AISB Journal **1**(1), © 2001
http://www.aisb.org.uk

Market Diversity and Market Efficiency: The Approach Based on Genetic Programming

Chia-Hsuan Yeh* and Shu-Heng Chen†

* Department of Information Management, Yuan Ze University, Chungli, Taoyuan 320, Taiwan, *imcyeh@saturn.yzu.edu.tw*

† Department of Economics, National Chengchi University, Taipei 11623, Taiwan, *chchen@nccu.edu.tw*

Abstract

The relation between market diversity and market efficiency has been studied. Economic heterogeneity is a fundamental driving force and an essential property in the economic systems. People who have different perspectives, technologies, or endowments may benefit from their trading behavior which constitutes economic activities. In this paper, economic simulation based on the growing field of *artificial stock market* is employed to study this issue. Market size and different learning styles are used to discuss the influence of heterogeneity. Simulation results have demonstrated that more participants and individual learning cause higher degree of traders' diversity, which, in turn, enhances market efficiency.

1 Introduction

The relationship between *competitiveness* and market performance has been discussed for a long time. In a competitive economic environment, each firm or individual is unable to influence the market. It has been mentioned in the economics courses that the competitive market is more efficient and has higher social welfare. Therefore, it is the desirable picture that economists intend to draw. The concept of competitiveness is related to market size, i.e., the number of market participants. The idea here is that the larger economy contributes to microeconomic heterogeneity, for example, behavior and strategies, profitability and market shares, production technology and efficiency. People who have different perspectives about the future implies that there exits room for the economic activity and they may benefit from their trading behavior. In other words, the higher degree of heterogeneity may provide more opportunities for trading. It is also an important seed of innovation.

However, few papers addressed the influence of the number of agents in the economic simulation literature. Usually, the number of agents in the simulated economies is determined arbitrarily or under the consideration of computational load. In (Arifovic et al. (1997)), they mentioned that the minimal number of strings (agents) for effective search is usually taken to be 30 according to the artificial intelligence literature. However, the point of view from optimization is quite different from the agent-based simulated economies (and/or the real world). (Aoki (1999)) pointed out that there exists different statistical properties when the number of agents (N) goes to infinity. (Egenter et al. (1999)) have shown that the behavior of the simulated market became quite smooth or periodic and thus predictable if $N \to \infty$. (Den Haan (2001)) also described the importance of the number of *different* agents. In the framework of asset-pricing model, several properties depend on the number of types of agents. For example, the dynamics of interest rate, investment behavior, and agent's welfare. In this paper, we try to restudy its relation to the market efficiency. This issue is very important in the financial market because it is well known that the thin and thick markets may have different financial properties and phenomena.

Economic simulations have been widely used in the study of economics. In order to model the interaction between many *heterogeneous* agents, the techniques of evolutionary computation were employed, for example, artificial neural nets, genetic algorithms (GAs) and genetic programming (GP). Different representation about agents' behavior will influence what they can learn, which may explain different scenarios. Genetic programming serves this purpose better than other techniques. Therefore, GP is applied to modeling the evolution of traders' behavior in this paper.

However, there are two ways to implement GP, namely, single-population GP and multi-population GP. The former is a way to model social learning, whereas the latter focuses on individual learning ((Chen and Yeh (2001))). In this paper, we also attempt to figure out how differently the relation between market size and efficiency may emerge under these two different versions of GP. All these issues are studied within the context of *agent-based artificial stock markets*, which is probably the most powerful tool to study behavioral finance.

The organization of this paper is as follows. In Section 2, we present the framework of the agent-based artificial stock market. The implementation of social learning and individual learning is described in Section 3. Experimental designs and simulation results are shown in Section 4 and 5 respectively. Section 5 concludes.

2 Model Description

The basic framework of the artificial stock market considered in this paper is the standard asset pricing model employed in (Grossman and Stiglitz (1980)). The dynamics of market is determined by an interaction of many heterogeneous agents. Each of them, based on his forecast of the future, maximizes his expected utility.

