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Information Sciences 170 (2005) 75–100

INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL

www.elsevier.com/locate/ins

Agent-based computational modeling of the stock price–volume relation

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Received 10 March 2001; accepted 18 March 2003

Abstract

From the perspective of the agent-based model of stock markets, this paper examines the possible explanations for the presence of the causal relation between stock returns and trading volume. Using the agent-based approach, we find that the explanation for the presence of the stock price–volume relation may be more fundamental. Conventional devices such as information asymmetry, reaction asymmetry, noise traders or tax motives are not explicitly required. In fact, our simulation results show that the stock price–volume relation may be regarded as a generic property of a financial market, when it is correctly represented as an evolving decentralized system of autonomous interacting agents. One striking feature of agent-based models is the rich profile of agents' behavior. This paper makes use of the advantage and investigates the micro–macro relations within the market. In particular, we trace the evolution of agents' beliefs and examine their consistency with the observed aggregate market behavior. We argue that a full understanding of the price–volume relation cannot be accomplished unless the feedback relation between individual behavior at the bottom and aggregate phenomena at the top is well understood.

Keywords: Agent-based model; Artificial stock markets; Genetic programming; Granger causality test; Stock price–volume relation; Micro–macro relation

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doi:10.1016/j.ins.2003.03.026

1. Motivation and introduction

The agent-based modeling of stock markets, which originated at the Santa Fe Institute [2,47], is a fertile and promising field that can be thought of as a subfield of agent-based computational economics (ACE). [1]Up to the present, most of the research efforts have been devoted to the analysis of the *price dynamics* and/or *market efficiency* of the artificial markets (e.g. [13,14,44,57]). Some studies have focused on the price deviation or mispricing in the artificial stock markets (e.g. [2,8,10,12,43,44,47,56]). Some have gone further to explore the corresponding *micro-structure* of the markets, such as the aspect of *traders' beliefs and behavior* (e.g. [11,13,14]). Nevertheless, few have ever visited the univariate dynamics of trading volume series [43,56], and, to our best knowledge, none has addressed joint dynamics with prices. [2]

As Ying [58] noted almost 40 years ago, stock prices and trading volume are *joint products* from *one single market mechanism*. He argued that ''any model of the stock market which separates prices from volume or vice versa will inevitably yield incomplete if not erroneous results'' [58, p. 676]. In similar vein, Gallant et al. [25] also asserted that researchers can learn more about the very nature of stock markets by studying the *joint dynamics* of prices in conjunction with volume, instead of focusing on price dynamics alone. As a result, the stock price–volume relation has been an interesting subject in financial economics for many years. [3]

While most of the earlier empirical work focused on the *contemporaneous* relation between trading volume and stock returns, some more recent studies began to address the *dynamic relation*, i.e. *causality*, between daily stock returns and trading volume following the notion of *Granger causality* proposed by Wiener [55] and Granger [27]. In many cases, a bi-directional Granger causality (or a *feedback* relation) was found to exist in the stock price–volume relation, although some other studies could only find evidence of a uni-directional causality: Either returns would *Granger-cause* trading volume, or the opposite situation would prevail [1,37,48,49,51].

As noted by Granger [28], Hsieh [35], and many others, we live in a world which is ''almost certainly nonlinear''. We cannot be satisfied with only

[1] As Farmer and Lo [22] mentioned, ''Evolutionary and ecological models of financial markets is truly a new frontier whose exploration has just begun''. By modeling financial markets ''as evolving systems of autonomous interacting agents'', the agent-based approach in finance, indeed, follows this evolutionary paradigm [54]. Visit *the ACE website* maintained by Leigh Tesfatsion for a comprehensive guide to the field of *ACE*. <*URL:*`http://www.econ.iastate.edu/tesfatsi/ace.htm`>.

[2] See Chen [9] or LeBaron [42] for reviews of the field of artificial financial markets.

[3] See the survey article by Karpoff [40].

exploring the linear causality between stock prices and trading volume. Nonlinear causality would naturally be the next step to pursue. Baek and Brock [3] argued that traditional Granger causality tests based on vector autoregression (VAR) models might overlook significant nonlinear relations. As a result, they proposed a nonlinear Granger causality test by using *nonparametric estimators of temporal relations within and across time series*. This approach can be applied to any two stationary, mutually independent and individually *i.i.d.* series. Hiemstra and Jones [32] modified their test slightly to allow the two series under consideration to display "weak (or short-term) temporal dependence". Several researchers have already adopted this modified Baek and Brock test to uncover price and volume causal relation in real world financial markets [23,32,50]. In most of the cases, they found bi-directional *nonlinear* Granger causality in the prices and trading volume. In other words, not only did stock returns Granger-cause trading volume, but trading volume also Granger-caused stock returns. The significance of this finding is that trading volume can help predict stock returns, or as an old Wall Street adage goes, "It takes volume to make price move".

There are several possible explanations for the presence of a causal relation between stock returns and trading volume in the literature. First, Epps [20] gave an explanation based on the *asymmetric reaction* of two groups of investors—"bulls" and "bears"—to the positive information and negative information.

The second explanation, which is referred to as the *mixture of distributions hypothesis*, considers special distributions of speculative prices. For example, Epps and Epps [21] derived a model in which trading volume is used to measure disagreement among traders concerning their beliefs with regard to the variance of the price changes. On the other hand, in Clark's [16] mixture of distributions model, the speed of information flow is a latent common factor which influences stock returns and trading volume simultaneously.

A third explanation is the *sequential arrival of information models* (see, for example, Copeland [17], He and Wang [31], Jennings et al. [38], and Morse [46]). In this asymmetric information world, traders possess differential pieces of new information in the beginning. Before the final complete information equilibrium is achieved, the information is disseminated to different traders only gradually and sequentially. This implies a positive relationship between price changes and trading volume.

Lakonishok and Smidt [41] proposed still another model which involves *tax- and nontax-related motives* for trading. For the sake of window dressing, portfolio rebalancing, or the optimal timing for capital gains, traders may have some special kinds of trading behavior. As a result, Lakonishok and Smidt [41] showed that current trading volume can be related to past price changes because of these motives.

In moving away from the traditional representative agent models stated above, recent theoretical works have started to model financial markets with *heterogeneous traders*. Besides informed traders (insiders), DeLong et al. [18] introduced *noise traders* with positive-feedback trading strategies into their model. Noise traders do not have any information about the fundamentals and trade solely based on past price movements. As a result, a positive causal relation from stock returns to trading volume appears. In Brock's [5] nonlinear theoretical noise trading model, the estimation errors made by different groups of traders are correlated. Under these settings, he could find that stock price movements and volatilities are related nonlinearly to volume movements. Campbell et al. [6] developed another heterogeneous agent model, in which there are two different types of risk-averse traders. In their frameworks, they are able to explain the autocorrelation properties of stock returns in terms of a nonlinear relation with trading volume.

