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A dynamic portfolio choice model of tax evasion: Comparative statics of tax rates and its implication for economic growth

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Abstract

This paper extends the portfolio choice model of tax evasion from a static to a dynamic setting. It is shown that while higher tax rates *repress* tax evasion in the static model, they *encourage* tax evasion in the dynamic model. We explore a novel implication of this result and show that while growth is *decreasing* in tax rates in the absence of tax evasion, it is *U-shaped* in tax rates in the presence of tax evasion. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Starting from the seminal paper by Allingham and Sandmo (1972), the standard approach to tax evasion has been based on the economics of decision

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making under risk. In particular, declaring income is viewed as being analogous to purchasing a safe asset, while concealing income is viewed as being analogous to purchasing a risky asset. The tax evasion problem facing an individual then essentially becomes a portfolio selection issue.

In the Allingham–Sandmo model, taxpayers are assumed to pay a penalty on their *concealed income* if the act of evasion is caught. This assumption leads to the result that a tax rate change will have an ambiguous impact on evasion, for there exist two opposing effects, an income effect and a substitution effect. Following Yitzhaki's (1974) modification of the Allingham–Sandmo model, most models of tax evasion now assume that the penalty is levied on the *evaded tax*, as it is under most tax laws. This modification yields no substitution effect, and hence leads to an unambiguous result that an increase in tax rates reduces tax evasion.

It is intuitively appealing, however, to speculate that higher tax rates will *encourage* rather than *repress* evasion. This commonly held view comes from both academic and nonacademic circles. In a report by *The Economist* (May 28, 1994) on how to reduce tax evasion, lowering tax rates is regarded as one of the best ways of accomplishing this goal. Ching (1996) reported in the *Far Eastern Economic Review* (April 20, 1996) that tax rates would be greatly reduced to lessen the incentives for tax evasion, according to a tax reform bill that Philippine President Fidel Ramos had in mind. Roubini and Sala-i-Martin (1995, Eq.(4)) simply impose a positive relation between evasion and tax rates in their study of the effect of financial repression on growth. Using panel data that trace the tax returns of the same individuals before and after the 1986 Tax Reform Act, Feldstein (1995, p. 555) suggests: "high marginal tax rates may induce taxpayers to take more 'aggressive' interpretations of tax rules (e.g., claiming questionable deductions) or even to evade taxes by understating income or claiming unjustified deductions".

Though not unanimous, there is much evidence in support of the intuition that higher tax rates encourage rather than repress tax evasion.[1] Several authors, including Cowell and Gordon (1988), Gordon (1989), Yaniv (1994), and Myles and Naylor (1996) have tried to build theoretical models which are capable of producing a prediction that is consistent with this evidence.

In this paper we extend the portfolio choice model of tax evasion from a static to a dynamic setting. The dynamic model, as will be seen, enables us to reverse the counterintuitive prediction of the static model regarding the impact of a tax rate change on evasion. A novel implication of our finding for economic growth is also explored.

[1] See, for example, Clotfelter (1983), Tanzi (1983), Crane and Nourzad (1986), Baldry (1987), Poterba (1987), Alm et al. (1991), Pommerehne and Weck-Hannemann (1996), and Blacksmore et al. (1996).

The remainder of the paper is organized as follows. Section 2 introduces a static portfolio choice model of tax evasion, which provides a benchmark for the paper. Section 3 extends the portfolio choice model of tax evasion from statics to dynamics. Contrary to the static model in which an increase in tax rates represses evasion, we show that an increase in tax rates encourages evasion. Section 4 then explores an implication of our finding for economic growth. We show that tax evasion can actually be conducive to economic growth. Section 5 explores the issue of optimal tax evasion and Section 6 concludes.

2. A static portfolio choice model of tax evasion

Consider a closed economy populated by a continuum of ex ante identical individuals whose size is normalized to unity. Each individual seeks to maximize the instantaneous utility function

$$u = \ln(c^{1-\theta}G^{\theta}) = (1-\theta)\ln c + \theta \ln G, \tag{1}$$

where $1 < \theta < 1$, c denotes consumption per person and G the level of the public good. This setup follows Barro (1990, Eq. (23)). Note that (1) implies the unitary coefficient of relative risk aversion. The use of other constant coefficients of relative risk aversion does not alter our main results, and hence is not presented for simplicity.[2]

Each individual is assumed to be a household-producer who has access to the Ak production function

$$y = Ak, \tag{2}$$

where y is output per person net of depreciation of capital, k is capital per person, and A is the technology parameter.

