

# Linear and nonlinear Granger causality in the stock price-volume relation: A perspective on the agent-based model of stock markets

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## Abstract

From the perspective of the agent-based model of stock markets, this paper examines the possible explanations for the presence of the causal relation between stock returns and trading volume. The implication of this result is that the presence of the stock price-volume causal relation does not require any explicit assumptions like information asymmetry, reaction asymmetry, noise traders, or tax motives. In fact, it suggests that the causal relation may be a generic property in a market modeled as evolving decentralized system of autonomous interacting agents.

Keyword: Agent-based model, Artificial stock markets, Genetic programming, Granger causality test, Stock price-volume relation

## 1 Motivation and introduction

Agent-based modeling of stock markets, originated in Santa Fe Institute [44, 2], is a fertile and promising field, which can be thought as a subfield of *agent-based computational economics* (ACE).<sup>1</sup> Up to the present, most of the research efforts have been devoted to the analysis of the *price dynamics* and/or *market efficiency* of the artificial markets (e.g. [13, 14, 41, 52]). Some focused their study on the price deviation or mispricing in the artificial stock markets (e.g. [2, 8, 10, 12, 40, 41, 44, 51]). Some of them went further to explore the corresponding *microstructure* of the markets, such as *aspect traders' beliefs and behaviors* (e.g. [11, 13, 14]). Nevertheless, few have ever visited the univariate dynamics of trading volume series [40, 51], and, to our best knowledge, none has addressed its joint dynamics with prices.<sup>2</sup>

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<sup>1</sup>As Farmer and Lo [22] mentioned, “Evolutionary and ecological models of financial markets is truly a new frontier whose exploration has just begun.” By modeling financial markets “as evolving systems of autonomous interacting agents,” the agent-based approach in finance, indeed, follows this evolutionary paradigm [49]. Visit *the ACE website* maintained by Leigh Tesfatsion for a comprehensive guide to the field of *agent-based computational economics*. <URL: <http://www.econ.iastate.edu/tesfatsi/ace.htm>>

<sup>2</sup>See Chen [9] or LeBaron [39] for reviews of the field of artificial financial markets.

As Ying [53] noted almost forty years ago, stock prices and trading volume are *joint products* from *one single market mechanism*. He argued that “any model of the stock market which separates prices from volume or vice versa will inevitably yield incomplete if not erroneous results” [*ibid.*, p. 676]. In similar vein, Gallant et al. [24] also asserted that researchers can learn more about the very nature of stock markets by studying the *joint dynamics* of prices in conjunction with volume, instead of focusing price dynamics alone. As a result, the stock price-volume relation has been an interesting subject in financial economics for many years.<sup>3</sup>

While most of the earlier empirical work focused on the *contemporaneous* relation between trading volume and stock returns, some recent studies began to address the *dynamic relation*, i.e., *causality*, between daily stock returns and trading volume following the notion of *Granger causality* proposed by Wiener [50] and Granger [26]. In many cases, a bi-directional Granger causality (or a *feedback* relation) was found to exist in the stock price-volume relation, although some other works could only find evidence of a uni-directional causality: either returns would *Granger-cause* trading volume, or vice versa [1, 34, 45, 46, 48].

As noted by Granger [27], Hsieh [33], and many others, we live in a world which is “almost certainly nonlinear.” We can not be satisfied with only exploring the linear Causality between stock prices and trading volume. Non-linear causality would naturally be the next step to pursue. Baek and Brock [3] argued that traditional Granger causality tests based on VAR models might overlook significant nonlinear relations. As a result, they proposed a nonlinear Granger causality test by using *nonparametric estimators of temporal relations within and across time series*. This approach can be applied to any two stationary, mutually independent and individually *i.i.d.* series. Hiemstra and Jones [31] modified their test slightly to allow the two series under considerations to display “weak (or short-term) temporal dependence.” Several researchers have already adopted this modified Baek and Brock test to uncover price and volume causal relation in real world financial markets [23, 31, 47]. In most of the cases, they could found bi-directional *nonlinear* Granger causality in the prices and trading volume. In other words, not only did stock returns Granger-cause trading volume, but trading volume also Granger-cause stock returns. The significance of this finding is that trading volume can help predict stock returns, as an old Wall Street adage goes, “*It takes volume to make price move.*”

There are several possible explanations for the presence of a causal relation between stock returns and trading volume in the literatures. First, Epps [20] gave their explanation based on the *asymmetric reaction* of two groups of investors — “bulls” and “bears” — to the positive information and negative information.

The second explanation, which is called the *mixture of distributions hypothesis*, considers special distributions of speculative prices. For example, Epps and Epps [21] derived a model in which trading volume is used to measure disagreement of traders’ beliefs on the variance of the price changes. On the other hand, in Clark’s [16] mixture of distributions model, the speed of information flow is a latent common factor which influences stock returns and trading volume simultaneously.

A third explanation is the *sequential arrival of information models* (see, for example, Copeland [17], He and Wang [30], Jennings et al. [35], and Morse [43]). In this asym-

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<sup>3</sup>See the survey article by Karpoff [37].

metric information world, traders possess differential pieces of new information in the beginning. Before the final complete information equilibrium is achieved, the information is disseminated to different traders only gradually and sequentially. This implies a positive relationship between price changes and trading volume.

Lakonishok and Smidt [38] proposed still another model which involves *tax- and non-tax-related motives* for trading. For the sake of window dressing, portfolio rebalancing, or the optimal timing for capital gains, traders may have some special kinds of trading behaviors. As a result, Lakonishok and Smidt [38] showed that current trading volume can be related to past price changes owing to these motives.

Away from traditional representative-agent models stated above, recent theoretical works have started to model financial markets with *heterogeneous traders*. Besides informed traders (insiders), DeLong et al. [18] introduced *noise traders* with positive-feedback trading strategies in their model. Noise traders do not have any information about the fundamentals and trade solely based on the past price movements. As a result, positive causal relation from stock returns to trading volume appears. In Brock’s [5] nonlinear theoretical noise trading model, the estimation errors made by different groups of traders are correlated. Under these settings, he could find that stock price movements and volatilities are related nonlinearly to volume movements. Campbell et al. [6] developed another heterogenous agent model, in which there are two different types of risk-averse traders. In their frameworks, they could explain the autocorrelation properties of stock returns as a nonlinear relation with trading volume.

In light of these explanations, this paper attempts to see *whether we can replicate the causal relation between stock returns and the trading volume via the agent-based stock markets (ABSMs)*. We consider the agent-based model of stock markets highly relevant to this issue. First, the existing explanations mentioned above based their assumptions either on the information dissemination schemes or the traders’ reaction styles to information arrival. Since both of these factors are well encapsulated in agent-based stock markets, it is interesting to see whether ABSMs are able to replicate the casual relation. Secondly, information dissemination schemes and traders’ behavior are known as the *emergent phenomena* in ABSMs. In other words, these factors are endogenously generated rather than exogenously imposed. This feature can allows us to search for a fundamental explanation for the causal relation. For example, we can ask: *without the assumption of information asymmetry, reaction asymmetry, or noise traders, and so on, can we still have the causal relation?* Briefly, is the causal relation a generic phenomenon?