In light of these explanations, this paper attempts to see whether we can replicate the causal relation between stock returns and trading volume via the agent-based stock markets (ABSMs). We consider the agent-based model of stock markets to be highly relevant to this issue. First, the existing explanations mentioned above were based on assumptions either related to the information dissemination schemes or to the traders' reaction styles in regard to information arrival. Since both of these factors are well encapsulated in ABSMs, it is interesting to see whether ABSMs are able to replicate the causal relation. Secondly, information dissemination schemes and traders' behavior are known as *emergent phenomena* in ABSMs. In other words, these factors are endogenously generated rather than exogenously imposed. This feature can allow us to search for a fundamental explanation for the causal relation. For example, we can ask, "without the assumption of information asymmetry, reaction asymmetry, or noise traders, and so on, can we still have the causal relation?" Briefly, "is the causal relation a generic phenomenon?"

Thirdly, we claim that the agent-based models of financial markets are "true" heterogeneous agent models, which depict the real markets more faithfully. We might think of the models proposed by DeLong et al. [18] and their successors as having pre-specified representative agents of two different types, say, a representative rational informed trader and a representative uninformed noise trader. These settings might overlook some important features of financial markets, for example, the interaction and feedback dynamics of traders. In the agent-based approach, we, however, do not assign any agent as being of any specific type exogenously. As a matter of fact, we do not even have the device of representative agents. Hundreds of agents in the model can all have different behavioral rules which they themselves evolve (adapt) over time. [4] How many types there are by which they can be distinguished and what these types should be named are difficult issues to be addressed within this highly dynamical evolving environment. However, this is the reality of the real world, isn't it?

Finally, in ABSMs, we can also observe what agents (artificial traders) really believe in the depths of their minds when they are trading. This exploration is probably the most striking feature of the *agent-based social simulation paradigm*. Not only can we observe the *macro-phenomena* of our artificial society, e.g., the joint dynamics of prices and trading volume, but we can also watch the *micro-behavior* of every heterogeneous agent down to the details of their thought processes, e.g., the forecasting models or trading strategies that these agents use. Via this feature, we can then trace how the behavior and interaction of agents at the mirco-level can generate macro-level phenomena. Furthermore, we may see whether the agents observing macro-phenomena would change their behavior, and hence may transform the whole financial dynamics into different scenarios (the so-called *regime change*). These complex feedback relations cannot be well captured by the traditional representative agent model.

The rest of the paper is organized as follows: Section 2 describes the ABSM considered in this paper. Section 3 outlines the experimental designs. Section 4 introduces the concept of Granger causality and two different econometric tests used in this paper. Section 5 gives the simulation and testing results both for the "top" and the "bottom", followed by the concluding remarks in Section 6.

2. The agent-based artificial stock market

The ABSM considered in this paper is the AIE-ASM, Version 3, developed by the AI-ECON Research Center [13,15]. The basic framework of the AIE-ASM is the standard asset pricing model in the vein of Grossman and Stiglitz [29]. The dynamics of the market are determined by the interactions of many heterogeneous agents. Each of them, based on his forecast of the future, maximizes his expected utility.

[4] This model of agents follows the notion mentioned by Lucas [45, p. S401], "...we view or model an individual as a collection of decision rules...These decision rules are *continuously under review and revision*; new decision rules are tried and tested against experience, and *rules that produce desirable outcomes supplant those that do not*" (italics added). To model these kinds of adaptive agents, agent-based computational economists borrow multi-agent techniques and artificial intelligence (AI) tools from the field of computer science. See Holland and Miller [33] for the foundations for building *artificial adaptive agents* in economic models.

2.1. Traders

For simplicity, we assume that all traders share the same constant absolute risk aversion (CARA) utility function,

$$U(W_{i,t}) = -\exp(-\lambda W_{i,t}), \tag{1}$$

where $W_{i,t}$ is the wealth of trader i in period t, and λ is the degree of absolute risk aversion. Traders can accumulate their wealth by making investments. There are two assets available for traders to invest in. One is the riskless interest-bearing asset called *money*, and the other is the risky asset known as the *stock*. In other words, in each period, each trader has two ways of keeping his wealth, i.e.

$$W_{i,t} = M_{i,t} + P_t h_{i,t}, \tag{2}$$

where $M_{i,t}$ and $h_{i,t}$ denote the money and shares of the stock held by trader i in period t, respectively, and P_t is the price of the stock in period t. Given this portfolio ($M_{i,t}$,$h_{i,t}$), a trader's total wealth $W_{i,t+1}$ is thus

$$W_{i,t+1} = (1 + r_{\mathrm{f}})M_{i,t} + h_{i,t}(P_{t+1} + D_{t+1}), \tag{3}$$

where D_t denotes the per-share *cash dividends* paid by the companies issuing the stocks and r_{f} is the riskless interest rate. D_t can follow *a stochastic process* not known to traders. Given these wealth dynamics, the goal of each trader is to myopically maximize the one-period expected utility function,

$$E_{i,t}(U(W_{i,t+1})) = E(-\exp(-\lambda W_{i,t+1})|I_{i,t}), \tag{4}$$

subject to Eq. (3), where $E_{i,t}(\cdot)$ is trader i's conditional expectations of W_{t+1} given his information up to t (the information set $I_{i,t}$).

It is well known that, under CARA utility and Gaussian distributions for the forecasts, trader i's desired demand, $h^*_{i,t+1}$, for holding shares in the risky asset is linear in terms of the expected *excess return*:

$$h^*_{i,t} = \frac{E_{i,t}(P_{t+1} + D_{t+1}) - (1 + r_{\mathrm{f}})P_t}{\lambda \sigma^2_{i,t}}, \tag{5}$$

where $\sigma^2_{i,t}$ is the conditional variance of $(P_{t+1} + D_{t+1})$ given $I_{i,t}$.

The key point in relation to the agent-based artificial stock market is the formation of $E_{i,t}(\cdot)$. In this paper, the expectation is modeled by *genetic programming*. The details are described in the next subsection.

2.2. Price determination

Given $h^*_{i,t}$, the market mechanism is described as follows: Let $b_{i,t}$ be the number of shares trader i would like to submit a bid to buy in period t, and let

$o_{i,t}$ be the number of shares trader i would like to offer to sell in period t. It is clear that

$$b_{i,t} = \begin{cases} h^*_{i,t} - h_{i,t-1}, & h^*_{i,t} \geqslant h_{i,t-1}, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

and

$$o_{i,t} = \begin{cases} h_{i,t-1} - h^*_{i,t}, & h^*_{i,t} < h_{i,t-1}, \\ 0, & \text{otherwise}. \end{cases} \tag{7}$$

Furthermore, let

$$B_t = \sum_{i=1}^{N} b_{i,t} \quad \text{and} \quad O_t = \sum_{i=1}^{N} o_{i,t}$$

be the totals of the bids and offers for the stock in period t, where N is the number of traders. Following Palmer et al. [47], we use the following simple rationing scheme:

$$h_{i,t} = \begin{cases} h_{i,t-1} + b_{i,t} - o_{i,t} & \text{if } B_t = O_t, \\ h_{i,t-1} + \frac{O_t}{B_t} b_{i,t} - o_{i,t} & \text{if } B_t > O_t, \\ h_{i,t-1} + b_{i,t} - \frac{B_t}{O_t} o_{i,t} & \text{if } B_t < O_t. \end{cases} \tag{8}$$

All of these cases can be subsumed into

$$h_{i,t} = h_{i,t-1} + \frac{V_t}{B_t} b_{i,t} - \frac{V_t}{O_t} o_{i,t}, \tag{9}$$

where $V_t \equiv \min(B_t, O_t)$ is the volume of trade in the stock.