Let $\tau > 0$ be the flat income tax rate imposed. The level of the public good in the absence of tax evasion would be given by the equality $G = \tau y$, which implies a balanced budget. In what follows we allow for the possibility of tax evasion.

Let e be the portion of income concealed from the tax authority, π be the probability that tax evasion will be discovered and convicted, and s be the surcharge rate levied on the evaded tax if the evader is caught. The random return on a dollar of evaded tax is then given by[3]

$$r = 1 \text{ with probability } 1-\pi; \quad r = -s \text{ with probability } \pi.$$

[2] Details are available from the authors upon request.

[3] This is the standard setup in the tax evasion literature; see Cowell (1990).

Both the technology parameter A and the capital per person k are fixed in a static setting, and hence the level of income per person y is fixed as well (see Eq. (2)). The evasion decision faced by each individual is how to allocate the given income between declaring it (purchasing a safe asset) and concealing it (purchasing a risky asset). This scenario is in the tradition of the portfolio choice model of tax evasion. It can be shown in this static framework that the compliance of taxpayers responds positively to tax rates (τ) as well as to punishment (π and s). It is also known that tax evasion will occur if and only if $\pi(1+s) < 1$,[4] an inequality which is assumed satisfied here. Since individuals are identical ex ante, all of the individuals in the population evade tax in our model.

The economy, as stated above, is populated by a continuum of ex ante identical individuals whose size is normalized to unity. Because all of the population evade tax, the probability of detection π can be taken as the proportion of the individuals whose evasion is actually detected by the tax authority. This ex ante identical but ex post differential setup is similar to that of Kolm (1973).

For the economy as a whole, the random rate of return on evaded tax can be replaced by its expected value. After allowing for tax evasion, the level of the public good is modified to be

$$G = \tau(1 - \bar{r}e)y, \tag{3}$$

where $\bar{r} = 1 - \pi - \pi s$, i.e. the expected rate of return on a dollar of evaded tax. The expression $\tau(1 - \bar{r}e)$ represents the *effective* tax rate, which may be different from the *statutory* tax rate τ. When we use the term 'tax rate' in what follows, it will always mean 'statutory tax rate'.

3. An extension from statics to dynamics

The static model above is embedded in a dynamic/growing context in this section.

First, instead of seeking the maximization of an instantaneous utility function as in (1), all individuals are assumed to be infinitely-lived and to maximize the overall utility

$$U = \int_0^\infty [(1-\theta)\ln c + \theta \ln G]\exp(-\rho t)\,\mathrm{d}t, \tag{4}$$

where $\rho > 0$ is the constant rate of time preference.

[4] See Fishburn (1979).

The enforcement parameters (π and s) will be assumed to be fixed all the time. As a result, there is no possibility for the exploitation of prior tax-evasion records by the tax authority as time elapses. This appears defective. However, as argued by Barro (1991), the infinite horizon expressed in Eq. (4) applies naturally if parents are altruistic toward their children, who are in turn altruistic toward their children, and so on. The rate of time preference can be regarded as capturing the degree of altruism toward successive generations, rather than the influence of time, per se. It seems not inappropriate to rule out the possible exploitation of prior tax-evasion records under the 'dynasty' interpretation of (4). This is because each generation lives only for a short period of time even though the agent representing all generations never passes away.

In a dynamic/growing economy, one should allow for saving and hence the accumulation of capital. Tax evasion results in random income for an individual, since the act of evasion may or may not be detected. This in turn leads to randomness in the evolution of individual capital. We show in the appendix that, in the presence of tax evasion, each individual's accessed capital can be modeled to evolve according to a stochastic differential equation

$$\mathrm{d}k = ([(1-\tau) + \bar{r}\tau e]y - c)\,\mathrm{d}t + (ey\sigma)\,\mathrm{d}z,\ k(t=0) = k_0, \tag{5}$$

where σ is a positive parameter, and z is a standard Brownian motion whose increment $\mathrm{d}z$ has zero mean and variance $\mathrm{d}t$. Eq. (5) indicates that changes in individual capital have the expected value $([(1-\tau) + \bar{r}\tau e]y - c)\,\mathrm{d}t$ and variance $(ey\sigma)^2\,\mathrm{d}t$.