Thirdly, we claim that agent-based modeling of financial markets are “*true*” heterogeneous agent models, which depict the real markets more faithfully. We might think that the models proposed by DeLong et al. [18] and their successors as having pre-specified representative agents of two different types, say, a representative rational informed trader and a representative uninformed noise trader. These settings might overlook some important features of financial markets, for example, interaction and feedback dynamics of traders. In the agent-based approach, we, however, do not assign any agent to be any specific type exogenously. As a matter of fact, we don’t even have the device of representative agents. Hundreds of agents in the model can all have different behavioral rules which themselves shall evolve (adapt) over time.<sup>4</sup> How many types by which they can be

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<sup>4</sup>This model of agents follows the notion mentioned by Lucas [42, p. S401], “. . . we view or model an individual as a collection of decision rules. . . . These decision rules are *continuously under review*

distinguished and what these types should be are difficult issues to be addressed within this highly dynamical evolving environment.

Finally, in agent-based stock markets, we can also observe what agents (artificial traders) really believe in the deep of their mind when they are trading. This exploration is probably the most striking feature of the *agent-based social simulation paradigm*. Not only can we observe the *macro-phenomena* of our artificial society, e.g. the joint dynamics of prices and trading volume; but we can also watch the *micro-behavior* of every heterogeneous agents to the details of their thought processes, e.g. the forecasting models or trading strategies these agents used. Via this feature, we can then trace how the behaviors and interaction of agents in the micro-level could generate the macro-level phenomena. Furthermore, we may see whether the agents' watching macro-phenomena would change their behaviors, and hence may transform the whole financial dynamics into different scenarios (the so-called *regime change*). These complex feedback relations can not be well captured by the traditional representative agent model.

The rest of the paper is organized as follows. Section 2 describes the agent-based stock market considered in this paper. In Section 3, we briefly depicts experimental designs we adopted. Section 4 introduces the concept of Granger causality and two different econometric test used in this paper. Section 5 gives the simulation and testing results both of the "top" and of the "bottom", followed by the concluding remarks in Section 6.

## 2 The agent-based artificial stock market

The agent-based stock markets considered in this paper is AIE-ASM, version 3, developed by AI-ECON Research Center [13, 15]. The basic framework of the AIE-ASM is the standard asset pricing model in the vein of Grossman and Stiglitz [28]. The dynamics of the market is determined by interactions of many heterogeneous agents. Each of them, based on his forecast of the future, maximizes his expected utility.

### 2.1 Traders

For simplicity, we assume that all traders share the same *constant absolute risk aversion* (CARA) utility function,

$$U(W_{i,t}) = -\exp(-\lambda W_{i,t}), \quad (1)$$

where  $W_{i,t}$  is the wealth of trader  $i$  at period  $t$ , and  $\lambda$  is the degree of absolute risk aversion. Traders can accumulate their wealth by making investments. There are two assets available for traders to invest. One is the riskless interest-bearing asset called *money*, and the other is the risky asset known as the *stock*. In other words, at each period, each trader has two ways to keep his wealth, i.e.,

$$W_{i,t} = M_{i,t} + P_t h_{i,t}, \quad (2)$$

where  $M_{i,t}$  and  $h_{i,t}$  denote the money and shares of the stock held by trader  $i$  at period  $t$  respectively, and  $P_t$  is the price of the stock at period  $t$ . Given this portfolio  $(M_{i,t}, h_{i,t})$ , a

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*and revision*; new decision rules are tried and tested against experience, and *rules that produce desirable outcomes supplant those that do not.*" (Italics added.) To model this kind of adapting agents, agent-based computational economists borrow multi-agent techniques and artificial intelligence (AI) tools from the field of computer science.

trader's total wealth  $W_{i,t+1}$  is thus

$$W_{i,t+1} = (1+r)M_{i,t} + h_{i,t}(P_{t+1} + D_{t+1}), \quad (3)$$

where  $D_t$  is per-share *cash dividends* paid by the companies issuing the stocks and  $r$  is the riskless interest rate.  $D_t$  can follow a *stochastic process* not known to traders. Given this wealth dynamics, the goal of each trader is to myopically maximize the one-period expected utility function,

$$E_{i,t}(U(W_{i,t+1})) = E(-\exp(-\lambda W_{i,t+1})|I_{i,t}), \quad (4)$$

subject to Equation (3), where  $E_{i,t}(\cdot)$  is trader  $i$ 's conditional expectations of  $W_{t+1}$  given his information up to  $t$  (the information set  $I_{i,t}$ ).

It is well known that under CARA utility and Gaussian distribution for forecasts, trader  $i$ 's desire demand,  $h_{i,t+1}^*$  for holding shares of risky asset is linear in the expected *excess return*:

$$h_{i,t}^* = \frac{E_{i,t}(P_{t+1} + D_{t+1}) - (1+r)P_t}{\lambda\sigma_{i,t}^2}, \quad (5)$$

where  $\sigma_{i,t}^2$  is the conditional variance of  $(P_{t+1} + D_{t+1})$  given  $I_{i,t}$ .

The key point in the agent-based artificial stock market is the formation of  $E_{i,t}(\cdot)$ . In this paper, the expectation is modeled by *genetic programming*. The detail is described in the next subsection.

## 2.2 Price Determination

Given  $h_{i,t}^*$ , the market mechanism is described as follows. Let  $b_{i,t}$  be the number of shares trader  $i$  would like to submit a bid to buy at period  $t$ , and let  $o_{i,t}$  be the number of shares trader  $i$  would like to offer to sell at period  $t$ . It is clear that

$$b_{i,t} = \begin{cases} h_{i,t}^* - h_{i,t-1}, & h_{i,t}^* \geq h_{i,t-1}, \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

and

$$o_{i,t} = \begin{cases} h_{i,t-1} - h_{i,t}^*, & h_{i,t}^* < h_{i,t-1}, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Furthermore, let

$$B_t = \sum_{i=1}^N b_{i,t}, \quad \text{and} \quad O_t = \sum_{i=1}^N o_{i,t}$$

be the totals of the bids and offers for the stock at period  $t$ , where  $N$  is the number of traders. Following Palmer et al. [44], we use the following simple rationing scheme:

$$h_{i,t} = \begin{cases} h_{i,t-1} + b_{i,t} - o_{i,t}, & \text{if } B_t = O_t, \\ h_{i,t-1} + \frac{O_t}{B_t}b_{i,t} - o_{i,t}, & \text{if } B_t > O_t, \\ h_{i,t-1} + b_{i,t} - \frac{B_t}{O_t}o_{i,t}, & \text{if } B_t < O_t. \end{cases} \quad (8)$$

All these cases can be subsumed into

$$h_{i,t} = h_{i,t-1} + \frac{V_t}{B_t} b_{i,t} - \frac{V_t}{O_t} o_{i,t}, \quad (9)$$

where  $V_t \equiv \min(B_t, O_t)$  is the volume of trade in the stock.

