has already incurred traveling time associated with work. This is the reason why the same term x is deducted in both equations. Treating expected utility as the asset value corresponding to a given state, the above two equations then state that the rate of return times the asset value equals dividends plus expected capital gains (or losses).

Workers will choose not to shirk only if the expected lifetime utility of being a non-shirker is no smaller than that of being a shirker. That is, a condition referred to as the no-shirking condition (NSC) must be fulfilled as:

$$V_E^N \geq V_E^S.$$

For an agent who is in the state of unemployment, he or she has an instantaneous probability a of reobtaining a job. Similarly, we can show that the corresponding expected utility of an unemployed agent, V_u, exhibits the following relationship:

$$rV_u = \bar{w} + a(V_E - V_u), \tag{3}$$

where V_E is the expected utility of an employed worker. Since employed workers are non-shirkers in equilibrium ($V_E^N \geq V_E^S$), this implies that V_E is equal to V_E^N. From (1)–(3), we can rewrite the NSC as:[9]

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[8] For the sake of simplicity, we assume here that the firm pays the worker's pecuniary traffic costs. It will be easy to see later that our result remains unchanged when the worker bears these costs himself or herself. Regarding this point, please see Footnote 10.

[9] A detailed mathematical derivation of the main results in this paper can be obtained from the authors upon request (email: cclin@econ.sinica.edu.tw).

$$w^{1/2} \geq \bar{w}^{1/2} + e + x + e(a + b + r)/q. \tag{NSC}$$

Evidently, as Shapiro and Stiglitz showed, the wage w required to prevent shirking is higher whenever an increase in $\bar{w}$, e, a, b, and r or a decrease in q occur. In this paper the NSC also indicates that the more time that the worker requires to get to his or her workplace, the higher will be the wage that the firm must pay in order to motivate the worker not to shirk.

2.2. The firm

Firms offer wage packages $\bar{w}$ and w, subject to the NSC that workers should be induced not to shirk. An individual firm has no difficulty attracting labor in an economy where there are involuntarily unemployed workers, and an increase in $\bar{w}$ requires a higher w to meet the NSC. Any profit-maximizing firm will offer as little unemployment benefits as required by law or expected by social norms. For simplicity, we assume that $\bar{w}$ is set at the minimum legal level. Given that $\bar{w}$ is offered at the minimum allowable level (let $\bar{w} = 0$ in what follows), the lowest or no-shirking wage w that the firm must pay to induce work effort can then be directly set at a level that meets the NSC. That is, $w = [e + x + e(a + b + r)/q]^2$.

Having determined the firm's wage package, the manager has to choose the plant's location x and employment n so as to maximize its profit, π. Let p be the output price in the output market, and t_1 and t_2 the transport rates for output and input, respectively. Both rates are constant regardless of how great the distance is. Assuming that the firm pays the transportation costs of both output and input, and that the distance between the two cities is normalized to be unity, the firm's optimization problem can be expressed as:

$$\begin{aligned} \max_{x,n} \quad & \pi = [p - (1 - x)t_1]f(n) - (w + xt_2)n, \\ & s.t. w = [1 + x + (a + b + r)/q]^2, \end{aligned} \tag{4}$$

where $e = 1$ has been imposed and $f(\cdot)$ is the production function with the usual property of diminishing marginal returns ($f' > 0$ and $f'' < 0$).[10]

It is assumed that the firm's scale is small, so it makes its employment decision under the belief that it cannot affect the output price and the job acquisition rate a. The corresponding first-order conditions with respect to x and n are, respectively,

$$\pi_x = t_1 f(n) - (w_x + t_2)n = 0 \quad \text{where} \quad w_x = 2w^{1/2} > 0, \tag{5}$$

$$\pi_n = [p - (1 - x)t_1]f'(n) - (w + xt_2) = 0. \tag{6}$$

Now we are ready to check whether the second-order conditions for an interior solution of x and n are satisfied. From (5) and (6), we obtain the following conditions:

[10] If the worker's traffic costs are paid by himself or herself, then the NSC and the profit function, respectively, become: $\pi = [p - (1 - x)t_1]f(n) - wn$, and $w \geq [xt_2 + e + x + (a + b + r)/q]^2$. It is very clear that both specifications will come to the same conclusions.

$$\pi_{xx} = -w_{xx}n = -2n < 0 \text{ where } \quad w_{xx} = 2(\frac{1}{2}w^{-1/2})w_x = 2 > 0,$$

$$\pi_{nn} = [p - (1-x)t_1]f'' < 0,$$

$$D_{xn} \equiv \pi_{nn}\pi_{xx} - \pi_{nx}\pi_{xn} = -2[p - (1-x)t_1]nf'' - [t_1 f' + (2w^{1/2} + t_2)]^2 \gtreqless 0.$$

Since the interior solution requires $\pi_{xx} < 0$, $\pi_{nn} < 0$, and $D_{xn} > 0$, an intermediate location is thus possible in this involuntary unemployment model.

For ease of explaining the above result, it is useful to examine the case where there is no workers' moral hazard problem. That is, $e = 1$ and w is determined by the market-clearing condition rather than by the no-shirking condition. To be more specific, the firm is a wage-taker and takes the market-clearing wage as given, and its optimal problem is to choose x and n to maximize the profit in (4). The corresponding first-order conditions are (note that $w_x = 0$ when w is given):

$$\pi_x = t_1 f(n) - t_2 n \gtreqless 0; \quad t_1 f(n) \gtreqless t_2 n, \tag{7}$$

$$\pi_n = [p - (1-x)t_1]f'(n) - (w + xt_2) = 0. \tag{8}$$

The second-order conditions for an interior solution of (x, n) are not fulfilled in this case due to:

$$\pi_{xx} = 0,$$

$$\pi_{nn} = [p - (1-x)t_1]f'' < 0,$$

$$D_{xn} \equiv \pi_{nn}\pi_{xx} - \pi_{nx}\pi_{xn} = -(\pi_{xn})^2 < 0.$$

When there is no workers' moral hazard problem, given n, it follows from (7) that $\pi_x > 0$ if $t_1 f(n) > t_2 n$. That is, wherever the firm chooses a greater x, i.e. it locates its plant at a greater distance away from the labor market, the firm's profit will increase. This is because the saving in the output transportation cost due to a marginal increase in x is always greater than the increase in the labor transportation cost. The firm's marginal profit associated with x will always increase and the firm will therefore locate its plant in the output city $(x = 1)$.[11] This is the so-called "exclusion theorem" as found in Mathur (1979), and others.

On the other hand, when faced with the shirking problem, the firm will need to pay a wage according to the NSC to prevent shirking. A greater transportation distance will decrease the expected utilities of the employed workers, and so the firm will need to offer a higher wage to motivate workers not to shirk. Due to this increasing wage effect (the term $w_x = 2w^{1/2} > 0$ in Eq. (5)), by locating its plant at a greater distance away from the labor market (a greater x), the firm will not always be able to increase its profit. This gives rise to a tradeoff between the transportation cost and the wage payment. This tradeoff provides an incentive for the firm not to construct its plant in the output city.

[11] By contrast, when $t_1 f(n) < t_2 n$, the optimal location is then in the input city $(x = 0)$.