2.1 Traders

For simplicity, we assume that all traders share the same *constant absolute risk aversion* (CARA) utility function,

$$U(W_{i,t}) = -exp(-\lambda W_{i,t}), \tag{1}$$

where $W_{i,t}$ is the wealth of trader i at period t, and λ is the degree of relative risk aversion. Traders can accumulate their wealth by making investments. There are two assets available for traders to invest. One is the riskless interest-bearing asset called *money*, and the other is the risky asset known as the *stock*. In other words, at each period, each trader has two ways to keep his wealth, i.e.,

$$W_{i,t} = M_{i,t} + P_t h_{i,t} \tag{2}$$

where $M_{i,t}$ and $h_{i,t}$ denote the money and shares of the stock held by trader i at period t respectively, and P_t is the price of the stock at period t. Given this portfolio ($M_{i,t}$,$h_{i,t}$), a trader's total wealth $W_{i,t+1}$ is thus

$$W_{i,t+1} = (1 + r)M_{i,t} + h_{i,t}(P_{t+1} + D_{t+1}) \tag{3}$$

where D_t is per-share *cash dividends* paid by the companies issuing the stocks and r is the riskless interest rate. D_t can follow *a stochastic process* not known to traders. Given this wealth dynamics, the goal of each trader is to myopically maximize the one-period expected utility function,

$$E_{i,t}(U(W_{i,t+1})) = E(-exp(-\lambda W_{i,t+1}) \mid I_{i,t}) \tag{4}$$

subject to Equation (3), where $E_{i,t}(.)$ is trader i's conditional expectations of W_{t+1} given his information up to t (the information set $I_{i,t}$).

It is well known that under $CARA$ utility and Gaussian distribution for forecasts, trader i's desire demand, $h^*_{i,t+1}$ for holding shares of risky asset is linear in the expected *excess return*:

$$h^*_{i,t} = \frac{E_{i,t}(P_{t+1} + D_{t+1}) - (1+r)P_t}{\lambda \sigma^2_{i,t}}, \tag{5}$$

where $\sigma^2_{i,t}$ is the conditional variance of $(P_{t+1} + D_{t+1})$ given $I_{i,t}$.

The key point in the agent-based artificial stock market is the formation of $E_{i,t}(.)$. In this paper, the expectation is modeled by genetic programming. The detail is described in the next section.

2.2 Price Determination

Given $h^*_{i,t}$, the market mechanism is described as follows. Let $b_{i,t}$ be the number of shares trader i would like to submit a bid to buy at period t, and let $o_{i,t}$ be the number of shares trader i would like to offer to sell at period t. It is clear that

$$b_{i,t} = \begin{cases} h^*_{i,t} - h_{i,t-1}, & h^*_{i,t} \geq h_{i,t-1}, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

and

$$o_{i,t} = \begin{cases} h_{i,t-1} - h^*_{i,t}, & h^*_{i,t} < h_{i,t-1}, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Furthermore, let

$$B_t = \sum_{i=1}^{N} b_{i,t}, \ \text{ and } \ O_t = \sum_{i=1}^{N} o_{i,t} \tag{8}$$

be the totals of the bids and offers for the stock at period t, where N is the number of traders. Following (Palmer et al. (1994)), we use the following simple rationing scheme:

$$h_{i,t} = \begin{cases} h_{i,t-1} + b_{i,t} - o_{i,t}, & \text{if } \ B_t = O_t, \\ h_{i,t-1} + \frac{O_t}{B_t} b_{i,t} - o_{i,t}, & \text{if } \ B_t > O_t, \\ h_{i,t-1} + b_{i,t} - \frac{B_t}{O_t} o_{i,t}, & \text{if } \ B_t < O_t. \end{cases} \tag{9}$$

All these cases can be subsumed into

$$h_{i,t} = h_{i,t-1} + \frac{V_t}{B_t} b_{i,t} - \frac{V_t}{O_t} o_{i,t} \tag{10}$$

where $V_t \equiv \min(B_t, O_t)$ is the volume of trade in the stock.