According to Palmer et al.'s *rationing scheme*, we can have a very simple price adjustment scheme, based solely on the *excess demand* $B_t - O_t$:

$$P_{t+1} = P_t(1 + \beta(B_t - O_t)), \tag{10}$$

where β is a function of the difference between B_t and O_t. β can be interpreted as the speed of adjustment of prices. The β function we consider is

$$\beta(B_t - O_t) = \begin{cases} \tanh(\beta_1(B_t - O_t)) & \text{if } B_t \geqslant O_t, \\ \tanh(\beta_2(B_t - O_t)) & \text{if } B_t < O_t, \end{cases} \tag{11}$$

where tanh is the hyperbolic tangent function:

$$\tanh(x) \equiv \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

The price adjustment process introduced above implicitly assumes that the total number of shares of the stock circulated in the market is fixed, i.e.

$$H_t = \sum_{i=1}^{N} h_{i,t} = H. \tag{12}$$

In addition, we assume that dividends and interest are all paid with cash, so that

$$M_{t+1} = \sum_{i=1}^{N} M_{i,t+1} = M_t(1 + r_{\mathrm{f}}) + H_t D_{t+1}. \tag{13}$$

2.3. Formation of expectations

As to the formation of traders' expectations, $E_{i,t}(P_{t+1} + D_{t+1})$, we assume the following functional form for $E_{i,t}(\cdot)$: [5]

$$E_{i,t}(P_{t+1} + D_{t+1}) = \begin{cases} (P_t + D_t)(1 + \theta_1 f_{i,t} \times 10^{-4}) & \text{if } -10^4 \leqslant f_{i,t} \leqslant 10^4, \\ (P_t + D_t)(1 + \theta_1) & \text{if } f_{i,t} > 10^4, \\ (P_t + D_t)(1 - \theta_1) & \text{if } f_{i,t} < -10^4. \end{cases} \tag{14}$$

The population of $f_{i,t}$ $(i = 1, \ldots, N)$ is formed by genetic programming. That means the value of $f_{i,t}$ is decoded from its GP tree $gp_{i,t}$. [6]

As to the subjective risk equation, we modified the equation originally used by Arthur et al. [2]:

$$\sigma^2_{i,t} = (1 - \theta_2)\sigma^2_{t-1|n_1} + \theta_2(P_t + D_t - E_{i,t-1}(P_t + D_t))^2, \tag{15}$$

where

$$\sigma^2_{t-1|n_1} = \frac{\sum_{j=0}^{n_1-1}(P_{t-j} - \overline{P}_{t|n_1})^2}{n_1 - 1}$$

and

$$\overline{P}_{t|n_1} = \frac{\sum_{j=0}^{n_1-1} P_{t-j}}{n_1}.$$

In other words, $\sigma^2_{t-1|n_1}$ is simply the *historical volatility* based on the past n_1 observations.

Given each trader's expectations, $E_{i,t}(P_{t+1} + D_{t+1})$, according to Eq. (5) and his own subjective risk equation, we can obtain each trader's desired demand, $h^*_{i,t+1}$ shares of the stock, and then determine how many shares of the stock each trader intends to bid or offer based on Eq. (6) or (7).

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[5] There are several alternatives to model traders' expectations. The interested reader is referred to Chen et al. [15].

[6] See Chen and Yeh [13] for more details about the GP-based evolutionary forecasting processes.

3. Experimental designs and data description

3.1. Experimental designs

As mentioned earlier, our simulations are based on the software, AIE-ASM, Version 3. A tutorial on this software can be found in [15]. This tutorial explains most of the parameters shown in Table 1, the details of which we shall skip except for mentioning that most parameter values are taken from [13]. The simulations presented in this paper are mainly based on three different designs. These designs are motivated by our earlier studies on the ABSM, in particular [10,13,14]. These three designs differ in two key economic parameters, namely, dividend processes and risk attitude.

In Market A, the baseline market, the dividend process is assumed to be *iid* Gaussian and the traders' degree of absolute risk aversion (λ) is assumed to be 0.1. In Market B, the traders are assumed to be more risk-averse, which is characterized by a higher degree of absolute risk aversion ($\lambda = 0.5$). As to Market C, the dividends are assumed to be *iid* uniformly distributed, while traders' attitudes toward risk are assumed to be the same as those in the baseline market. Three runs were conducted for each of the three markets, each with 5000 generations. Table 2 is a summary of our experimental designs.

3.2. Data description

The data generated from each run of the simulations is then used to test for the existence of the price–volume relation. As we mentioned in Section 1, Granger causality is employed to define the dynamic relation between prices and trading volume. Following the standard econometric procedure, we first applied the augmented Dickey–Fuller unit root test to examine the stationarities of the price series, P_t, and the trading volume series, V_t. Based on the testing results, the difference transformation was taken to make sure that all time series were stationary:

$$r_t = \ln(P_t) - \ln(P_{t-1}), \quad v_t = V_t - V_{t-1},^{7}$$

where r_t is also known as the stock return. We then examined the causal relation between r_t and v_t. To test whether there is any uni-directional causality from one variable to the other, we followed the conventional approach in econometrics, i.e. the linear Granger causality test and, for the nonlinear case,

[7] The reason why we did not take the log-difference transformation for volume is that trading volume may be zero in some trading periods.

Table 1
Parameters of the stock market

The stock market	
Shares per capita (h)	1
Initial money supply per capita (m)	100
Risk-free interest rate (r_f)	0.1
Stochastic process (D_t)	*i.i.d.* Normal ($\mu = 10, \sigma^2 = 4$) [Market A, Market B]
	i.i.d. Uniform (5, 15) [Market C]
Price adjustment function	tanh
Price adjustment (β_1)	10^{-4}
Price adjustment (β_2)	0.2×10^{-4}
Traders	
Number of traders	500
Degree of ARA (λ)	0.1 [Market A, Market C], 0.5 [Market B]
Criterion of fitness (traders)	`Increments in wealth`
Sample size of σ_t^2	10
Evaluation cycle	1
Sample size	10
Search intensity	5
θ_1	0.5
θ_2	10^{-4}
θ_3	0.0133
Business school	
Number of faculty members	500
Proportion of trees initiated	
By the full method	0.5
By the grow method	0.5
Function set	$\{+, -, \times, \div, \surd, \sin, \cos, \exp, \mathrm{Rlog}, \mathrm{abs}\}$
Terminal set	$\{P_t, P_{t-1}, \ldots, P_{t-10}, V_{t-1}, \ldots, V_{t-10}, P_{t-1} + D_{t-1}, \ldots, P_{t-10} + D_{t-10}\}$
Selection scheme	`Tournament selection`
Tournament size	2
Proportion of offspring trees created	
By reproduction (p_r)	0.1
By crossover (p_c)	0.7
By mutation (p_m)	0.2
Probability of mutation	0.0033
Mutation scheme	`Tree mutation`
Replacement scheme	`Tournament selection`
Maximum depth of tree	17
Number of generations	5000
Maximum in the domain of RExp	1700
Criterion of fitness (faculty)	`MAPE`
Evaluation cycle	20
Sample size (MAPE)	10

the modified Baek and Brock test. We briefly present these notions of causality and the associated implementation procedures in the next section.