At first glance, the economic intuition behind our model seems quite different from that of the above-mentioned increasing transport rate literature (e.g., Mai and Shieh 1984, 1991; Hwang and Mai 1990). Upon further inspection, we find that both approaches are dependent on a marginal cost that increases with distance to ensure an intermediate location. In their full employment models where the wage is exogenously determined, the increasing rate assumption is a necessary condition for attaining an increasing marginal cost with respect to the distance. In our involuntary unemployment model where the transportation distance has an impact on the price of the input (the wage), a prerequisite that requires increasing marginal costs with respect to the distance is that the marginal input cost (the marginal wage w_x) is increasing with the distance ($w_{xx} > 0$). It is very easy to confirm this argument by recognizing that the total labor cost is $C = w + xt_1$. When the wage is an exogenous constant, to obtain the condition of increasing marginal cost in terms of the distance ($C_{xx} > 0$), it is necessary to rely on the increasing rate assumption in relation to the input transportation ($C_{xx} = \partial t_1/\partial x > 0$). When the wage is set to discipline workers, obtaining the condition $C_{xx} > 0$ may depend on the condition that the input cost (the wage) is increasing at an increasing rate with the distance ($C_{xx} = w_{xx} > 0$).

In the above basic model we only focus on the cost side to obtain an intermediate location. In the next section we will extend the basic model to show that the shirking perspective of efficiency wage models may result in an intermediate location by influencing the firm's revenue through its effect on labor productivity.

3. Extended models

The previous analysis operated within the basic framework of Shapiro and Stiglitz (1984), which assumes that workers are *homogeneous* and that work effort is a *binary* variable ($e = 0$ or $e = 1$). Nevertheless, workers are in practice *heterogeneous* rather than homogeneous and effort is a *continuous* rather than a binary variable. In what follows, we will show under both situations that an interior location may still be an optimal choice for the firm.

3.1. A heterogeneous workers model

We first consider a model where workers are heterogeneous, but where work effort is still a binary variable. We focus on an example where workers are heterogeneous in the sense that the marginal disutility in relation to effort varies across workers. Let λ represent the worker's marginal disutility of effort, so that the higher the λ is, the more onerous the worker will find putting forth effort, and thus the higher will be the worker's propensity to choose to shirk. To capture this heterogeneity, the instantaneous utility in the previous section for a type λ worker becomes:

$$u(w, e; x, \lambda) = w^{1/2} - \lambda e - x.$$

Due to asymmetric information, firms cannot observe the worker's *hidden characteristic* λ. They thus can neither reject the higher λ workers when recruiting their employees, nor pay them differential wages. It is well-known that when workers are homogeneous, the NSC is used to find the no-shirking wage so as to prevent *all* workers from shirking. Workers are heterogeneous in this section and firms are unable to screen workers to pay them according to their particular no-shirking wage. Given that workers receive the same wage, the NSC cannot determine the lowest wage to prevent all employees from shirking, but can only distinguish the *marginal* type of workers who are indifferent regardless of whether or not they are shirking at the given wage.

The marginal or critical type of workers $\hat{\lambda}$ can be solved from the no-shirking condition $V_E^N \geq V_E^S$ under $u(w, e; x, \lambda) = w^{1/2} - \lambda e - x$. This turns out to be:

$$\hat{\lambda} \geq \frac{(w^{1/2} - \bar{w}^{1/2} - x)q}{(a + b + r + q)e}. \tag{9}$$

Workers with λ less than $\hat{\lambda}$ will choose not to shirk. For simplicity, λ is assumed to be uniformly distributed between 0 and 1. Therefore, $\hat{\lambda}$ is the proportion of non-shirking workers. Since $e = 1$, so $\hat{\lambda}$ also stands for the average effort (productivity) of the workers. According to (9), the proportion of non-shirkers rises whenever there is an increase in w or a decrease in x (other variables that are irrelevant in the following analysis are omitted).