According to Palmer et al.'s *rationing scheme*, we can have a very simple price adjustment scheme, based solely on the *excess demand* $B_t - O_t$:

$$P_{t+1} = P_t(1 + \beta(B_t - O_t)) \tag{11}$$

where β is a function of the difference between B_t and O_t. β can be interpreted as speed of adjustment of prices. The β function we consider is:

$$\beta(B_t - O_t) = \begin{cases} \texttt{tanh}(\beta_1(B_t - O_t)) & \text{if } B_t \geq O_t, \\ \texttt{tanh}(\beta_2(B_t - O_t)) & \text{if } B_t < O_t \end{cases} \tag{12}$$

where `tanh` is the *hyperbolic tangent function*:

$$\texttt{tanh}(x) \equiv \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{13}$$

The price adjustment process introduced above implicitly assumes that the total number of shares of the stock circulated in the market is fixed, i.e.,

$$H_t = \sum_i h_{i,t} = H. \tag{14}$$

In addition, we assume that dividends and interests are all paid by cash, so

$$M_{t+1} = \sum_i M_{i,t+1} = M_t(1+r) + H_t D_{t+1}. \tag{15}$$

2.3 Formation of Expectations

As to the formation of traders' expectations, $E_{i,t}(P_{t+1}+D_{t+1})$, we assume the following functional form for $E_{i,t}(.)$.[1]

$$E_{i,t}(P_{t+1} + D_{t+1}) = \begin{cases} (P_t + D_t)(1 + \theta_1 f_{i,t} * 10^{-4}), & \text{if } -10^4 \leq f_{i,t} \leq 10^4, \\ (P_t + D_t)(1 + \theta_1), & \text{if } f_{i,t} > 10^4, \\ (P_t + D_t)(1 - \theta_1), & \text{if } f_{i,t} < -10^4 \end{cases} \tag{16}$$

The population of $f_{i,t}$ (i=1,...,N) is formed by genetic programming. That means, the value of $f_{i,t}$ is decoded from its GP tree $gp_{i,t}$.

As to the subjective risk equation, we modified the equation originally used by (Arthur et al. (1997)),

$$\sigma^2_{i,t} = (1-\theta_2)\sigma^2_{t-1|n_1} + \theta_2[(P_t + D_t - E_{i,t-1}(P_t + D_t))^2]. \tag{17}$$

where

$$\sigma^2_{t-1|n_1} = \frac{\sum_{j=0}^{n_1-1}(P_{t-j} - \overline{P}_{t|n_1})^2}{n_1 - 1} \tag{18}$$

and

$$\overline{P}_{t|n_1} = \frac{\sum_{j=0}^{n_1-1} P_{t-j}}{n_1} \tag{19}$$

In other words, $\sigma^2_{t-1|n_1}$ is simply the *historical volatility* based on the past n_1 observations.

Given each trader' expectations, $E_{i,t}(P_{t+1} + D_{t+1})$, according to Equation (5) and his own subjective risk equation, we can obtain each trader's desire demand, $h^*_{i,t+1}$ shares of the stock, and then how many shares of stock each trader intends to bid or offer based on Equation (6) or (7).

[1] There are several alternatives to model traders' expectations. The interested reader is referred to (Chen et al. (2001)).

3 The Framework of Social and Individual Learning

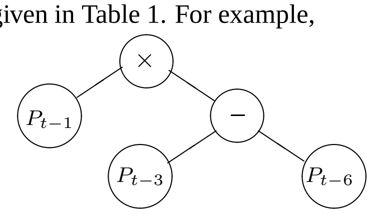
In this paper, genetic programming is employed to model the formation of traders' expectations. In agent-based computational economics, there are two different styles to implement GP, namely, single-population GP and multi-population GP. While, from the optimization point of view, the choice of the two styles (architectures) may have an effect on the search efficiency, it brings different implications for the agent-based economic modeling. In the framework of single-population GP (SGP), each agent is represented by a *tree* (forecasting model, computer program). The evolution of this population of agents is performed through natural selection and genetic operations (reproduction, crossover, and mutation) on all of these trees (agents). In this case, agents can learn from other agents' experiences, and hence is called *social learning*. On the other hand, in the framework of multi-population GP (MGP), each agent is endowed with a population of trees. Evolution is respectively manipulated in terms of the population of trees for each agent. In this case, agents learn only from their own experiences and reasoning, and hence is called *individual learning*.