Table 2
Experimental designs

Market	Case	Stochastic process of dividends	Measure of ARA[a]
Market A	A1, A2, A3	*i.i.d.* Normal ($\mu = 10, \sigma^2 = 4$)	0.1
Market B	B1, B2, B3	*i.i.d.* Normal ($\mu = 10, \sigma^2 = 4$)	0.5
Market C	C1, C2, C3	*i.i.d.* Uniform (5, 15)	0.1

[a] Note that ARA stands for "absolute risk aversion".

4. Wiener–Granger causality: definition and testing

The concept of *causality* plays a crucial role in many empirical economic studies, and is particularly important for our understanding and interpretation of dynamic economic phenomena. Nevertheless, it is difficult to give a formal notion of causality. This issue, in fact, is a philosophical one (see, e.g., Geweke [26]). Wiener [55], however, proposed a widely accepted concept of causality based on the *predictive relation* between the two time series in question. This notion of causality, known as *Wiener–Granger causality* (or simply *Granger causality*), was then introduced to economists by Granger [27].

In this section, we first review the definition of causality in the Wiener–Granger sense, followed by introducing two different versions of Granger-causality tests proposed by Granger himself [27] and Hiemstra and Jones [32]. The former can only be applied to test the *linear* causal relation, whereas the latter is the extension of the former to the *nonlinear* case.

4.1. Definition

Suppose that we have two *stationary* time series, i.e. $\{X_t\}$ and $\{Y_t\}$, where $t = 1, 2, \ldots$, in hand. Without loss of generality, we shall illustrate Wiener–Granger's definition and testing procedures by showing how to conduct unidirectional causality tests from $\{Y_t\}$ to $\{X_t\}$. Based on Wiener–Granger's definition [27,55], $\{Y_t\}$ *fails to cause* $\{X_t\}$ if we remove the past values of Y_t from the information set, we can get *no worse prediction* of present and future values of X_t, only provided by lagged values of X_t itself. Formally, it is defined as follows:

Definition 1 (*Wiener–Granger causality*). Let $F(X_t|\mathbf{I}^*)$ be the conditional probability distribution of X_t given some information set $\mathbf{I}^*$. Under certain lag lengths of Lx and Ly, $\{Y_t\}$ fails to cause $\{X_t\}$ in the Wiener–Granger sense if

$$F(X_t|\mathbf{I}_{t-1}) = F\big(X_t|\big(\mathbf{I}_{t-1} - \mathbf{Y}_{t-Ly}^{Ly}\big)\big), \quad t = 1, 2, 3, \ldots, \tag{16}$$

where $\mathbf{I}_{t-1} \equiv \left(\mathbf{X}_{t-Lx}^{Lx}, \mathbf{Y}_{t-Ly}^{Ly}\right)$ is the bi-variate information set consisting of an *Lx*-length lag vector of X_t and an *Ly*-length lag vector of Y_t, i.e. $\mathbf{X}_{t-Lx}^{Lx} \equiv (X_{t-Lx}, X_{t-Lx+1}, \ldots, X_{t-1})$ and $\mathbf{Y}_{t-Ly}^{Ly} \equiv (Y_{t-Ly}, Y_{t-Ly+1}, \ldots, Y_{t-1})$.

Conversely, if the lagged values of Y_t have *predictive power* for the present and future values of X_t, then we conclude that the time series $\{Y_t\}$ *Wiener–Granger-causes* (or simply *Granger-causes*) the time series $\{X_t\}$.

4.2. Linear Granger causality testing: vector autoregression (VAR) approach

Based on the definition given above, Wiener–Granger causality refers to a historical path of one time series which influences the probability distribution of the present and future path of another time series. However, the definition in Eq. (16) is not easy to test. Granger [27], therefore, proposed a testable form by restricting the original concept to a *linear* prediction model. In other words, he assumed that predictors are least-squares projections, and the mean square error (MSE) is adopted to be the criterion for comparing predictive power.

Definition 2 (*Linear Granger causality*). Given certain lag lengths of *Lx* and *Ly*, $\{Y_t\}$ fails to linearly Granger-cause $\{X_t\}$ (denoted by $Y_t \nrightarrow X_t$) if

$$\mathrm{MSE}\left(\widehat{E}(X_t|\mathbf{I}_{t-1})\right) = \mathrm{MSE}\left(\widehat{E}\left(X_t|\left(\mathbf{I}_{t-1} - \mathbf{Y}_{t-Ly}^{Ly}\right)\right)\right), \tag{17}$$

where $\mathrm{MSE}\left(\widehat{E}(X_t|\mathbf{I}^*)\right)$ denotes the MSE for a prediction of X_t based on some information set $\mathbf{I}^*$.

According to the definition of (linear) Granger causality given above, we now consider the following well-known bi-variate VAR equations:

$$X_t = c + \sum_{i=1}^{Lx} \alpha_i X_{t-i} + \sum_{j=1}^{Ly} \beta_j Y_{t-j} + \varepsilon_t, \tag{18}$$

$$Y_t = c' + \sum_{i=1}^{Ly'} \alpha_i' Y_{t-i} + \sum_{j=1}^{Lx'} \beta_j' X_{t-j} + \eta_t, \tag{19}$$

where the disturbances, $\{\varepsilon_t\}$ and $\{\eta_t\}$, are two uncorrelated series following the conventional assumptions of *white noises*, say, they are *i.i.d.* with zero mean and some common variance of σ^2 such that

$$E(\varepsilon_t\varepsilon_s) = E(\eta_t\eta_s) = 0 \quad \forall\ s \neq t,$$

and

$$E(\varepsilon_t\eta_s) = 0 \quad \forall\ s, t.$$

It has been shown by Granger [27] that if $\{Y_t\}$ does not Granger-cause $\{X_t\}$ (linearly), then this is equivalent to saying that $\beta_j = 0$ for all $j = 1, 2, \ldots, Ly$ (Eq. (18)). Similarly, $\{X_t\}$ does not Granger-cause $\{Y_t\}$ (linearly) if, and only if, $\beta'_j = 0$ for all $j = 1, 2, \ldots, Lx'$ (Eq. (19)).

In this linear framework, we can then conduct the *Wald test* (an F or an asymptotically equivalent χ^2-test) for the null hypothesis:

$$\mathrm{H}_0 : \beta_1 = \beta_2 = \cdots = \beta_{Ly} = 0$$

in Eq. (18), or equivalently,

$$\mathrm{H}_0 : Y_t \nrightarrow X_t.$$ [8]

If the coefficients of the Ly-length lagged series of Y_t are jointly significantly different from zero, then we can conclude that the time series $\{Y_t\}$ Granger-causes the time series $\{X_t\}$, or, equivalently, that *lagged Y_t has statistically significant linear forecasting power for current X_t*. By following the same procedure, we can also test whether $\{X_t\}$ Granger-causes $\{Y_t\}$ (denoted by $X_t \rightarrow Y_t$) or not.