The marginal type of workers, $\hat{\lambda}$, can then be expressed as:

$$\begin{aligned} \hat{\lambda} = \hat{\lambda}(w, x); \quad \hat{\lambda}_w &= \frac{1}{2} w^{-\frac{1}{2}} \phi > 0, \;\; \hat{\lambda}_x = -\phi < 0, \\ \hat{\lambda}_{ww} &= -\frac{1}{4} w^{-\frac{3}{2}} \phi < 0, \;\; \hat{\lambda}_{wx} = \hat{\lambda}_{xx} = 0 \end{aligned}, \tag{10}$$

where $\phi \equiv q/(a + b + r + q)e > 0$. The results $\hat{\lambda}_w > 0$ and $\hat{\lambda}_x < 0$ stem from the fact that an increase in w or a decrease in x raises the opportunity cost of being fired. It therefore increases the proportion of no-shirking workers and the average effort. This implies that the higher the wage is, the higher the effort will be. This is the basic tenet of the efficiency wage theory. Additionally, $\hat{\lambda}_{ww} < 0$ and $\hat{\lambda}_{wx} = \hat{\lambda}_{xx} = 0$ are relevant for the discussion that follows in this section.

Since firms cannot distinguish among different types of workers, they recruit their employees from the labor market at random. A typical firm recruiting n workers expects the average labor quality to be $\hat{\lambda}(w, x)$ and the corresponding effective labor force is supposed to be $\hat{\lambda}(w, x)n$. The firm's profit becomes:

$$\pi = [p - (1 - x)t_1] f[\hat{\lambda}(w, x)n] - (w + xt_2)n. \tag{11}$$

In order to highlight the role of the firm's location x in this efficiency wage model, we first address the special situation where x is not taken into account. In this efficiency wage model when the firm's location x is ignored, the profit becomes $\pi = pf[\hat{\lambda}(w)n] - wn$, and the firm's employment n and wage w can be regarded as two *substitute* inputs. On the output side, the wage determines the *quality of labor* (average effort $\hat{\lambda}(w)$) while employment relates to the quantity of labor (n). The product of the quality and quantity of labor is the firm's actual or effective labor force ($\hat{\lambda}n$). On the cost side, the marginal or average cost of employment is w, and the marginal or average cost of the wage is n. The product of both average costs is the firm's total cost (wn). When the choice of location is taken into account, the distance x then plays a similar role to n and w. A decrease in x raises the work effort and decreases the expenditure on labor transportation, while increasing the cost of transportation for delivering output.

Because the wage can be regarded as an input in this shirking model rather than merely a cost (as in the neo-classical competitive labor model), the firm may trade off the cost of and the benefit from adjusting the wage to set the wage at a level that exceeds the worker's opportunity cost or the competitive market wage. In this situation the firm is a wage-maker, and its optimization problem is to choose its plant's location, employment, and wage so as to maximize (11). The corresponding first-order conditions with respect to x, w, and n are

$$\pi_x = t_1 f[\hat{\lambda}(w,x)n] + [p - (1-x)t_1]\hat{\lambda}_x(w,x)nf'[\hat{\lambda}(w,x)n] - t_2 n = 0, \tag{12}$$

$$\pi_w = [p - (1-x)t_1]\hat{\lambda}_w(w,x)nf'[\hat{\lambda}(w,x)n] - n = 0, \tag{13}$$

$$\pi_n = [p - (1-x)t_1]\hat{\lambda}(w,x)f'[\hat{\lambda}(w,x)n] - (w + xt_2) = 0. \tag{14}$$

Since the second-order conditions for an interior solution require that $\pi_{ii} < 0$, $D_{ij} > 0$, and $D_{ijk} < 0$, for $i, j, k = x$, w, or n, then an intermediate location is possible due to:

$$\pi_{xx} = 2t_1\hat{\lambda}_x nf' + K(\hat{\lambda}_x)^2 n^2 f'' < 0,$$

$$\pi_{ww} = K[\hat{\lambda}_{ww} nf' + (\hat{\lambda}_w)^2 n^2 f''] < 0,$$

$$\pi_{nn} = K(\hat{\lambda})^2 f'' < 0,$$

$$D_{xw} = 2t_1 K\hat{\lambda}_{ww}\hat{\lambda}_x n^2 (f')^2 + K^2\hat{\lambda}_{ww}(\hat{\lambda}_x)^2 n^3 f'f'' - (t_1\hat{\lambda}_x nf'')^2 \gtreqless 0,$$