According to (Vriend (2000)) and (Vriend (2001)), there is an essential difference between individual and social learning, and the underlying cause for this is the so-called *spite effect*. The spite effect may occur in a social learning GA, but can never occur in an individual learning GA. To see how the spite effect can influence the outcome of the evolutionary process, Vriend uses the two different GAs to simulate the learning process of an oligopoly game. The simulation results show that while the individual learning GA moves close to the Cournot-Nash output level, the social learning GA converges to the competitive Walrasian output level.

Vriend's finding pushes us to think harder of the choice of the two styles of learning. But, SGP (social learning) has another issue which is generally overlooked. (Harrald (1998)) criticized the traditional implementation of social learning. He mentioned the traditional distinction between the *phenotype* and *genotype* in biology and doubted whether the adaptation can be directly operated on the genotype via the phenotype in social processes. In particular, it is not easy to justify why we can learn or imitate other agents' strategies (genotype) by means of their actions (phenotype).

Motivated by this criticism, (Chen and Yeh (2001)) proposed a modified version of social learning. The idea is to include a mechanism, called the "*business school*". The business school serves as a faculty of researchers or a library. Knowledge in the business school or library is *open for everyone*. Traders can consult the researchers when they are under great peer pressure or economic loss. More precisely, the business school can be viewed as a collection of forecasting models. SGP (social learning) can then be applied to model the evolution of the business school without inviting Haralld's criticism.[2]

Each researcher (forecasting model) is represented by a GP parse tree. Initially, the forecasting models ($GP_{t=1}$) are randomly generated based on the function set and terminal set given in Table 1. For example,



Each tree in the initial population of GP trees (forecasting models) is randomly generated

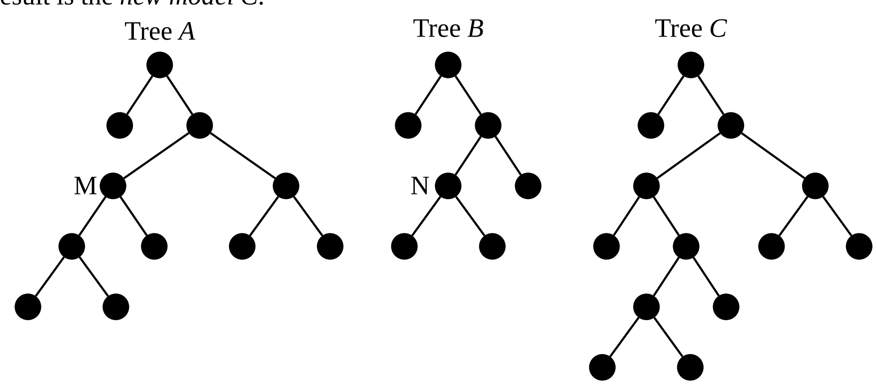
[2] For more details, please see (Chen and Yeh (2001)).

by either the growth method or the full method based on a toss of fair coin.[3] In the initialization stage, the maximum depth of a tree is restricted to 6. The performance of each forecasting model is determined by the pre-assigned fitness function, i.e., *mean absolute percentage error* (MAPE).

In the business school, the forecasting models will be evaluated with a pre-specified schedule, say once for every m_1 trading days. The evaluation procedure proceeds as follows. At the evaluation date t, each old model (model at period $t-1$, GP_{t-1}) shall compete with a *new model* which is generated from GP_{t-1} itself by using one of the following four genetic operators: reproduction, mutation, crossover, and immigration, each with a probability p_r, p_m, p_c, and p_I (Table 1). The winner, the one with lower MAPE, will be selected as a model of next generation, GP_t. When the evaluation is done over all the old models, a new generation of forecasting models (GP_t) is then born. The four genetic operators are detailed as below.

- Reproduction:
  Two forecasting models are randomly selected from GP_{t-1}. The one with lower MAPE over the last m_2 days' forecasts is chosen as the *new model*.