According to Palmer et al.'s *rationing scheme*, we can have a very simple price adjustment scheme, based solely on the *excess demand*  $B_t - O_t$ :

$$P_{t+1} = P_t(1 + \beta(B_t - O_t)) \quad (10)$$

where  $\beta$  is a function of the difference between  $B_t$  and  $O_t$ .  $\beta$  can be interpreted as the speed of adjustment of prices. The  $\beta$  function we consider is:

$$\beta(B_t - O_t) = \begin{cases} \tanh(\beta_1(B_t - O_t)), & \text{if } B_t \geq O_t, \\ \tanh(\beta_2(B_t - O_t)), & \text{if } B_t < O_t, \end{cases} \quad (11)$$

where  $\tanh$  is the *hyperbolic tangent function*:

$$\tanh(x) \equiv \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

The price adjustment process introduced above implicitly assumes that the total number of shares of the stock circulated in the market is fixed, i.e.,

$$H_t = \sum_{i=1}^N h_{i,t} = H. \quad (12)$$

In addition, we assume that dividends and interests are all paid by cash, so

$$M_{t+1} = \sum_{i=1}^N M_{i,t+1} = M_t(1 + r) + H_t D_{t+1}. \quad (13)$$

### 2.3 Formation of Expectations

As to the formation of traders' expectations,  $E_{i,t}(P_{t+1} + D_{t+1})$ , we assume the following functional form for  $E_{i,t}(\cdot)$ .<sup>5</sup>

$$E_{i,t}(P_{t+1} + D_{t+1}) = \begin{cases} (P_t + D_t)(1 + \theta_1 f_{i,t} \times 10^{-4}), & \text{if } -10^4 \leq f_{i,t} \leq 10^4, \\ (P_t + D_t)(1 + \theta_1), & \text{if } f_{i,t} > 10^4, \\ (P_t + D_t)(1 - \theta_1), & \text{if } f_{i,t} < -10^4. \end{cases} \quad (14)$$

The population of  $f_{i,t}$  ( $i=1, \dots, N$ ) is formed by genetic programming. That means, the value of  $f_{i,t}$  is decoded from its GP tree  $gp_{i,t}$ .<sup>6</sup>

As to the subjective risk equation, we modified the equation originally used by Arthur et al. [2],

$$\sigma_{i,t}^2 = (1 - \theta_2)\sigma_{t-1|n_1}^2 + \theta_2(P_t + D_t - E_{i,t-1}(P_t + D_t))^2, \quad (15)$$

<sup>5</sup>There are several alternatives to model traders' expectations. The interested reader is referred to Chen et al. [15].

<sup>6</sup>See Chen and Yeh [13] for more details about the GP-based evolutionary forecasting processes.

where

$$\sigma_{t-1|n_1}^2 = \frac{\sum_{j=0}^{n_1-1} (P_{t-j} - \bar{P}_{t|n_1})^2}{n_1 - 1},$$

and

$$\bar{P}_{t|n_1} = \frac{\sum_{j=0}^{n_1-1} P_{t-j}}{n_1}.$$

In other words,  $\sigma_{t-1|n_1}^2$  is simply the *historical volatility* based on the past  $n_1$  observations.

Given each trader's expectations,  $E_{i,t}(P_{t+1} + D_{t+1})$ , according to equation (5) and his own subjective risk equation, we can obtain each trader's desire demand,  $h_{i,t+1}^*$  shares of the stock, and then how many shares of stock each trader intends to bid or offer based on equation (6) or (7).

### 3 Experimental designs and data description

#### 3.1 Experimental designs

As mentioned earlier, our simulations are based on the software, AIE-ASM, version 3. A tutorial on this software can be found in Chen et al. [15]. This tutorial would help explain most of the parameters shown in Table 1 and 2, which we shall skip its details except mentioning that most parameter values are taken from Chen and Yeh [13]. The simulations presented in this paper are mainly based on three different designs. These designs are motivated by our earlier studies on the ABSM, in particular, Chen and Yeh [13, 14] and Chen and Liao [10]. These three designs differs in two key economic parameters, namely, dividend processes and risk attitude.

In *Market A*, the baseline market, the dividend process is assumed to be *iid* Gaussian distribution and the traders' measure of absolute risk aversion ( $\lambda$ ) are assumed to be 0.1. In *Market B*, the traders are assumed to be more risk-averse, which is characterized by a higher degree of absolute risk aversion ( $\lambda = 0.5$ ). As to *Market C*, the dividends are assumed to be *iid* uniform distribution, while the traders' attitude toward risk are assumed to be the same as that of the baseline market. Three runs each with 5000 generations was conducted for each of the three markets. Table 2 is a summary of our experimental designs.

#### 3.2 Data description

The data generated from each run of simulation is then used to test the existence of price-volume relation. As we mentioned in Section 1, Granger causality is used to define the dynamic relation between prices and trading volume. Following the standard econometric procedure, we first applied the augmented Dickey-Fuller unit root test to examine the stationarities of the price series,  $P_t$ , and trading volume series,  $V_t$ . Based on the testing results, difference transformation was taken to make sure that all time series are stationary:

$$r_t = \ln(P_t) - \ln(P_{t-1}), \quad v_t = V_t - V_{t-1},^7$$

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<sup>7</sup>The reason that we did not take the log-difference transformation for volume is that trading volume may be zero in some trading periods.

Table 1: Parameters of the stock market

<b>The Stock Market</b>	
Share per capita ( $h$ )	1
Initial money supply per capita ( $m$ )	100
Interest Rate ( $r$ )	0.1
Stochastic process ( $D_t$ )	<i>i.i.d.</i> Normal( $\mu = 10, \sigma^2 = 4$ ) [Market A, Market B]
	<i>i.i.d.</i> Uniform(5,15) [Market C]
Price adjustment function	tanh
Price adjustment ( $\beta_1$ )	$10^{-4}$
Price adjustment ( $\beta_2$ )	$0.2 \times 10^{-4}$
<b>Traders</b>	
Number of traders	500
Degree of ARA ( $\lambda$ )	0.1 [Market A, Market C], 0.5 [Market B]
Criterion of fitness (traders)	Increments in wealth
Sample size of $\sigma_t^2$	10
Evaluation cycle	1
Sample size	10
Search intensity	5
$\theta_1$	0.5
$\theta_2$	$10^{-4}$
$\theta_3$	0.0133

where  $r_t$  is also known as stock return. We then examined the causal relation between  $r_t$  and  $v_t$ . To test whether there is any uni-directional causality from one variable to the other, we followed the conventional approach in econometrics, i.e. linear Granger causality test and, for nonlinear case, the modified Baek and Brock test. There are several different ways to conduct the Granger causality test: some tests require an arbitrary choice of filtering processes, and others require an arbitrary choice of lags. We shall briefly present these notions of causality and the procedures of tests in the next section.

## 4 Wiener-Granger causality: definition and testing

The concept of *causality* plays a crucial role in many empirical economic studies, and is particularly important for our understanding and interpretation of dynamic economic phenomena. Nevertheless, it is difficult to give a formal notion of causality. This issue, in fact, is a philosophical one (see, e.g. Geweke [25]). Wiener [50], however, proposed a widely accepted concept of causality based on *predictive relation* between the two time series in question. This notion of causality, known as *Wiener-Granger causality* (or simply *Granger causality*), was then introduced to economists by Granger [26].