$$D_{wn} = K^2(\hat{\lambda})^2\hat{\lambda}_{ww} nf'f'' > 0,$$

$$D_{nx} = 2t_1 K\hat{\lambda}\hat{\lambda}_x ff'' - (t_1)^2(\hat{\lambda})^2\left(f' - \frac{f}{\hat{\lambda}n}\right)^2 > 0, \left(\because f' > 0, f'' < 0, \therefore f' > \frac{f}{\hat{\lambda}n}\right),$$

$$\begin{aligned} D_{xwn} =& 2t_1 K^2\hat{\lambda}\hat{\lambda}_x\hat{\lambda}_{ww} nff'f'' - (t_1)^2 K\hat{\lambda}(\hat{\lambda}_w)^2 nff'f'' \\ &+ t_1 K(\hat{\lambda})^2(\hat{\lambda}_w)^2 n^2 f'f''\left(f' - \frac{f}{\hat{\lambda}n}\right) \\ &- (t_1)^2 K(\hat{\lambda})^2\hat{\lambda}_{ww} nf'\left(f' - \frac{f}{\hat{\lambda}n}\right)^2 \\ &- (t_1)^2 K(\hat{\lambda})^2(\hat{\lambda}_w)^2 n^2 f''\left(f' - \frac{f}{\hat{\lambda}n}\right)^2 \gtreqless 0, \end{aligned}$$

where $K \equiv p - (1-x)t_1$, $D_{xn} \equiv \pi_{xx}\pi_{nn} - \pi_{xn}\pi_{nx}$, and D_{xw}, D_{wx}, and D_{xnw} has a similar meaning to D_{xn}.

It is easy to see that when there is no workers' shirking problem, the employers do not need to pay a premium wage to reduce shirking. The wage will be set at the level of the market-clearing wage and all workers will provide the required effort (that is, $\hat{\lambda} = e = 1$). The corresponding first-order conditions with respect to x and n in (12)–(13) will then degenerate to (7) and (8), and the exclusion theorem will hold again.

By contrast, in the presence of the shirking problem, the result $\hat{\lambda}_x < 0$ in (10) indicates that a greater transportation distance that increases the worker's traveling time will cause more workers to choose to shirk, thereby lowering labor productivity. Due to this negative productivity effect, by locating its plant further away from the labor market (a greater x), the firm cannot always increase its profit. This gives rise to a possible explanation that the firm may not construct its plant at the end-point, the output city.

3.2. A continuous effort model

We now turn to briefly consider a simple model where workers are homogeneous, while effort is a continuous rather than a binary variable. The idea of replacing the binary effort setting with a continuous effort one is not new, and some authors have already provided shirking models with homogeneous workers and continuous effort (e.g., Chatterji and Sparks 1991; Pisauro 1991; Rasmussen 1998). All of these models show that the worker effort is related to the payment system and working conditions. These include the wage, the unemployment rate, unemployment benefit, working hours, and so on. In line with this literature, by taking into account the worker's unwillingness to spend time traveling to his or her workplace, it can be shown that the worker's effort will be a negative function of the distance x.[12]

In other words, the effort function embodying the location consideration can be written as (other irrelevant variables such as the unemployment rate are omitted):

$$e(w,x); \quad e_w > 0, \;\; e_x < 0. \tag{15}$$

From the shirking viewpoint, this effort function can be explained in common sense terms as follows. The utility maximizing worker balances the disutility of providing effort against the expected loss of income from being fired for enjoying on-the-job leisure (shirking). The higher the wage is, the greater will be the expected cost to the worker of losing his or her job. The worker will thus exert a greater effort to reduce the probability of being caught shirking and fired. This leads to the result $e_w > 0$. Similarly, a greater distance between home and the workplace will lengthen transportation time and reduce the satisfaction of being employed. The worker will consequently care less about being fired and will reduce his or her work effort. This results in the outcome $e_x < 0$. Since (15) is similar to (10), as a result, the intermediate location is a possible solution in the continuous effort model.