- Mutation:
  Two forecasting models are randomly selected from GP_{t-1}. The one with lower MAPE over the last m_2 days is chosen as a candidate, and has a probability of p_M (Table 1) being mutated. There are several ways to implement mutation. What implemented here is known as *tree mutation*. If a tree is chosen to be mutated, one of its nodes is randomly selected, and the subtree originating from the node is replaced by a new subtree which is randomly generated. For example, consider the forecasting model *A*, whose tree structure is shown below. Suppose that the point M is chosen as the mutation point. Then the subtree originated from point M is removed and is replaced by a subtree *B*, which is randomly generated. And the result is the *new model C*.[4]

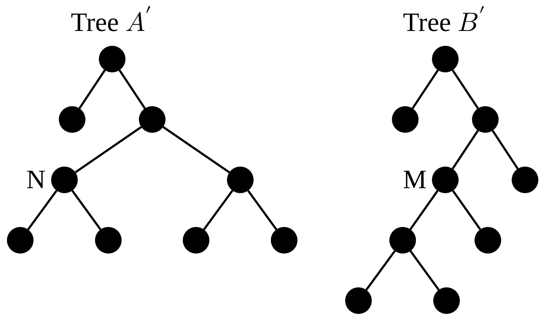


- Crossover:
  Two pairs of forecasting models are randomly chosen, ($gp_{j_1,t-1}$, $gp_{j_2,t-1}$) and ($gp_{k_1,t-1}$, $gp_{k_2,t-1}$). The one with lower MAPE in each pair is chosen as a *parent*. The crossover operation is a sexual operation that starts with two parental models. One of the nodes for each parent is selected randomly, and the subtrees originating

[3] For more details, please refer to (Koza (1992)).

[4] But, if Tree *A* is not mutated (with a probability $1 - P_M$), then *A* will automatically be the *new model*.

from the nodes are exchanged. Two offspring are then produced. One of them is randomly chosen as the *new model*. For example, consider Tree *A* and *B* shown above as the parents, and points M and N are the crossover points. After crossover by exchanging the subtrees originating from M and N, the two offspring are Tree $A^{'}$ and $B^{'}$ shown below.



- Immigration:
  A forecasting model is randomly created as the *new model*. This operator is used to approximate the concept of *imagination*.

As to the evolution of traders' behavior, each trader is endowed with N_I forecasting models. These forecasting models are used to predict $E_{i,t}(U(W_{i,t+1}))$. In the beginning, they are also randomly generated. The performance of these forecasting models is measured by *profits*, not MAPE, under a *validation* process. Traders' adaptation proceeds as follows. At the evaluation date t, each trader has to make a decision. Should he change his forecasting model used in the previous period? This decision is affected by two psychological factors, namely, *peer pressure* and *a sense of self-realization*. Generally speaking, if a trader's profits earned in the last period is inferior to many other traders, or his profits comes to a historical low, then he will have a higher motivation, and hence a higher probability, to change.

The probabilities that traders consider to change a model can be mathematically described as follows. First, suppose that traders are ranked by *the net change of wealth* over the last n_2 trading days. Let $\Delta W_{i,t}^{n_2}$ be this net change of wealth of trader i at time period t, i.e.,

$$\Delta W_{i,t}^{n_2} \equiv W_{i,t} - W_{i,t-n_2}, \tag{20}$$

and, let $R_{i,t}$ be her rank. Then, the probability that trader i will change a model at the end of period t is assumed to be determined by

$$p_{i,t} = \frac{R_{i,t}}{N}. \tag{21}$$

The choice of the function $p_{i,t}$ is quite intuitive. It simply means that

$$p_{i,t} < p_{j,t}, \text{ if } R_{i,t} < R_{j,t}. \tag{22}$$

In words, the traders who come out top shall suffer less peer pressure, and hence have less motivation to change models than those who are ranked at the bottom.