In this section, we first review the definition of causality in Wiener-Granger's sense, followed by introducing two different versions of Granger-causality tests proposed by Granger

Table 1: Parameters of the stock market

<b>Business School</b>	
Number of faculty members	500
Proportion of trees initiated	
by the full method	0.5
by the grow method	0.5
Function set	{+, −, ×, ÷, √, sin, cos, exp, Rlog, abs}
Terminal set	{ $P_t, P_{t-1}, \dots, P_{t-10}, V_{t-1}, \dots, V_{t-10}$ $P_{t-1} + D_{t-1}, \dots, P_{t-10} + D_{t-10}$ }
Selection scheme	Tournament selection
Tournament size	2
Proportion of offspring trees created	
by reproduction ( $p_r$ )	0.1
by crossover ( $p_c$ )	0.7
by mutation ( $p_m$ )	0.2
Probability of mutation	0.0033
Mutation scheme	Tree mutation
Replacement scheme	Tournament selection
Maximum depth of tree	17
Number of generations	5,000
Maximum in the domain of RExp	1,700
Criterion of fitness (faculty)	MAPE
Evaluation cycle	20
Sample size (MAPE)	10

himself [26] and Hiemstra and Jones [31]. The former can only be applied to test the *linear* causal relation, whereas the latter is the extension of the former to the *nonlinear* cases.

#### 4.1 Definition

Suppose that we have two *stationary* time series, i.e.  $\{X_t\}$  and  $\{Y_t\}$ ,  $t = 1, 2, \dots$ , in hand. Without loss of generality, we shall illustrate Wiener-Granger's definition and testing procedures by showing how to conduct uni-directional causality tests from  $\{Y_t\}$  to  $\{X_t\}$ . In the Wiener-Granger's definition [50, 26],  $\{Y_t\}$  *fails to cause*  $\{X_t\}$  if we remove the past values of  $Y_t$  from the information set, we can get *no worse prediction* of present and future values of  $X_t$ , only provided by lagged values of  $X_t$  itself. Formally, it is defined as follows:

**Definition 1 (Wiener-Granger causality)** *Let  $F(X_t|\mathbf{I}^*)$  be the conditional probability distribution of  $X_t$  given some information set  $\mathbf{I}^*$ . Under certain lag lengths of  $L_x$  and  $L_y$ ,  $\{Y_t\}$  fails to cause  $\{X_t\}$  in Wiener-Granger's sense if:*

$$F(X_t|\mathbf{I}_{t-1}) = F(X_t|(\mathbf{I}_{t-1} - \mathbf{Y}_{t-L_y}^{L_y})), t = 1, 2, 3, \dots, \quad (16)$$

where  $\mathbf{I}_{t-1} \equiv (\mathbf{X}_{t-L_x}^{L_x}, \mathbf{Y}_{t-L_y}^{L_y})$  is the bivariate information set consisting of an  $L_x$ -length lag vector of  $X_t$  and an  $L_y$ -length lag vector of  $Y_t$ , i.e.  $\mathbf{X}_{t-L_x}^{L_x} \equiv (X_{t-L_x}, X_{t-L_x+1}, \dots, X_{t-1})$  and  $\mathbf{Y}_{t-L_y}^{L_y} \equiv (Y_{t-L_y}, Y_{t-L_y+1}, \dots, Y_{t-1})$ .

Table 2: Experimental Designs

Market	Case	Stochastic Process of Dividends	Measure of ARA <sup>‡</sup>
Market A	A1, A2, A3	<i>i.i.d.</i> Normal( $\mu = 10, \sigma^2 = 4$ )	0.1
Market B	B1, B2, B3	<i>i.i.d.</i> Normal( $\mu = 10, \sigma^2 = 4$ )	0.5
Market C	C1, C2, C3	<i>i.i.d.</i> Uniform(5,15)	0.1

<sup>‡</sup> Note that ARA stands for absolute risk aversion.

Conversely, if the lagged values of  $Y_t$  have *predictive power* for the present and future values of  $X_t$ , then we conclude that the time series  $\{Y_t\}$  *Wienr-Granger-cause* (or simply *Granger-cause*) the time series  $\{X_t\}$ .

## 4.2 Linear Granger causality testing: vector autoregression (VAR) approach

Based on the definition given above, Wiener-Granger causality refers to a historical path of one time series which influences the probability distribution of the present and future path of another time series. However, the definition in equation (16) is not easy to test. Granger [26], therefore, proposed a testable form by restricting the original concept to a *linear* prediction model. In other words, he assumed that predictors are *least-squares projections*, and *mean square error* (MSE) is adopted to be the criterion for comparing predictive power:

**Definition 2 (linear Granger causality)** *Given certain lag lengths of  $Lx$  and  $Ly$ ,  $\{Y_t\}$  fails to LINEARLY Granger-cause  $\{X_t\}$  (denoted by  $Y_t \nrightarrow X_t$ ) if:*

$$\text{MSE}(\hat{\text{E}}(X_t|\mathbf{I}_{t-1})) = \text{MSE}(\hat{\text{E}}(X_t|(\mathbf{I}_{t-1} - \mathbf{Y}_{t-Ly}^{Ly}))), \quad (17)$$

where  $\text{MSE}(\hat{\text{E}}(X_t|\mathbf{I}^*))$  denotes the mean square error for a prediction of  $X_t$  based on some information set  $\mathbf{I}^*$ .

According to the definition of (linear) Granger causality given above, we now consider the following well-known bivariate *vector autoregression* (VAR) equations:

$$X_t = c + \sum_{i=1}^{Lx} \alpha_i X_{t-i} + \sum_{j=1}^{Ly} \beta_j Y_{t-j} + \varepsilon_t, \quad (18)$$

$$Y_t = c' + \sum_{i=1}^{Ly'} \alpha'_i Y_{t-i} + \sum_{j=1}^{Lx'} \beta'_j X_{t-j} + \eta_t, \quad (19)$$

where the disturbances,  $\{\varepsilon_t\}$  and  $\{\eta_t\}$ , are two uncorrelated series following the conventional assumptions of *white noises*, say, they are *i.i.d.* with zero mean and some common variance of  $\sigma^2$  such that

$$\text{E}(\varepsilon_t \varepsilon_s) = \text{E}(\eta_t \eta_s) = 0, \quad \forall s \neq t,$$

and

$$\text{E}(\varepsilon_t \eta_s) = 0, \quad \forall s, t.$$

It has been shown by Granger [26] that if  $\{Y_t\}$  does not Granger-cause  $\{X_t\}$  (linearly), then it is equivalent to say that  $\beta_j = 0$ , for all  $j = 1, 2, \dots, Ly$  (Equation 18). Similarly,  $\{X_t\}$  does not Granger-cause  $\{Y_t\}$  (linearly) if, and only if,  $\beta'_j = 0$ , for all  $j = 1, 2, \dots, Lx'$  (Equation 19).

In this linear framework, we can then conduct the *Wald test* (an F- or an asymptotically equivalent  $\chi^2$ -test) for the null hypothesis:

$$H_0 : \beta_1 = \beta_2 = \dots = \beta_{Ly} = 0$$

in equation (18), or equivalently,

$$H_0 : Y_t \not\rightarrow X_t.^8$$

If the coefficients on the  $Ly$ -length lagged series of  $Y_t$  are jointly significantly different from zero, then we can conclude that the time series  $\{Y_t\}$  Granger-cause the time series  $\{X_t\}$ , or, equivalently, that *lagged  $Y_t$  has statistically significant linear forecasting power for current  $X_t$* . By following the same procedure, we can also test whether  $\{X_t\}$  Granger-cause  $\{Y_t\}$  (denoted by  $X_t \rightarrow Y_t$ ) or not.