Individual optimization is solved by employing the stochastic optimal control. With the setup of (2)–(5), the corresponding stochastic Bellman equation is[5]

$$\rho I(k) = \max_{c,\,e} \{(1-\theta)\ln c + I'(k)([(1-\tau) + \bar{r}\tau e]y - c) + \tfrac{1}{2}I''(k)(ey\sigma)^2\}, \tag{6}$$

where $I(k)$ is a value function with $J(k,t) = I(k)\exp(-\rho t)$ and

$$J(k_0,0) = \max_{c,e}\ \mathrm{E}_0\left[\int_0^t (1-\theta)\ln c\,\exp(-\rho t)\,\mathrm{d}t + J(k,t)\right]$$

(E_0 denotes the conditional expectation operator, given k_0).

Note that the term associated with G is left out in (6). This is due to the fact that each individual takes the level of the public good as given in a large economy.

The maximization values of c and e for (6) satisfy (in terms of the unknown function $I(.)$)

$$c = 1/[I'(k)],\ e = -[(\bar{r}\tau)I'(k)]/[(y\sigma^2)I''(k)]. \tag{7}$$

[5] See Merton (1990), Kamien and Schwartz (1991), or Dixit and Pindyck (1994).

Substituting (7) back into (6) gives

$$\rho I(k) = \ln[1/I'(k)] - 1 + [(1-\tau)k]I'(k) - [(\bar{r}\tau)I'(k)]^2/2\sigma^2\, I''(k)], \tag{8}$$

which is a nonlinear second-order differential equation. The solution of (8) is of the form

$$I(k) = \frac{1}{\rho} \ln Ak + B, \tag{9}$$

where B is a constant.

From (7) and (9), the chosen consumption and evasion are

$$c = \rho Ak, \tag{10}$$

$$e = (\bar{r}\tau)/A\sigma^2. \tag{11}$$

The result of (10) is standard and not different from that without tax evasion.[6] As to the result of (11), it needs a bit of elaboration. First, note that $e > 0$ if and only if $\bar{r}\tau > 0$. Since $\tau > 0$ and $\bar{r} = 1 - \pi - \pi s$, tax evasion will occur if and only if $\pi(1+s) < 1$, a result which is the same as in the static case. Second, the random return on a dollar of *evaded tax* is given by r, and hence the random return on a dollar of *concealed income* is $r\tau$. Eq. (11) thus asserts that the concealed portion of income varies directly with the expected return from the concealed income. If concealing income is viewed as purchasing a risky asset while declaring income is viewed as purchasing a safe asset, this result is in essence the same as that of the dynamic portfolio analysis (see Merton, 1969).

An increase in the certainty of punishment π or in the severity of punishment s results in greater compliance on the part of taxpayers, according to (11). This comparative statics result does not differ from that in the static case. A comparative statics result from (11) which contrasts with that of the static case is that an increase in the tax rate τ leads to an increase rather than a decrease in tax evasion. The dynamic portfolio choice model of tax evasion thus enables us to reverse the counterintuitive prediction of the static model regarding the impact of a tax rate change on evasion.

The key here, of course, lies in extending the model from statics to dynamics. To understand the fundamental difference between the static and the dynamic setting, consider the discrete-time version of the Bellman equation corresponding to (6)[7]

$$I(k(t)) = \max_{c(t),\, e(t)} \left\{(1-\theta)\ln c(t) + \frac{1}{1+\rho}\, \mathrm{E}_t[I(k(t+1))]\right\} \tag{6'}$$

[6] See Turnovsky (1995, pp. 244–245).

[7] See Dixit and Pindyck (1994, Chapter 4).

where E_t denotes the conditional expectation operator, and the term $E_t[I(k(t+1))]$ is known as the continuation value. At any time t, the agent is assumed to maximize the *sum* of the immediate payoff $(1-\theta)\ln c(t)$ plus the continuation value $E_t[I(k(t+1))]$ (discounted). By contrast, there is no continuation value in the static setting and hence the agent's objective is simply to maximize the immediate payoff $(1-\theta)\ln c(t)$. Note that a positive expected value of $r\tau$ (the random return on a dollar of concealed income or risky asset) implies a favorable gamble faced by individuals. It is well known that a risk averter takes no part of an unfavorable or barely fair gamble, but always takes some part of a favorable gamble (see Arrow, 1970, pp. 99–100). Facing a favorable gamble with infinite continuation as in our model, it is only reasonable that the degree to which an individual will take part in the favorable gamble varies directly with the expected return from the gamble. That is, the portion of income concealed e varies directly with the expected return on a dollar of concealed income $\bar{r}\tau$. This positive relationship between e and $\bar{r}\tau$ is precisely captured by Eq. (11). To sum up, we state:

Proposition 1. While higher tax rates repress evasion in the static portfolio choice model of tax evasion, they encourage evasion in the dynamic model.