In addition to peer pressure, a trader may also decide to change a model out of a sense of *self-realization*. Let the growth rate of wealth over the last n_2 days be

$$\delta_{i,t}^{n_2} = \frac{W_{i,t} - W_{i,t-n_2}}{\mid W_{i,t-n_2} \mid}, \tag{23}$$

Table 1: Parameters of the Stock Market

The Stock Market	
Shares of the stock (H)	100 (500)
Initial money supply (M_1)	100 (500)
Interest rate (r)	0.1
Stochastic process (D_t)	Uniform distribution, U(5.01,14.99)
Price adjustment function	`tanh`
Price adjustment (β_1)	0.2×10^{-4}
Price adjustment (β_2)	0.2×10^{-4}
Parameters of Genetic Programming	
Function set	$\{+, -, \times, \%,$ `sin,cos,exp,Rlog, Abs,sqrt`$\}$
Terminal set	$\{P_t, P_{t-1}, \cdots, P_{t-10}, P_t + D_t, \cdots, P_{t-10} + D_{t-10}\}$
Selection scheme	Tournament selection
Tournament size	2
Probability of creating a tree by reproduction (p_r)	0.10
Probability of creating a tree by immigration (p_I)	0.20
Probability of creating a tree by crossover (p_c)	0.35
Probability of creating a tree by mutation (p_m)	0.35
Probability of mutation (P_M)	0.30
Probability of leaf selection under crossover	0.5
Mutation scheme	Tree mutation
Replacement scheme	(1+1) Strategy
Maximum depth of tree	17
Maximum number in the domain of Exp	1700
Number of generations	4000
Business School	
Number of faculty members (F)	500
Criterion of fitness (Faculty members)	MAPE
Evaluation cycle (m_1)	20
Sample size (MAPE) (m_2)	10
Search intensity in Business School (I_s^*)	5
Traders	
Number of traders (N)	100
Number of ideas for each trader (N_I)	1 (A), 10 (B), 25 (C)
Degree of RRA (λ)	0.5
Criterion of fitness (Traders)	Increments in wealth (Income)
Evaluation cycle (n_2)	1
Search intensity by trader himself (I_h^*)	5
θ_1	0.5
θ_2	0.0133

and let $q_{i,t}$ be the probability that trader i will change a model at the end of the tth trading day, then it is assumed that

$$q_{i,t} = \frac{1}{1 + exp^{\delta^{n_2}_{i,t}}}. \tag{24}$$

The choice of this density function is also straightforward. Notice that

$$\lim_{\delta^{n_2}_{i,t} \to \infty} q_{i,t} = 0, \tag{25}$$

and

$$\lim_{\delta^{n_2}_{i,t} \to -\infty} q_{i,t} = 1. \tag{26}$$

Therefore, the traders who have made great progress will naturally be more confident and hence have little need for changing ideas, whereas those who suffer devastating regression will have a strong desire for changing ideas.

In sum, for trader i, the decision to change a model can be considered as a result of *a two-stage independent Bernoulli experiments*. The success probability of the first experiment is $p_{i,t}$. If the outcome of the first experiment is success, the trader will change a model. If, however, the outcome of the first experiment is failure, the trader will continue to carry out the second experiment with the success probability $q_{i,t}$. If the outcome of the second experiment is success, then the trader will also change a model. Otherwise, the trader will keep her model unchanged. Based on the description above, the probability of changing the idea ($r_{i,t}$) for trader i at period t is

$$r_{i,t} = p_{i,t} + (1 - p_{i,t})q_{i,t} = \frac{R_{i,t}}{N} + \frac{N - R_{i,t}}{N} \frac{1}{1 + exp^{\delta^{k}_{i,t}}} \tag{27}$$