Unfortunately, in order to conduct the tests illustrated above, we face a knotty problem of lag-length selection. More specifically, we need to choose appropriate lag lengths of  $X_t$  and  $Y_t$ , that is, the values of  $Lx$ ,  $Ly$ ,  $Lx'$  and  $Ly'$ . In the earlier empirical studies, researchers often chose the lag-length by some rules of thumb (*ad hoc* methods). Nevertheless, as Hsiao [32] has shown, it would often be the case that the distributions of test statistics, and hence the results of the (linear) Granger causality tests, are sensitive to the choice of lag lengths. To cope with this technical issue, several *statistical search criteria*, viz. AIC, FPE, BEC, etc., are used to determine the optimal lag structure in equation (18) and (19).<sup>9</sup>

### 4.3 Nonlinear Granger causality testing: modified Baek and Brock approach

The test procedures stated in the last subsection has been widely adopted by economists in the empirical studies to detect causal relationships between two time-dependent variables of interests. As a result, the earlier studies on the price-volume relation focused exclusively on *linear* causalities [1, 34, 45, 46, 48]. Such VAR approach, nevertheless, has low power in uncovering *nonlinear* causalities (see Brock [4] and Baek and Brock [3]).

Following the definition of Weiner-Granger causality presented in equation (16), Baek and Brock [3] proposed a nonparametric statistical counterpart for detecting *nonlinear* causal relations. To do so, their technique is based on the *correlation integral*, which is an estimator of spatial dependence across time. By first filtering out linear predictive power with the VAR model in equation (18) and (19), they argued that any remaining predictive power existing between the two residual series of  $\{\hat{\varepsilon}_t\}$  and  $\{\hat{\eta}_t\}$  can be considered as nonlinear one. Their test is built upon the assumptions of mutually independent and individually *i.i.d.* for the two series of residuals. This method was modified by Hiemstra and Jones [31] to allow for the residuals being weakly dependent.

<sup>8</sup>Those who are not familiar with these test procedures are referred to Hamilton [29, pp. 302–309] for a comprehensive reference.

<sup>9</sup>See Jones [36] for a survey of the non-statistical *ad hoc* methods and those statistical criteria for specifying optimal lag lengths in (linear) Granger causality testing.

**Definition 3 (modified Baek and Brock approach)** Consider the two estimated residual series from the VAR model in equation (18) and (19), i.e.  $\{\hat{\varepsilon}_t\}$  and  $\{\hat{\eta}_t\}$ . Assume that they are strictly stationary and weakly dependent. We then define the following notations:

$$\begin{aligned}\mathbf{E}_t^m &\equiv (\hat{\varepsilon}_t, \hat{\varepsilon}_{t+1}, \dots, \hat{\varepsilon}_{t+m-1}), \quad m = 1, 2, \dots, \quad t = 1, 2, \dots, \\ \mathbf{E}_{t-Lx}^{Lx} &\equiv (\hat{\varepsilon}_{t-Lx}, \hat{\varepsilon}_{t-Lx+1}, \dots, \hat{\varepsilon}_{t-1}), \quad Lx = 1, 2, \dots, \quad t = Lx + 1, Lx + 2, \dots, \\ \mathbf{H}_{t-Ly}^{Ly} &\equiv (\hat{\eta}_{t-Ly}, \hat{\eta}_{t-Ly+1}, \dots, \hat{\eta}_{t-1}), \quad Ly = 1, 2, \dots, \quad t = Ly + 1, Ly + 2, \dots\end{aligned}$$

Given certain lag lengths of  $Lx$  and  $Ly \geq 1$ ,  $\{Y_t\}$  fails to NONLINEARLY Granger-cause  $\{X_t\}$  (denoted by  $Y_t \not\Rightarrow X_t$ ) if:

$$\begin{aligned}\Pr(\|\mathbf{E}_t^m - \mathbf{E}_s^m\| < e \mid \|\mathbf{E}_{t-Lx}^{Lx} - \mathbf{E}_{s-Lx}^{Lx}\| < e, \|\mathbf{H}_{t-Ly}^{Ly} - \mathbf{H}_{s-Ly}^{Ly}\| < e) \\ = \Pr(\|\mathbf{E}_t^m - \mathbf{E}_s^m\| < e \mid \|\mathbf{E}_{t-Lx}^{Lx} - \mathbf{E}_{s-Lx}^{Lx}\| < e),\end{aligned}\quad (20)$$

for some pre-designated values of lead length  $m$  and distance  $e > 0$ . Note that  $\Pr(\cdot)$  denotes probability and  $\|\cdot\|$  denotes the sup norm.

In order to transform equation (20) into a testable form, we denote the joint and marginal probabilities by :

$$\begin{aligned}C_1(m + Lx, Ly, e) &\equiv \Pr(\|\mathbf{E}_{t-Lx}^{m+Lx} - \mathbf{E}_{s-Lx}^{m+Lx}\| < e, \|\mathbf{H}_{t-Ly}^{Ly} - \mathbf{H}_{s-Ly}^{Ly}\| < e), \\ C_2(Lx, Ly, e) &\equiv \Pr(\|\mathbf{E}_{Lx}^{t-Lx} - \mathbf{E}_{Lx}^{s-Lx}\| < e, \|\mathbf{H}_{t-Ly}^{Ly} - \mathbf{H}_{s-Ly}^{Ly}\| < e), \\ C_3(m + Lx, e) &\equiv \Pr(\|\mathbf{E}_{t-Lx}^{m+Lx} - \mathbf{E}_{s-Lx}^{m+Lx}\| < e), \\ C_4(Lx, e) &\equiv \Pr(\|\mathbf{E}_{t-Lx}^{Lx} - \mathbf{E}_{s-Lx}^{Lx}\| < e).\end{aligned}\quad (21)$$

By the definition of conditional probability, say  $\Pr(A|B) = \Pr(A \cap B)/\Pr(B)$ , we can modify equation (20) slightly into

$$\frac{C_1(m + Lx, Ly, e)}{C_2(Lx, Ly, e)} = \frac{C_3(m + Lx, e)}{C_4(Lx, e)},\quad (22)$$

for some given values of  $m$ ,  $Lx$ , and  $Ly \geq 1$  and  $e > 0$ . This implies that  $\{Y_t\}$  does not Granger-cause  $\{X_t\}$  (nonlinearly) if equation (22) holds.