4. An implication for economic growth

This section explores a novel implication of Proposition 1 for economic growth within our framework.

Substituting (10) and (11) into (5) yields

$$dk/k = [(1-\tau)A + (\bar{r}\tau/\sigma)^2 - \rho]\,dt + (\bar{r}\tau/\sigma)\,dz, \tag{12}$$

where the growth of individual capital follows a geometric Brownian motion.[8] Tax evasion results in uncertain income for an individual. However, there is no aggregate risk for the economy as a whole since the size of the population is large. The per capita growth rate of the economy in the presence of tax evasion is simply the 'expected value' part of (12), that is

$$\gamma_e = \dot{c}/c = (1-\tau)A + (\bar{r}\tau/\sigma)^2 - \rho. \tag{13}$$

From the viewpoint of the economy, $\dot{k}/k = \dot{y}/y = \dot{c}/c$ always holds by (2) and (10).

If there were no tax evasion, our model would have the following per capita growth rate (see Barro, 1990):

$$\gamma = \dot{c}/c = (1-\tau)A - \rho. \tag{13'}$$

[8] See Dixit (1993, Section 1.3) for details on the geometric Brownian motion.

Contrasting (13) with (13′), we can see that the growth rate with evasion is higher by a factor $(\bar{r}\tau/\sigma)^2$ than it would be without evasion. This higher rate of growth is obviously due to the existence of tax evasion. It is interesting to observe that the growth rate γ_e displays a U-curve in terms of tax rates.[9] This U-curve relation is in stark contrast to the monotonic decreasing relation without tax evasion (see Eq. (13′))[10].

The intuition behind the U-curve is as follows. Note first that public goods are nonproductive (though possibly utility-enhancing) in our model. This implies that 'diverting' resources from the nonproductive public sector to the productive private sector (through saving) will be conducive to economic growth. The factor $(\bar{r}\tau/\sigma)^2$ captures the tax evasion effect of this resource diversion on growth. At low tax rates, the extent of tax evasion is small according to Proposition 1. Since the adverse impact of distorting taxation dominates, growth rates decline with tax rates. At higher tax rates, however, the extent of tax evasion becomes more significant according to Proposition 1. The positive effect of tax evasion on growth eventually overruns the negative effect of taxation as the tax rate progressively increases. This results in growth rates rising with tax rates.

The result of (13) can also be understood from another perspective. Let us define $\eta = \tau(1 - \bar{r}e)$, the effective tax rate of the economy. Using (11), one can rewrite the γ_e of (13) as

$$\gamma_e = \dot{c}/c = (1 - \eta)A - \rho, \tag{13″}$$

which shows that the rate of growth with evasion is decreasing in effective tax rates. Note that the functional form of (13′) is identical to that of (13″), except that the expression is now presented in terms of *effective* rather than *statutory* tax rates. At low tax rates, increases in tax rates result in higher effective tax rates since the extent of evasion is small. At higher tax rates, the extent of tax evasion becomes more significant. Once the tax rate reaches a critical level, further increases actually result in lower rather than higher effective tax rates.

To sum up, we state within our framework:

Proposition 2. While growth is decreasing in tax rates in the absence of tax evasion, it is U-shaped in tax rates in the presence of tax evasion.

When tax rates are low, the sign predictions for the effect of taxation on growth are the same with or without tax evasion. Growth rates decline with tax

[9] $\partial\gamma_e/\partial\tau \gtreqless$ if $\tau \gtreqless 0.5A(\sigma/\bar{r})^2$, and hence γ_e is minimized at $\tau = 0.5A(\sigma/\bar{r})^2$.