The economic intuition behind this continuous effort model is similar to that in the heterogeneous workers model. A greater distance decreases the workers' opportunity cost when getting fired and reduces their effort levels. This negative relationship between the transportation distance and labor productivity may make it profitable for the firm to choose an intermediate location.

If the distance between the worker's home and the workplace affects the firm's choice of location only on the cost side, as concluded in the previous section, then the prerequisite for attaining an interior location is that the (total) marginal cost of the input has to increase with the distance ($C_{xx} > 0$). Alternatively, when the distance affects the firm's location decisions on the

[12] To save space, we do not explicitly derive the effort function here.

revenue side, then the condition for an interior solution is that of diminishing marginal revenue with respect to the distance. By integrating both conditions, the general rule for an intermediate location is that of diminishing marginal profit with respect to the distance (i.e., $\pi_{xx} < 0$).[13]

4. Concluding remarks

We have shown that an intermediate location is possible if a greater labor transportation distance raises a firm's wage offer and/or decreases its labor productivity. It is worth noting that the negative impact of a greater distance on labor productivity can result from the workers' shirking problem as well as from the workers' adverse selection problem. Suppose now that the information problem arises from the workers' *hidden characteristics* regarding their ability to work (or productivity) rather than their propensity to shirk. Moreover, in line with the literature on labor adverse selection models, let us suppose that a higher ability worker has a higher opportunity cost or reservation wage.[14] The most able or highest quality worker is therefore the very one who is most unwilling to work at a more distant plant. The firm cannot exactly observe the ability of any job candidate due to imperfect information, but it knows that, the shorter the distance between its plant and the town in which the workers reside, the better will be the average ability of workers that it may recruit. In order to recruit more able workers, the firm should avoid constructing its plant in the output city, even though the city is its optimal choice without taking the heterogeneity regarding labor ability into account.

Our insights can be obtained not only by focusing on the impact of the transportation distance on the quality of workers, but also by emphasizing the effect of the transportation time on the quality of the output or other inputs. When the transportation distance decreases the quality of the goods (agricultural products or seafood, for example), an intermediate location may be the best option for keeping the goods fresh. Finally, even though the model in this paper is admittedly rudimentary, we hope that this explanation of an intermediate location in terms of emphasizing the negative effect of transportation distance on labor quality can enhance our understanding of the choice of location of the firm.[15]

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[13] The setting of the utility function in this paper is designed to ensure that the first- and second-order conditions in the basic and extended models are all satisfied. For example, in the basic model the second-order condition requires that $C_{xx} = w_{xx} > 0$; therefore, the utility function form must ensure that $w_{xx} > 0$ by using the NSC. A simpler setup such as $u(w, e; x) = w - e - x$ is not suitable, since under this setting the no-shirking wage is $w = e + x + e(a + b + r)/q$ and thus $C_{xx} = w_{xx} = 0$. Moreover, the simpler setting $u(w, e; x, \lambda) = w - \lambda e - x$ is also not suitable in the heterogeneous workers model. In this situation, the marginal type of workers in (9) becomes $\hat{\lambda} \geq (w - \bar{w} - x)q/(a + b + r + q)e$ and $\hat{\lambda}_{ww} = 0$, the second-order condition with respect to $D_{wn} = K^2(\hat{\lambda})^2\hat{\lambda}_{ww}nf'f''$ then turns out to be zero, and so an intermediate location is impossible. As a result, we do not adopt a simpler utility function such as $u(w, e; x) = w - e - x$ in our analysis.

[14] The setting of the positive relationship between labor ability and the reservation wage can be traced at least back to Weiss (1980).

[15] Even though this paper focuses on the possibility of an intermediate location and does not explore the determination of the equilibrium unemployment as in Shapiro and Stiglitz (1984), our model may constitute a simple framework for analyzing unemployment problems in a spatial economy.

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