Baek and Brock [3] suggested correlation-integral estimators for the joint and marginal probabilities in equation (21) — denoted as  $\hat{C}_1(m+Lx, Ly, e)$ ,  $\hat{C}_2(Lx, Ly, e)$ ,  $\hat{C}_3(m+Lx, e)$ , and  $\hat{C}_4(Lx, e)$  — to test the condition (22).<sup>10</sup>

Then Baek and Brock [3] constructed the following asymptotic test statistic, for given values of  $m$ ,  $Lx$ , and  $Ly \geq 1$  and  $e > 0$ ,<sup>11</sup>

$$\sqrt{n} \left( \frac{\hat{C}_1(m + Lx, Ly, e)}{\hat{C}_2(Lx, Ly, e)} - \frac{\hat{C}_3(m + Lx, e)}{\hat{C}_4(Lx, e)} \right) \overset{a}{\sim} N(0, \sigma^2(m, Lx, Ly, e)).\quad (23)$$

<sup>10</sup>For the definition and details of these correlation-integral estimators, see Hiemstra and Jones [31, p. 1647].

<sup>11</sup>Rather than asymptotic distribution theory, Diks and DeGoede [19] proposed another approach to test the equivalence in equation (22) based on *bootstrap methods*. They reported that their bootstrap tests and the modified Baek and Brock test performed almost equally well.

The asymptotic *Gaussian distribution* of this test statistic holds under the null hypothesis that  $\{Y_t\}$  does not Granger-cause  $\{X_t\}$  (nonlinearly), i.e.  $H_0 : Y_t \not\Rightarrow X_t$ . By further using the *delta method*,<sup>12</sup> Hiemstra and Jones [31, pp. 1660–1662] suggested a consistent estimator for  $\sigma^2(m, Lx, Ly, e)$  in equation (23) to conduct the test empirically.

Note that a significant positive value in equation (23) suggests that  $\{Y_t\}$  does Granger-cause  $\{X_t\}$  (nonlinearly). Nevertheless, a significant negative test statistic represents that “knowledge of the lagged values of  $Y$  *confounds* the prediction of  $X$ ” (*italics added*, see Hiemstra and Jones [31, p. 1648]). Thus, we conduct the modified Baek and Brock test with *right-tailed critical values*. Like the VAR approach in linear Granger-causality testing, we face the difficulties in choosing appropriate lagged length of  $Lx$  and  $Ly$ . Unfortunately, unlike linear Granger-causality testing, there is no literature discussing how to specify the optimal values of those parameters, i.e.  $m$ ,  $Lx$ ,  $Ly$ , and  $e$ . In this paper, we simply follow Hiemstra and Jones [31] to tackle this issue.

## 5 Experimental results

We first summarize some basic descriptive statistics of our simulation results in Table 3.<sup>13</sup> Some essential features, such as price deviation (or price discovery) and excess volatility, were already studied in our earlier paper [10, 12]. The summary statistics reported in this table shows nothing significantly different from what we already discussed. We, therefore, shall focus exclusively on the price-volume relation in this paper. The presentation of our results shall follow the sequence indicated below. First, we start from the aggregate data (the macro-level). At this level, the issue concerns us is whether price-volume causality exists. Second, we then go down to the “bottom” level, and examine the microstructure of traders. Finally, what we have found at the “top” is compared to what we found at the “bottom”, to see whether the micro-macro relation can be consistent.

### 5.1 Aggregate outcomes: Granger causality at the “top”

Table 4 gives the testing result of linear causality. The result is mixing. In some cases, the causal relation is not found in both directions. In some other cases, the uni-direction causality is found. Clearly, the existence of the causal relation is not definite. This picture is somewhat in line with what we learned from the literature: some found the existence of linear causality, while some didn’t.

Table 5 shows the result of nonlinear causality, and the result is also inconclusive, which

<sup>12</sup>The delta method is a prevailing tool in econometric studies. It helps to derive asymptotic distributions for arbitrary nonlinear functions of an estimator. See Campbell et al. [7, p. 540] for a brief illustration.

<sup>13</sup>Note that HREEP stands for *homogeneous rational expectation equilibrium price*. In the model which we construct in section 2, it can be derived that

$$\text{HREEP} = \frac{1}{r}(\bar{d} - \lambda\sigma_d^2 \frac{H}{N}),$$

by further incorporating the assumptions of a representative-agent with rational expectation and perfect foresight. See Chen and Liao [10] for the proof. We further define  $\bar{P} = \frac{1}{T} \sum P_t$ ,  $\text{MAPE} = \frac{1}{T} \sum \left| \frac{P_t - \text{HREEP}}{\text{HREEP}} \right|$ , and  $\text{MPE} = \frac{1}{T} \sum \left( \frac{P_t - \text{HREEP}}{\text{HREEP}} \right)$  to show how far the artificial stock prices deviate from the HREE price. Also note that  $\sigma_P$ , the standard deviation of prices, shows the price volatility of the artificial stock markets.

Table 3: Basic Descriptive Statistics

Case	HREEP	$\bar{P}$	MAPE	MPE	$\sigma_P$
A1	96	104.72	9.21%	9.08%	5.004
A2	96	103.84	8.39%	8.16%	5.085
A3	96	104.54	9.07%	8.90%	5.054
B1	80	84.25	6.01%	5.31%	3.967
B2	80	84.53	6.16%	5.66%	3.762
B3	80	84.21	5.93%	5.26%	3.838
C1	91.667	108.32	18.16%	18.16%	5.350
C2	91.667	108.24	18.08%	18.08%	5.359
C3	91.667	108.54	18.42%	18.41%	5.666

is also consistent with what we experienced from the literature. The bi-directional non-linear causality is found only in case B2 and B3, while the uni-directional causality from return to volume exists in many cases. The return-to-volume casual relation is in general much stronger than the volume-to-return causality.

Table 4: Linear Granger causality test

Case	$H_0$ : Volume changes do not cause stock returns ( $v_t \nrightarrow r_t$ )			$H_0$ : Stock returns do not cause volume changes ( $r_t \nrightarrow v_t$ )		
	# of Lags	F-value	p-value	# of Lags	F-value	p-value
A1	16	1.942	0.0134*	20	1.030	0.4218
A2	7	1.154	0.3261	18	1.243	0.2166
A3	16	1.398	0.1324	18	1.246	0.2145
B1	10	1.262	0.2459	20	1.020	0.4331
B2	10	4.832	0.0000*	20	1.074	0.3701
B3	10	2.510	0.0052*	18	1.314	0.1672
C1	7	0.579	0.7733	20	0.503	0.9671
C2	14	0.650	0.8244	20	0.987	0.4744
C3	8	2.519	0.0099*	17	0.897	0.5778

Note that a \* represents statistical significance at the 5% significance level, whereas a † represents statistical significance at the 10% significance level.

## 5.2 Traders' behavior: Granger causality at the “bottom”

Coming down to the “bottom” of the ABSM, we are interested in knowing the belief of agents. Did agents believe the price-volume relation? Did they actually apply volume to their forecasts of prices (returns)? To answer these questions, we have to check how many traders might in fact use past trading volume as useful information during forecasting

processes in the deep of their mind. That is to say, we have to check whether the traders incorporated trading volume into their expectation-generating formula (their GP trees).