[10] The finding that there exists a negative relation between taxation and growth in the case of no evasion holds under more general utility functions and is fairly robust in theory; see Alogoskoufis and van der Ploeg (1990) for a similar finding in the overlapping generations framework. However, empirical studies on the issue give mixed results (see Slemrod, 1995 and the references therein).

rates. When tax rates are high enough however, the prediction based on the no-evasion model will be opposite to that based on the evasion model. Growth rates still decline with tax rates if there is no evasion, whereas growth rates are reversed to rise with tax rates if there is evasion.

5. On optimal tax evasion

Assume that varying policy variables (τ, π, s) is not feasible for some exogenous reasons (a typical assumption in the second-best theory). We ask in this section the following question: how should the economy behave so as to maximize the overall utility of (4); in particular, is there an optimal degree of tax evasion?

There is no uncertainty from the viewpoint of the economy as a whole in our model, and hence the per capita capital of the economy evolves according to the 'expected value' part of (5), that is

$$\dot{k} = [(1-\tau) + \bar{r}\tau e]y - c. \tag{5'}$$

Individuals' decentralized decisions take the level of the public good as given in a large economy. For the economy as a whole, however, the level of the public good is no longer given and so the budget constraint (3) needs to be taken into account explicitly in deriving optimality. Maximizing (4) subject to (2), (3) and (5′) leads to the Hamiltonian setup:

$$H = (1-\theta)\ln c + \theta \ln[\tau(1-\bar{r}e)Ak] + \lambda\{[(1-\tau) + \bar{r}\tau e]Ak - c\}.$$

As a result, the conditions for the maximization are:

$$\frac{\partial H}{\partial c} = \frac{1-\theta}{c} - \lambda = 0, \tag{14}$$

$$\frac{\partial H}{\partial e} = -\frac{\theta}{G}\bar{r}\tau Ak + \lambda\bar{r}\tau Ak = -\frac{\theta}{G} + \lambda = 0, \tag{15}$$

$$\frac{\partial H}{\partial k} = \frac{\theta}{G}\tau(1-\bar{r}e)A + \lambda[(1-\tau) + \bar{r}\tau e]A = -\dot{\lambda} + \rho\lambda. \tag{16}$$

Eqs. (14) and (15) together give

$$\frac{G}{c} = \frac{\theta}{1-\theta}, \tag{17}$$

which indicates that the optimal consumption/evasion will enforce a G/c ratio precisely equal to $\theta/(1-\theta)$. The intuition behind this result is as follows. From (1) we have the marginal rate of substitution between consumption and public

good, $\text{MRS} \equiv -(\mathrm{d}c/\mathrm{d}G)_{u=\bar{u}} = [\theta/(1-\theta)](c/G)$. On the other hand, the marginal rate of transformation between consumption and public good, $\text{MRT} = 1$. The social optimal condition, $\text{MRS} = \text{MRT}$, leads to the result (17).

Using (14)–(16) yields

$$\gamma^* = A - \rho. \tag{18}$$

With the optimal consumption/evasion, the economy attains a growth rate that is higher than that without evasion (Eq. (13′)) and also that with non-optimal evasion (Eq. (13″)).[11]

6. Concluding remarks

The approach adopted in this paper is in the tradition of the portfolio choice model of tax evasion, except that we frame the problem in a dynamic/growing rather than static/stationary context. It is shown that: (i) the compliance of taxpayers responds negatively rather than positively to tax rates, and (ii) tax evasion can be conducive to economic growth.

All tax revenues collected are assumed to finance public goods in (3); in particular, there are no enforcement costs for the tax authorities. This setup is a useful first approximation, but certainly not the reality. For example, the US Internal Revenue Service spent $7.6 billion enforcing all varieties of federal taxation in fiscal year 1995, and total revenues collected in the same fiscal year amounted to about $1.27 trillion.[12]

This indicates that the enforcement cost is around 0.6% of the revenue collected. Based on this evidence, a simple way of incorporating enforcement costs is to let them exhaust a portion of the tax revenue collected. The balanced budget of (3) is modified to be

$$G = (1-\beta)[1-\bar{r}e)y], \tag{3′}$$

where β denotes the portion of tax revenues used in enforcement activities. The variable β possibly depends on the probability of detection π and/or the surcharge rate s. Since the level of G is taken as given in individual optimization, however, the modification (3′) will not alter our main results.