Unfortunately, in order to conduct the tests illustrated above, we face a knotty problem of lag-length selection. More specifically, we need to choose appropriate lag lengths of X_t and Y_t, that is, the values of Lx, Ly, Lx' and Ly'. In the earlier empirical studies, researchers often chose the lag-length by some rules of thumb (ad hoc methods). Nevertheless, as Hsiao [34] has shown, it would often be the case that the distributions of test statistics, and hence the results of the (linear) Granger causality tests, are sensitive to the choice of lag lengths. To cope with this technical issue, several *statistical search criteria*, viz. AIC, FPE, BEC, etc., are used to determine the optimal lag structure of Eqs. (18) and (19) in the related literature. [9]

4.3. Nonlinear Granger causality testing: modified Baek and Brock approach

The earlier studies on the price–volume relation focused exclusively on *linear* causalities [1,37,48,49,51], and the test procedure stated in the last subsection has been widely adopted by economists in empirical studies to detect causal relationships between two time-dependent variables of interest. Such a VAR

[8] Those who are not familiar with these test procedures are referred to Hamilton [30, pp. 302–309], for a comprehensive reference.

[9] See Jones [39] for a survey of the nonstatistical ad hoc methods and those statistical criteria for specifying optimal lag lengths in (linear) Granger causality testing.

approach, nevertheless, has low power in terms of uncovering *nonlinear* causalities (see Brock [4] and Baek and Brock [3]).

Following the definition of Wiener–Granger causality presented in Eq. (16), Baek and Brock [3] proposed a nonparametric statistical counterpart for detecting *nonlinear* causal relations. To this end, their technique was based on the *correlation integral*, which is an estimator of spatial dependence across time. By first filtering out linear predictive power with the VAR model in Eqs. (18) and (19), they argued that any remaining predictive power existing between the two residual series of $\{\hat{\varepsilon}_t\}$ and $\{\hat{\eta}_t\}$ could be considered to be nonlinear. Their test is built upon the assumptions that the two series of residuals are mutually independent and individually *i.i.d.* This method was modified by Hiemstra and Jones [32] to allow for the residuals being weakly dependent.

Definition 3 (*Modified Baek and Brock approach*). Consider the two estimated residual series from the VAR model in Eqs. (18) and (19), i.e. $\{\hat{\varepsilon}_t\}$ and $\{\hat{\eta}_t\}$. Assume that they are strictly stationary and weakly dependent. We then define the following notations:

$$\begin{aligned}
&\mathbf{E}_t^m \equiv \left(\hat{\varepsilon}_t, \hat{\varepsilon}_{t+1}, \ldots, \hat{\varepsilon}_{t+m-1}\right), \quad m = 1, 2, \ldots, \ \ t = 1, 2, \ldots, \\
&\mathbf{E}_{t-Lx}^{Lx} \equiv \left(\hat{\varepsilon}_{t-Lx}, \hat{\varepsilon}_{t-Lx+1}, \ldots, \hat{\varepsilon}_{t-1}\right), \quad Lx = 1, 2, \ldots, \ \ t = Lx+1, Lx+2, \ldots, \\
&\mathbf{H}_{t-Ly}^{Ly} \equiv \left(\hat{\eta}_{t-Ly}, \hat{\eta}_{t-Ly+1}, \ldots, \hat{\eta}_{t-1}\right), \quad Ly = 1, 2, \ldots, \ \ t = Ly+1, Ly+2, \ldots
\end{aligned}$$

Given certain lag lengths of Lx and $Ly \geqslant 1$, $\{Y_t\}$ fails to nonlinearly Granger-cause $\{X_t\}$ (denoted by $Y_t \not\Rightarrow X_t$) if

$$\begin{aligned}
&\Pr\left(\|\mathbf{E}_t^m - \mathbf{E}_s^m\| < e \big| \|\mathbf{E}_{t-Lx}^{Lx} - \mathbf{E}_{s-Lx}^{Lx}\| < e, \|\mathbf{H}_{t-Ly}^{Ly} - \mathbf{H}_{s-Ly}^{Ly}\| < e\right) \\
&\qquad = \Pr\left(\|\mathbf{E}_t^m - \mathbf{E}_s^m\| < e \big| \|\mathbf{E}_{t-Lx}^{Lx} - \mathbf{E}_{s-Lx}^{Lx}\| < e\right),
\end{aligned} \tag{20}$$

for some pre-designated values of lead length m and distance $e > 0$. Note that $\Pr(\cdot)$ denotes probability and $\|\cdot\|$ denotes the sup norm.

In order to transform Eq. (20) into a testable form, we denote the joint and marginal probabilities by

$$\begin{aligned}
&C_1(m+Lx, Ly, e) \equiv \Pr\left(\left\|\mathbf{E}_{t-Lx}^{m+Lx} - \mathbf{E}_{s-Lx}^{m+Lx}\right\| < e, \left\|\mathbf{H}_{t-Ly}^{Ly} - \mathbf{H}_{s-Ly}^{Ly}\right\| < e\right), \\
&C_2(Lx, Ly, e) \equiv \Pr\left(\left\|\mathbf{E}_{Lx}^{t-Lx} - \mathbf{E}_{Lx}^{s-Lx}\right\| < e, \left\|\mathbf{H}_{t-Ly}^{Ly} - \mathbf{H}_{s-Ly}^{Ly}\right\| < e\right), \\
&C_3(m+Lx, e) \equiv \Pr\left(\left\|\mathbf{E}_{t-Lx}^{m+Lx} - \mathbf{E}_{s-Lx}^{m+Lx}\right\| < e\right), \\
&C_4(Lx, e) \equiv \Pr\left(\left\|\mathbf{E}_{t-Lx}^{Lx} - \mathbf{E}_{s-Lx}^{Lx}\right\| < e\right).
\end{aligned} \tag{21}$$

By the definition of conditional probability, say $\Pr(A|B) = \Pr(A \cap B)/\Pr(B)$, we can modify Eq. (20) slightly into

$$\frac{C_1(m+Lx,Ly,e)}{C_2(Lx,Ly,e)} = \frac{C_3(m+Lx,e)}{C_4(Lx,e)} \tag{22}$$

for some given values of m, Lx, and $Ly \geqslant 1$ and $e > 0$. This implies that $\{Y_t\}$ does not Granger-cause $\{X_t\}$ (nonlinearly) if Eq. (22) holds.