To make the discussion convenient, we shall call those who believe trading volume as useful information to predict future prices as *price-volume believers*. Applying the technique invented by Chen and Yeh [14], we counted the number of price-volume believers. Since the counting work is very computational demanding, a census was made only after every 500 generations. This number is given in Table 6. In some cases, say B3, C1, and C2, the belief of price-volume relation prevails in the public from the beginning even to the end of the simulations. In some other cases, such as A1, A3, and C3, price-volume believers finally died out of the markets in the end. Note that the number of price-volume believers may fluctuate during the whole simulation periods, e.g. B1 and B2. A striking phenomena is that price-volume believers may revive even after some periods of noughts. A2 is a case in point. We now ready to check whether the marco-phenomena of price-volume relation we observed at the “top” matches what we observed at the “bottom”. This issue, called *consistency*, is checked in the next subsection.

### 5.3 The macro-micro relation

In the agent-based modeling framework, we are particularly interested in the so-called *macro-micro relation*. Based on the simulation results we have, four basic patterns stand out. They can be roughly divided into two categories, namely, consistent patterns and inconsistent ones. A pattern is called *consistent* if the macro behavior tends to lend support to what most individuals believe or come to believe. A pattern is called *inconsistent* if the macro behavior tends to invalidate what most individual believe or come to believe (see Table 7).

In a more technical way, let that the volume does not Granger-cause returns be the null hypothesis. If this null hypothesis is rejected (or failed to reject) by the aggregate market outcome based on econometric tests, then we say the pattern is *consistent* if it is also rejected (or failed to reject) by most or by an increasing number of market participants. Otherwise, it is called *inconsistent*.

According to the definition above, the case A2, A3, B1 and B3 exhibit consistent patterns (the main diagonal boxes on Table 7), whereas the case A1, B2, C1, C2 and C3 demonstrate inconsistent patterns (on the off-diagonal boxes on Table 7).

Among the consistent patterns, B3 is the case that the null hypothesis is consistently rejected by both macro and micro behavior. Its number of price-volume believers is persistently high during the entire simulation. In particular, for the second half of the trading session, almost all agents rejected the null by forecasting returns with volume (see Table 6).

A2, A3 and B1 are the other consistent patterns. In these three cases, the null was failed to reject in both linear and nonlinear tests, and our traders’ beliefs were in line with this test result. The number of participants who believe the null hypothesis continuously decreased. For example, consider the case A3. At the beginning, there are a great number of traders who used volume in their forecasts of returns. Nonetheless, after period 1500, the number dramatically drops down from 300 to 100, and further to nil.

Among the inconsistent patterns (patterns on the off-diagonal of Table 7), C1 and

C2 share the feature that the market is composed of hundreds of *price-volume believers*, while the causality test shows that the volume cannot help predict returns. This result is particularly striking for the case C2, where the market reached a state where all market participants are price-volume believers.

Equally interesting inconsistent patterns are cases A1, B2 and C3. In these cases, the causality test did indicate the significance of volume in return forecasting, but traders eventually gave up the use of this variable in their forecasts of returns.

## 5.4 Discussions

The analysis so far is mainly driven by the aggregate outcome. Basically, we are asking whether the individual behavior is consistent with our econometric tests. In other words, if our tests suggest the causal relation, did our “smart” and “adaptive” also notice so?

The real issue is whether those inconsistent patterns are unanticipated or puzzling us. The answer is no. There are, in effect, some arguments to predict why these inconsistent patterns may appear. For example, consider the cases C1 and C2. A supportive argument would be following: it is the intensive search, characterized by a large number of price-volume believers, over the hidden relation between volume and returns eventually nullify the effect of volume on returns and make volume be an useless variable. In this case, the micro and macro relation observed in cases A3 and B3 is actually also in harmony with each other. As a matter of fact, using this argument, one can question whether those consistent patterns are really consistent. For instance, if no one give the volume variable a try, would it possible that the volume-to-price relation can finally emerge, as a secret which has never been disclosed?

The argument which we have just been through points out one serious issue in our above-proposed analysis of micro and macro relation. In this analysis, we treat the whole micro process as one sample, and the whole macro process as the other sample. We then look into the consistence between the two. However, what was neglected is the complex dynamic feedback relation existing between aggregate outcome and individual behavior. As well depicted by Farmer and Lo [22, p. 9992],

*Patterns in the price tend to disappear as agents evolve profitable strategies to exploit them, but this occurs only over an extended period of time, during which substantial profits may be accumulated and new patterns may appear.*

As to the cases A1 and C3, we saw that there exists only linear Granger causality between returns and trading volume at the macro level. Nevertheless, from the micro viewpoint, traders were not aware of this. One possible explanation for observing such kind of inconsistency is the huge search space defined by GP. The set of linear function has only a measure of zero in it. If we restrict our attention only to the non-linear causality test, then there is no inconsistency for the cases A1 and C3. It follows that traders may overlook the usefulness of linear models, and spent most of their trials over the space of non-linear models. As we may expect, they eventually gave up their attempts, because non-linear causality does not exist. However, this explanation can not applied to Case B2, in which nonlinear-causal relation is also shown to exist statistically significantly.

To sum up, there is no definite relation between micro and macro behavior. The appearance of the patterns on the off-diagonal entries shows that the Neo-classical economic analysis, which generally assumes the consistency between the micro and macro behavior, does not have a solid ground. It is in this agent-based economic model we show how easily one can have aggregate result which is not anticipated from the individual behavior. The reason that one can have such a large variety of patterns is mainly because of complex dynamic interaction between individuals and the market.

Financial market dynamics is path-dependent, highly complex and nonlinear because it is the outcome of continuously evolving and interacting behavior, which is mainly driven by survival pressure. It is therefore difficult to make a simple conclusion on the relation between micro and macro behavior. To fully trace their interaction, the analysis based on high-frequency sampling (or census) of traders' behavior is required. Statistical analysis based on small samples is also useful to investigate the potential time variant relation, due to the real time survival pressure.

## 6 Conclusions

One distinguishing feature of ACE (and thus ABSMs) is that some interesting macro phenomena of financial markets could emerge (be endogenously generate) from interactions of adaptive agents without exogenously imposing any conditions like unexpected events, information cascade, noise or dumb traders, etc. In this paper, we show that the presence of the stock price-volume causal relation does not require any explicit assumptions like information asymmetry, reaction asymmetry, noise traders, or tax motives. In fact, it suggests that the causal relation may be a generic property in a market modeled as an evolving decentralized system of autonomous interacting agents.

We also show that our understanding of the appearance or disappearance of the price-volume relation can never be complete if the feedback relation between individual behavior and aggregate outcome is neglected. This feedback relation is, however, highly complex, which may defy any simple analysis, as the one we proposed initially. Consequently, econometric analysis which fails to take into account this complex feedback relation between micro and macro may produce misleading results. Unfortunately, we are afraid that is exactly the main stream financial econometrics did in a large pile of empirical studies.