The finding that tax evasion can be conducive to economic growth should be interpreted cautiously. A weakness of the portfolio choice approach to tax evasion is that it overlooks factors arising from social interactions. For example, tax evasion may be driven by perceived injustices or inequality in the tax system,

[11] The result (18) indicates that, with the optimal consumption/evasion, the possible distortion due to taxation will be completely offset.

[12] See *Office of Management and Budget* (1995).

or may be simply an epidemic. Some results obtained in this paper might need to be qualified after taking into account the social context. Another reason for interpreting our results cautiously is that 'transaction costs' arising from taxpayers' compliance and/or evasion activities, as in most models, are ignored in this paper. Increasing the extent of tax evasion raises evasion costs on the one hand, but at the same time it lowers compliance costs on the other hand. How the incorporation of these costs might modify our results is unclear at the present stage.[13]

Finally, our model considers only public consumption goods, leaving out public production goods. While public consumption goods enter utility directly, public production goods enter production directly. Following Barro (1990), the Ak production function (2) may be modified to be

$$y = Ak^{\alpha}g^{1-\alpha}, \tag{2'}$$

where $0 < \alpha < 1$ and g denotes the level of public capital available to the economy. In the case where g is treated as a stock variable (say, public infrastructure), one will need two transition equations to describe the evolutions of both k and g. Futagami et al. (1993) and Turnovsky (1997), both in the absence of tax evasion, provide attempts in this direction. However, in order to strike a balance between tractability and realism, most specifications introduce g as a flow as in Barro (1990). Turnovsky (2000) considers a stochastic version of the Barro (flow) model. He analyzes the role of productive government expenditure in a growing economy subject to stochastic productivity shocks. Since tax evasion results in stochastic income, this kind of stochastic model could be relevant for incorporating both tax evasion and public production goods within a framework.

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[13] The amount of tax evasion costs involved may not be small. It is worth emphasizing, however, that the magnitude of tax compliance costs is not trivial either. For a survey of recent research findings on tax compliance costs from around the world, see Sandford (1995).

Appendix A

Here, we show how the stochastic differential equation (5) can be derived as the continuous limit of a discrete-time problem. Our approach follows Dixit (1993, Section 1.1).

Step 1: Let $x \equiv r\tau$, the random return to a dollar of concealed income. The random variable x has a Bernoulli distribution with

$$x = \tau \text{ with probability } 1 - \pi, \quad x = -s\tau \text{ with probability } \pi.$$

Divide time into discrete periods of length Δt. The cumulated x with a time interval of length 1 is then a binomial variate with mean

$$[(1 - \pi - s\pi)\tau]/\Delta t \tag{A.1}$$

and variance

$$\{\pi(1 - \pi)[(1 + s)\tau]^2\}/\Delta t. \tag{A.2}$$

Note that if $s = 1$, our setting would be essentially identical to that in Dixit (1993).

Now set

$$\tau = \sigma(\Delta t)^{0.5} \tag{A.3}$$

$$s = 1 - \frac{(\bar{r}\Delta t)^2}{1 - \bar{r}\Delta t}, \tag{A.4}$$

$$\pi = \frac{(1 - \bar{r}\Delta t)^2}{1 + (1 - \bar{r}\Delta t)^2} \tag{A.5}$$

and substitute (A.3)–(A.5) into (A.1) and (A2). Letting Δt go to zero, then the binomial variate converges to the normal, with mean

$$\bar{r}\tau \tag{A.6}$$

and variance

$$\sigma^2. \tag{A.7}$$

Step 2: The change in capital stock Δk has a Bernoulli distribution with $\Delta k = [(1 - \tau + \tau e)y - c]\Delta t$ with probability $1 - \pi$; $\Delta k = [(1 - \tau - s\tau e)y - c]\Delta t$ with probability π.

This implies that the cumulated change $(k - k_0)$ with a time interval of length 1 is a binomial variate with mean

$$(1 - \tau)y + [(1 - \pi - s\pi)\tau e y]/\Delta t - c \tag{A.8}$$

and variance

$$\{\pi(1 - \pi)[(1 + s)\tau e y]^2\}/\Delta t. \tag{A.9}$$

Using the results obtained in Step 1, the binomial variate converges to the normal as Δt goes to zero, with mean

$$[(1-\tau)+\bar{r}\tau e]y - c \tag{A.10}$$

and variance

$$(ey\sigma)^2. \tag{A.11}$$

These are exactly the values we need for the Brownian motion (5).

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