Baek and Brock [3] suggested correlation-integral estimators for the joint and marginal probabilities in Eq. (21)—denoted as $\widehat{C}_1(m+Lx,Ly,e)$, $\widehat{C}_2(Lx,Ly,e)$, $\widehat{C}_3(m+Lx,e)$, and $\widehat{C}_4(Lx,e)$—to test the condition (22). [10]

Then Baek and Brock [3] constructed the following asymptotic test statistic for given values of m, Lx, and $Ly \geqslant 1$ and $e > 0$: [11]

$$\sqrt{n}\left(\frac{\widehat{C}_1(m+Lx,Ly,e)}{\widehat{C}_2(Lx,Ly,e)} - \frac{\widehat{C}_3(m+Lx,e)}{\widehat{C}_4(Lx,e)}\right) \overset{a}{\sim} N(0,\sigma^2(m,Lx,Ly,e)). \tag{23}$$

The asymptotic *Gaussian distribution* of this test statistic holds under the null hypothesis that $\{Y_t\}$ does not Granger-cause $\{X_t\}$ (nonlinearly), i.e. $H_0 : Y_t \nRightarrow X_t$. By further using the *delta method*, [12] Hiemstra and Jones [32, pp.1660–1662] suggested using a consistent estimator for $\sigma^2(m,Lx,Ly,e)$ in Eq. (23) to conduct the test empirically.

Note that a significant positive value in Eq. (23) suggests that $\{Y_t\}$ does Granger-cause $\{X_t\}$ (nonlinearly). Nevertheless, a significant negative test statistic is indicative that "knowledge of the lagged values of Y *confounds* the prediction of X" (see Hiemstra and Jones [32, p. 1648], italics added). Thus, we conduct the modified Baek and Brock test only with *right-tailed critical values*. Like the VAR approach in linear Granger-causality testing, we face the same difficulty in choosing appropriate lagged lengths of Lx and Ly. Unfortunately, unlike linear Granger-causality testing, there is no literature discussing how to specify the optimal values of those parameters, i.e. m, Lx, Ly, and e. In this paper, we simply follow Hiemstra and Jones [32] to tackle this issue.

[10] For the definition and details of these correlation-integral estimators, see Hiemstra and Jones [32, p. 1647].

[11] Instead of asymptotic distribution theory, Diks and DeGoede [19] proposed another approach to test the equivalence in Eq. (22) based on *bootstrap methods*. They reported that their bootstrap tests and the modified Baek and Brock test performed almost equally well.

[12] The delta method is a prevailing tool in econometric studies. It helps to derive asymptotic distributions for arbitrary nonlinear functions of an estimator. See Cambpell et al. [7, p. 540] for a brief illustration.

Table 3
Basic descriptive statistics

Case	HREEP	$\overline{P}$	MAPE (%)	MPE (%)	σ_P
A1	96	104.72	9.21	9.08	5.004
A2	96	103.84	8.39	8.16	5.085
A3	96	104.54	9.07	8.90	5.054
B1	80	84.25	6.01	5.31	3.967
B2	80	84.53	6.16	5.66	3.762
B3	80	84.21	5.93	5.26	3.838
C1	91.667	108.32	18.16	18.16	5.350
C2	91.667	108.24	18.08	18.08	5.359
C3	91.667	108.54	18.42	18.41	5.666

5. Results of experiments

We first summarize some basic descriptive statistics of our simulation results in Table 3. [13] Some essential features, such as price deviation (or price discovery) and excess volatility, were already studied in our earlier papers [10,12]. The summary statistics reported in this table show nothing significantly different from what we found there. We therefore focus exclusively on the price–volume relation in this paper. The presentation of our results proceeds as follows: First, we start from the aggregate data (the macro-level). At this level, the issue that concerns us is whether price–volume causality exists. Second, we then go down to the "bottom" level, and examine the micro-structure of traders. Finally, what we find at the "top" is compared with what we found at the "bottom" to see whether the micro–macro relation is consistent.

5.1. Aggregate outcomes: Granger causality at the "top"

Table 4 gives the test results for linear causality. The results are mixed. In some cases, the causal relation is not found to exist in both directions. In some other cases, uni-directional causality is found. Clearly, the existence of the causal relation is not definite. This picture is somewhat in line with what we

[13] Note that HREEP stands for *homogeneous rational expectations equilibrium price*. In the model which we construct in Section 2, it can be derived that

$$\text{HREEP} = \frac{1}{r_{\text{f}}}\left(\bar{d} - \lambda\sigma_d^2\frac{H}{N}\right),$$

by further incorporating the assumptions of a representative agent with rational expectations and perfect foresight. See Chen and Liao [10] for the proof. We further define $\overline{P} = \frac{1}{T}\sum P_t$, $\text{MAPE} = \frac{1}{T}\sum|\frac{P_t - \text{HREEP}}{\text{HREEP}}|$, and $\text{MPE} = \frac{1}{T}\sum(\frac{P_t - \text{HREEP}}{\text{HREEP}})$ to show how far the artificial stock prices deviate from the HREE price. Also note that σ_P, the standard deviation of prices, shows the price volatility of the artificial stock markets.

Table 4
Linear Granger causality test

Case	H_0: Volume changes do not cause stock returns ($v_t \nrightarrow r_t$)			H_0: Stock returns do not cause volume changes ($r_t \nrightarrow v_t$)		
	# of lags	F-value	p-value	# of lags	F-value	p-value
A1	16	1.942	0.0134*	20	1.030	0.4218
A2	7	1.154	0.3261	18	1.243	0.2166
A3	16	1.398	0.1324	18	1.246	0.2145
B1	10	1.262	0.2459	20	1.020	0.4331
B2	10	4.832	0.0000*	20	1.074	0.3701
B3	10	2.510	0.0052*	18	1.314	0.1672
C1	7	0.579	0.7733	20	0.503	0.9671
C2	14	0.650	0.8244	20	0.987	0.4744
C3	8	2.519	0.0099*	17	0.897	0.5778

Note that * represents the rejection of the null at the 5% significant level.

learned from the literature: Some found the existence of linear causality, while some did not.

Table 5 shows the results for nonlinear causality, and the results are also inconclusive, which is also in line with what we experienced in the literature. The bi-directional nonlinear causality is found only in cases B2 and B3, while the uni-directional causality from returns-to-volume exists in many cases. The returns-to-volume causal relation is in general much stronger than the volume-to-returns causality.

5.2. Traders' behavior: Granger causality at the "bottom"

Coming down to the "bottom" of the ABSM, we are interested in knowing the beliefs of agents. Did agents believe in the price–volume relation? Did they actually apply volume to their forecasts of prices (returns)? To answer these questions, we have to check how many traders in fact used past trading volume to forecast future prices. That is to say, we have to check whether the traders incorporated trading volume into their expectations-generating formula (their GP trees).