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Table 5: Nonlinear Granger causality test

Case	$H_0$ : Volume changes do not cause stock returns ( $v_t \nRightarrow r_t$ )			$H_0$ : Stock returns do not cause volume changes ( $r_t \nRightarrow v_t$ )		
	# of Lags	TVAL	p-value	# of Lags	TVAL	p-value
A1	1	0.091	0.4638	1	1.030	0.1515
	2	0.652	0.2573	2	1.932	0.0267*
	3	0.878	0.1899	3	1.502	0.0666 <sup>†</sup>
	4	1.169	0.1212	4	0.238	0.4060
	5	0.668	0.2520	5	1.029	0.1518
	6	0.117	0.4533	6	1.436	0.0755 <sup>†</sup>
	7	-0.314	0.6234	7	0.905	0.1827
	8	-0.382	0.6487	8	0.951	0.1708
	9	-0.067	0.5267	9	1.028	0.1519
	10	-0.109	0.5434	10	0.956	0.1695
A2	1	-0.582	0.7196	1	-0.409	0.6586
	2	-1.413	0.9212	2	1.114	0.1327
	3	-0.775	0.7809	3	0.273	0.3923
	4	-1.252	0.8947	4	0.300	0.3821
	5	-0.383	0.6493	5	-0.087	0.5345
	6	-0.428	0.6656	6	0.802	0.2112
	7	0.770	0.2207	7	0.187	0.4258
	8	1.046	0.1479	8	0.518	0.3022
	9	0.648	0.2585	9	0.152	0.4397
	10	0.575	0.2825	10	-0.122	0.5486
A3	1	-0.636	0.7377	1	2.180	0.0146*
	2	-0.678	0.7512	2	1.837	0.0331*
	3	-0.141	0.5560	3	2.191	0.0142*
	4	-0.213	0.5844	4	2.303	0.0106*
	5	-1.008	0.8432	5	1.230	0.1095
	6	-0.905	0.8173	6	0.654	0.2565
	7	-1.407	0.9202	7	1.841	0.0328*
	8	-1.569	0.9416	8	0.983	0.1628
	9	-1.811	0.9649	9	1.813	0.0350*
	10	-1.350	0.9115	10	2.907	0.0018*

Note that a \* represents statistical significance at the 5% significance level, whereas a <sup>†</sup> represents statistical significance at the 10% significance level.

Table 5: Nonlinear Granger causality test

Case	$H_0$ : Volume changes do not cause stock returns ( $v_t \not\Rightarrow r_t$ )			$H_0$ : Stock returns do not cause volume changes ( $r_t \not\Rightarrow v_t$ )		
	# of Lags	TVAL	p-value	# of Lags	TVAL	p-value
B1	1	-0.999	0.8412	1	-0.198	0.5783
	2	-1.043	0.8515	2	0.019	0.4925
	3	-1.534	0.9374	3	1.139	0.1274
	4	-0.933	0.8245	4	0.873	0.1914
	5	-0.447	0.6724	5	0.617	0.2685
	6	-0.094	0.5373	6	0.235	0.4071
	7	-0.158	0.5626	7	0.212	0.4161
	8	-0.052	0.5208	8	0.182	0.4278
	9	0.013	0.4950	9	-0.117	0.5468
	10	-0.227	0.5899	10	-0.916	0.8201
B2	1	4.193	0.0000*	1	3.490	0.0002*
	2	5.344	0.0000*	2	2.419	0.0078*
	3	4.938	0.0000*	3	2.377	0.0087*
	4	4.560	0.0000*	4	2.799	0.0026*
	5	5.564	0.0000*	5	2.917	0.0018*
	6	5.244	0.0000*	6	2.384	0.0086*
	7	5.083	0.0000*	7	2.294	0.0109*
	8	4.017	0.0000*	8	2.256	0.0120*
	9	3.256	0.0006*	9	2.650	0.0040*
	10	3.179	0.0007*	10	2.920	0.0018*
B3	1	1.492	0.0678 <sup>†</sup>	1	2.147	0.0159*
	2	1.847	0.0324*	2	2.204	0.0138*
	3	1.759	0.0393*	3	2.326	0.0100*
	4	2.375	0.0088*	4	3.724	0.0001*
	5	2.925	0.0017*	5	3.259	0.0006*
	6	2.297	0.0108*	6	3.699	0.0001*
	7	2.542	0.0055*	7	3.570	0.0002*
	8	1.467	0.0712 <sup>†</sup>	8	3.498	0.0002*
	9	1.166	0.1218	9	2.234	0.0128*
	10	1.259	0.1041	10	2.335	0.0098*

Table 5: Nonlinear Granger causality test

Case	$H_0$ : Volume changes do not cause stock returns ( $v_t \not\Rightarrow r_t$ )			$H_0$ : Stock returns do not cause volume changes ( $r_t \not\Rightarrow v_t$ )		
	# of Lags	TVAL	p-value	# of Lags	TVAL	p-value
C1	1	0.322	0.3738	1	0.451	0.3260
	2	0.885	0.1880	2	-0.512	0.6957
	3	1.234	0.1085	3	-0.436	0.6686
	4	1.377	0.0843 <sup>†</sup>	4	0.560	0.2877
	5	1.265	0.1030	5	0.625	0.2659
	6	1.003	0.1580	6	1.084	0.1393
	7	0.506	0.3065	7	0.239	0.4057
	8	0.537	0.2955	8	0.341	0.3664
	9	0.248	0.4022	9	0.097	0.4612
	10	0.089	0.4644	10	0.158	0.4371
C2	1	0.829	0.2035	1	-2.073	0.9809
	2	-0.462	0.6779	2	-0.149	0.5594
	3	-0.381	0.6484	3	-0.001	0.5003
	4	0.039	0.4845	4	0.282	0.3889
	5	0.518	0.3023	5	-0.358	0.6400
	6	0.219	0.4133	6	-0.518	0.6979
	7	0.178	0.4294	7	-0.421	0.6632
	8	-0.084	0.5334	8	0.709	0.2390
	9	-0.264	0.6040	9	1.238	0.1078
	10	-0.515	0.6967	10	0.860	0.1948
C3	1	-0.833	0.7975	1	-0.686	0.7536
	2	-0.247	0.5976	2	-1.797	0.9639
	3	-0.381	0.6483	3	-0.878	0.8100
	4	-0.854	0.8035	4	0.009	0.4964
	5	-0.983	0.8373	5	0.712	0.2382
	6	-0.751	0.7738	6	0.967	0.1667
	7	-0.177	0.5702	7	0.873	0.1912
	8	-0.084	0.5336	8	1.320	0.0934 <sup>†</sup>
	9	-0.316	0.6241	9	2.453	0.0071*
	10	-0.511	0.6955	10	2.069	0.0193*

Table 6: Number of price-volume believers in total 500 agents

Generation	Case								
	A1	A2	A3	B1	B2	B3	C1	C2	C3
1	200	179	278	180	187	241	188	187	250
500	45	106	322	387	176	431	325	317	263
1000	0	28	371	297	0	415	281	302	94
1500	0	0	170	124	69	398	313	313	192
2000	0	0	86	208	92	382	237	496	182
2500	0	3	23	134	470	496	350	500	7
3000	0	0	125	0	279	465	351	500	3
3500	0	3	32	0	133	500	477	499	0
4000	0	0	0	42	1	492	416	446	0
4500	0	0	0	16	0	476	325	498	0
5000	0	3	0	24	0	498	273	483	0

Table 7: Macro-micro interactions

	Market participants used the volume to forecast price	Market participants did not use the volume to forecast price
$v_t \rightarrow r_t$ or $v_t \Rightarrow r_t$	B3	A1, B2, C3
$v_t \nrightarrow r_t$ and $v_t \nRightarrow r_t$	C1, C2	A2, A3, B1