To make the discussion convenient, we refer to those who believe that trading volume is useful information for predicting future prices as *price–volume believers*. By applying the technique invented by Chen and Yeh [14], we counted the number of price–volume believers. Since the counting work is very computationally demanding, a concession was made only after every 500 generations. This number is given in Table 6. In some cases, say B3, C1, and C2, the belief in the price–volume relation prevails among the public from the beginning right through to the end of the simulations. In some other cases, such as A1, A3, and C3, price–volume believers finally die out. Note that the number of price–volume believers may fluctuate during the whole of the

Table 5
Nonlinear Granger causality test

Case	H_0: Volume changes do not cause stock returns ($v_t \not\Rightarrow r_t$)			H_0: Stock returns do not cause volume changes ($r_t \not\Rightarrow v_t$)		
	# of lags	TVAL	p-value	# of lags	TVAL	p-value
A1	1	0.091	0.4638	1	1.030	0.1515
	2	0.652	0.2573	2	1.932	0.0267*
	3	0.878	0.1899	3	1.502	0.0666†
	4	1.169	0.1212	4	0.238	0.4060
	5	0.668	0.2520	5	1.029	0.1518
	6	0.117	0.4533	6	1.436	0.0755†
	7	−0.314	0.6234	7	0.905	0.1827
	8	−0.382	0.6487	8	0.951	0.1708
	9	−0.067	0.5267	9	1.028	0.1519
	10	−0.109	0.5434	10	0.956	0.1695
A2	1	−0.582	0.7196	1	−0.409	0.6586
	2	−1.413	0.9212	2	1.114	0.1327
	3	−0.775	0.7809	3	0.273	0.3923
	4	−1.252	0.8947	4	0.300	0.3821
	5	−0.383	0.6493	5	−0.087	0.5345
	6	−0.428	0.6656	6	0.802	0.2112
	7	0.770	0.2207	7	0.187	0.4258
	8	1.046	0.1479	8	0.518	0.3022
	9	0.648	0.2585	9	0.152	0.4397
	10	0.575	0.2825	10	−0.122	0.5486
A3	1	−0.636	0.7377	1	2.180	0.0146*
	2	−0.678	0.7512	2	1.837	0.0331*
	3	−0.141	0.5560	3	2.191	0.0142*
	4	−0.213	0.5844	4	2.303	0.0106*
	5	−1.008	0.8432	5	1.230	0.1095
	6	−0.905	0.8173	6	0.654	0.2565
	7	−1.407	0.9202	7	1.841	0.0328*
	8	−1.569	0.9416	8	0.983	0.1628
	9	−1.811	0.9649	9	1.813	0.0350*
	10	−1.350	0.9115	10	2.907	0.0018*
B1	1	−0.999	0.8412	1	−0.198	0.5783
	2	−1.043	0.8515	2	0.019	0.4925
	3	−1.534	0.9374	3	1.139	0.1274
	4	−0.933	0.8245	4	0.873	0.1914
	5	−0.447	0.6724	5	0.617	0.2685
	6	−0.094	0.5373	6	0.235	0.4071
	7	−0.158	0.5626	7	0.212	0.4161
	8	−0.052	0.5208	8	0.182	0.4278
	9	0.013	0.4950	9	−0.117	0.5468
	10	−0.227	0.5899	10	−0.916	0.8201
B2	1	4.193	0.0000*	1	3.490	0.0002*
	2	5.344	0.0000*	2	2.419	0.0078*
	3	4.938	0.0000*	3	2.377	0.0087*

Table 5 (*continued*)

Case	H_0: Volume changes do not cause stock returns ($v_t \nRightarrow r_t$)			H_0: Stock returns do not cause volume changes ($r_t \nRightarrow v_t$)		
	# of lags	TVAL	*p*-value	# of lags	TVAL	*p*-value
	4	4.560	0.0000*	4	2.799	0.0026*
	5	5.564	0.0000*	5	2.917	0.0018*
	6	5.244	0.0000*	6	2.384	0.0086*
	7	5.083	0.0000*	7	2.294	0.0109*
	8	4.017	0.0000*	8	2.256	0.0120*
	9	3.256	0.0006*	9	2.650	0.0040*
	10	3.179	0.0007*	10	2.920	0.0018*
B3	1	1.492	0.0678†	1	2.147	0.0159*
	2	1.847	0.0324*	2	2.204	0.0138*
	3	1.759	0.0393*	3	2.326	0.0100*
	4	2.375	0.0088*	4	3.724	0.0001*
	5	2.925	0.0017*	5	3.259	0.0006*
	6	2.297	0.0108*	6	3.699	0.0001*
	7	2.542	0.0055*	7	3.570	0.0002*
	8	1.467	0.0712†	8	3.498	0.0002*
	9	1.166	0.1218	9	2.234	0.0128*
	10	1.259	0.1041	10	2.335	0.0098*
C1	1	0.322	0.3738	1	0.451	0.3260
	2	0.885	0.1880	2	−0.512	0.6957
	3	1.234	0.1085	3	−0.436	0.6686
	4	1.377	0.0843†	4	0.560	0.2877
	5	1.265	0.1030	5	0.625	0.2659
	6	1.003	0.1580	6	1.084	0.1393
	7	0.506	0.3065	7	0.239	0.4057
	8	0.537	0.2955	8	0.341	0.3664
	9	0.248	0.4022	9	0.097	0.4612
	10	0.089	0.4644	10	0.158	0.4371
C2	1	0.829	0.2035	1	−2.073	0.9809
	2	−0.462	0.6779	2	−0.149	0.5594
	3	−0.381	0.6484	3	−0.001	0.5003
	4	0.039	0.4845	4	0.282	0.3889
	5	0.518	0.3023	5	−0.358	0.6400
	6	0.219	0.4133	6	−0.518	0.6979
	7	0.178	0.4294	7	−0.421	0.6632
	8	−0.084	0.5334	8	0.709	0.2390
	9	−0.264	0.6040	9	1.238	0.1078
	10	−0.515	0.6967	10	0.860	0.1948
C3	1	−0.833	0.7975	1	−0.686	0.7536
	2	−0.247	0.5976	2	−1.797	0.9639
	3	−0.381	0.6483	3	−0.878	0.8100
	4	−0.854	0.8035	4	0.009	0.4964
	5	−0.983	0.8373	5	0.712	0.2382
	6	−0.751	0.7738	6	0.967	0.1667

(*continued on next page*)

Table 5 (*continued*)

Case	H_0: Volume changes do not cause stock returns ($v_t \nRightarrow r_t$)			H_0: Stock returns do not cause volume changes ($r_t \nRightarrow v_t$)		
	# of lags	TVAL	*p*-value	# of lags	TVAL	*p*-value
	7	−0.177	0.5702	7	0.873	0.1912
	8	−0.084	0.5336	8	1.320	0.0934†
	9	−0.316	0.6241	9	2.453	0.0071*
	10	−0.511	0.6955	10	2.069	0.0193*

Note that * represents the rejection of the null at the 5% significant level, whereas † represents the same at the 10% significant level.

Table 6
Number of price–volume believers among a total of 500 agents

Generation	Case								
	A1	A2	A3	B1	B2	B3	C1	C2	C3
1	200	179	278	180	187	241	188	187	250
500	45	106	322	387	176	431	325	317	263
1000	0	28	371	297	0	415	281	302	94
1500	0	0	170	124	69	398	313	313	192
2000	0	0	86	208	92	382	237	496	182
2500	0	3	23	134	470	496	350	500	7
3000	0	0	125	0	279	465	351	500	3
3500	0	3	32	0	133	500	477	499	0
4000	0	0	0	42	1	492	416	446	0
4500	0	0	0	16	0	476	325	498	0
5000	0	3	0	24	0	498	273	483	0

simulation period, e.g., B1 and B2. A striking phenomenon is that price–volume believers may revive even after some periods of noughts. A2 is a case in point. We are now ready to check whether the macro-phenomenon of the price–volume relation we observed at the "top" matches what we observed at the "bottom". This issue, called *consistency*, is discussed in the next subsection.

5.3. The micro–macro relation

In the agent-based modeling framework, we are particularly interested in the so-called *micro–macro relation*. [14] Based on the simulation results we have,

[14] This issue, however, is the key to unlock the mysteries of emergent properties in all kinds of *complex adaptive systems* (CASs). Trying to understand the aggregate phenomena via the adaptive behavior of interacting agents at the bottom level is the essence of the science of complexity. One may still wish to trace how individual agents could receive feedback from the macro-phenomena and further adapt themselves to form far richer micro–macro dynamic scenarios. Not too surprisingly, agent-based models in economics, following the paradigm of CASs, have paid great attention to the *micro–macro relation* in recent studies [11,24,36,52,53].

Table 7
Micro–macro relation

	Market participants used the volume to forecast price	Market participants did not use the volume to forecast price
$v_t \rightarrow r_t$ or $v_t \Rightarrow r_t$	B3	A1, B2, C3
$v_t \nrightarrow r_t$ and $v_t \nRightarrow r_t$	C1, C2	A2, A3, B1

four basic patterns stand out. These can be roughly divided into two categories, namely, consistent patterns and inconsistent ones. A pattern is called *consistent* if the macro-behavior tends to lend support to what most individuals believe or come to believe. A pattern is called *inconsistent* if the macro-behavior tends to invalidate what most individuals believe or come to believe (see Table 7).

In a more technical way, let the hypothesis that the volume does not Granger-cause returns be the null hypothesis. If this null hypothesis is rejected (or fails to be rejected) by the aggregate market outcome based on econometric tests, then we say that the pattern is *consistent* if it is also rejected (or fails to be rejected) by most or by an increasing number of market participants. Otherwise, it is called *inconsistent*.

According to the definition above, the cases A2, A3, B1 and B3 exhibit consistent patterns (the main diagonal boxes of Table 7), whereas the cases A1, B2, C1, C2 and C3 demonstrate inconsistent patterns (the off-diagonal boxes of Table 7).

Among the consistent patterns, B3 is the case where the null hypothesis is consistently rejected by both macro- and micro-behavior. Its number of price–volume believers is persistently high during the entire simulation. In particular, for the second half of the trading session, almost all agents rejected the null by forecasting returns with volume (see Table 6).

A2, A3 and B1 are the other consistent patterns. In these three cases, the null failed to be rejected in both the linear and nonlinear tests, and our traders' beliefs were in line with this test result. The number of participants who believed the null hypothesis continuously decreased. For example, consider case A3. At the beginning, there were a great number of traders who used volume in their forecasts of returns. Nonetheless, after period 1500, the number dramatically fell from 300 to 100, and further to nil.

Among the inconsistent patterns (patterns in the off-diagonal boxes of Table 7), C1 and C2 share the feature that the market is composed of hundreds of *price–volume believers*, while the causality test shows that the volume cannot help predict returns. This result is particularly striking in case C2, where the market reached a state where all market participants are price–volume believers.

Equally interesting inconsistent patterns are cases A1, B2 and C3. In these cases, the causality test did indicate the significance of volume in return forecasting, but traders eventually gave up the use of this variable in their forecasts of returns.

5.4. Discussion

The analysis so far is mainly driven by the aggregate outcome. Basically, we are asking whether the individual behavior is consistent with our econometric tests. In other words, if our tests suggest the existence of a causal relation, did our "smart" and "adaptive" agents also notice it to be so?

The real issue is whether these inconsistent patterns are unanticipated or whether they are puzzling us. The answer is not necessarily so. There are, in effect, some arguments that would predict why these inconsistent patterns may appear. For example, consider cases C1 and C2. A supportive argument would be the following: It is the intensive search, characterized by a large number of price–volume believers, over the hidden relation between volume and returns that eventually nullifies the effect of volume on returns and makes volume become a useless variable. In this case, the micro and macro relation observed in cases A3 and B3 is actually also in harmony. As a matter of fact, using this argument, one can question whether these "consistent" patterns are really consistent. For instance, if no one gives the volume variable a try, would it be possible for the volume-to-price relation to finally emerge as a secret which has never been disclosed?

The argument which we have just been through points out one serious limitation in the analysis of micro and macro relations that we proposed above. In this analysis, we treat the whole micro-process as one sample, and the whole macro-process as the other sample. We then look into the consistency between the two samples. However, what is neglected is the complex dynamic feedback relation existing between aggregate outcome and individual behavior, as aptly depicted by Farmer and Lo [22, p. 9992]:

> Patterns in the price tend to disappear as agents evolve profitable strategies to exploit them, but this occurs only over an extended period of time, during which substantial profits may be accumulated and new patterns may appear.

As for cases A1 and C3, we saw that there exists only linear Granger causality between returns and trading volume at the macro-level. Nevertheless, from the micro-viewpoint, traders were not aware of this. One possible explanation for observing such inconsistency is the huge search space defined by GP. The linear function set has only a measure of zero in it. If we restrict our attention only to the nonlinear causality test, then there is no inconsistency in cases A1 and C3. It follows that traders may overlook the usefulness of linear models, and perform most of their trials over the space for nonlinear models. As we may expect, they will eventually give up their attempts, because nonlinear causality does not exist. However, this explanation cannot be applied to case B2, in which the nonlinear causal relation is also shown to be statistically significant.

To sum up, there is no definite relation between micro- and macro-behavior. The appearance of the patterns in the off-diagonal entries shows that the Neoclassical economic analysis, which generally assumes consistency between the micro- and macro-behavior, does not have a solid basis. It is in this agent-based economic model that we show how easy it is to have aggregate results which are not anticipated based on individual behavior. The reason why one can have such a large variety of patterns is mainly because of complex dynamic interactions between individuals and the market.

Financial market dynamics is path-dependent, highly complex and nonlinear because it is the outcome of continuously evolving and interacting behavior, which is largely driven by survival pressure. It is therefore difficult to draw a simple conclusion on the relation between micro- and macro-behavior. To fully trace their interactions, an analysis based on high-frequency sampling (or a census) of traders' behavior is required. Statistical analysis based on small samples is also useful for investigating the potential time variant relation, due to the real time survival pressure.

6. Conclusions

One distinguishing feature of ACE (and thus ABSMs) is that some interesting macro-phenomena of financial markets could emerge (be endogenously generated) from interactions among adaptive agents without exogenously imposing any conditions like unexpected events, information cascades, noise or dumb traders, etc. In this paper, we show that the presence of the stock price–volume causal relation does not require any explicit assumptions like information asymmetry, reaction asymmetry, noise traders, or tax motives. In fact, it suggests that the causal relation may be a generic property in a market modeled as an evolving decentralized system of autonomous interacting agents.

We also show that our understanding of the appearance or disappearance of the price–volume relation can never be complete if the feedback relation between individual behavior and aggregate outcome is neglected. This feedback relation is, however, highly complex, and may defy any simple analysis, as in the case of the one we proposed initially. Consequently, econometric analysis which fails to take into account this complex feedback relation between the micro- and macro-aspects may produce misleading results. Unfortunately, we are afraid that this is exactly what mainstream financial econometrics ended up doing in a large number of empirical studies.

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