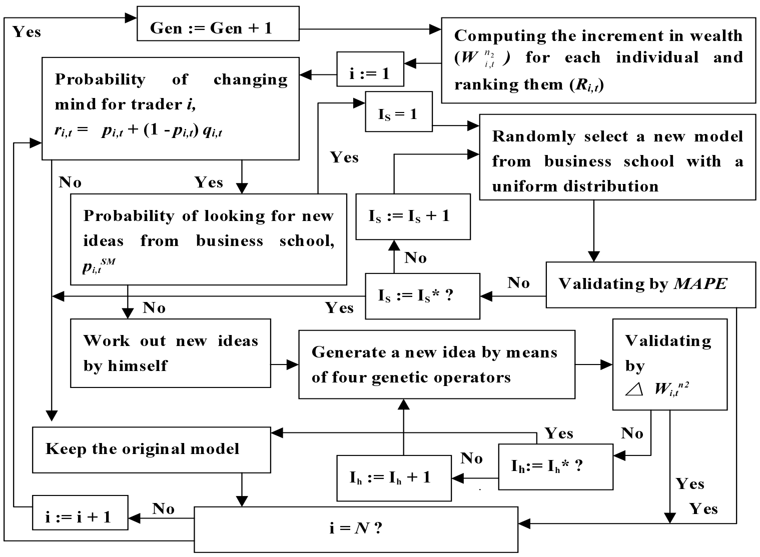
Once the trader decides to change, how he changes depends on the two styles of learning. In the case of social learning, he will register to the business school, and randomly catch a model there. In the case of individual learning, he will search a new model on his own by running genetic programming on the population of his N_I existing models (think tank).

Once he gets a new model, he will test the new model with the historical data and see whether the new model will bring him more profits than the old one. If it does, the old model will be replaced by the new model. Otherwise, he will keep on searching in the business school (in the case of social learning) or his think tank (in the case of individual learning) until either he succeeds or he fails for a pre-specified times, I^*_s or I^*_h (Table 1). The process of traders' evolution is also shown in Flowchart 1.

4 Experimental Designs

In order to examine the effect of market size on market efficiency, we consider experiments associated with two different *market size* (numbers of participants). The small market, M_1, has 100 traders, whereas the large market, M_2 has 500 traders. Furthermore, to see the effect of social and individual learning on market efficiency, three scenarios corresponding to different styles of learning are proposed. Traders in Market A follow the social learning scheme (driven by SGP), and Traders in Markets B and C follow the individual learning scheme (driven by MGP).

Markets B and C are distinguished by the number of models (N_I) (the size of think tank) assigned to each trader. In SGP, *market size* (number of traders) and *population size*



N **: Number of traders**

Flowchart 1: Traders' Search Process

(number of GP trees or models) refer to the same thing, because each trader is represented by one model in SGP. However, in MGP, market size and population size refer to two different things. Therefore, it is important to distinguish these two different size effects on market efficiency. In this paper, traders in Market B are endowed with 10 models, whereas traders in Market C are equipped with 25 models.

Therefore, totally, we conduct six distinct experiments, namely, (M_1, A), (M_1, B), (M_1, C), (M_2, A), (M_2, B), (M_2, C). Each experiment consists of 10 trials of 4000 trading periods. In terms of genetic programming, the number of *generations* is not necessary the same as the number of *trading periods*. In fact, the former is the latter divided by evaluation cycle. In the case of Markets B and C, since GP is applied to traders, and the evaluation cycle (n_2) is 1 (Table 1), the number of generations is 4000, i.e., the same as the number of trading periods. Nevertheless, in Market A, GP is not applied to traders but to the business school, and the evaluation cycle (m_1) for the business school is 20 (Table 1); therefore the number of generations is only 200.

Each simulation will generate a time series data with 4000 observations of price. To avoid the effect of *initialization*, we shall drop the first 1000 observations, and base our analysis only on the last 3000 observations.

5 Simulation Results

5.1 The Efficiency Measure

The purpose of our experiments is to examine the effect of market size and learning styles on market efficiency. Before proceeding further, we have to give *market efficiency* a technical notion so that our our numerical results can be presented and analyzed accordingly. In financial econometrics, the market is said to be efficient if the return series $\{r_t\}$ is *unpredictable*, where

$$r_t = \ln(P_t) - \ln(P_{t-1}). \tag{28}$$

Technically speaking, a series is unpredictable if there exists no linear and nonlinear structures in it, or the series is *independent*. To test whether a series is independent, we followed the procedure of (Chen et al. (2001)). This procedure is composed of two steps, namely, *the PSC filtering* and *the BDS testing*. We first applied the Rissanen's predictive stochastic complexity (**PSC**) to filter the linear process. The PSC criterion is frequently used as a model selection tool in time series analysis. By using the criterion, we can determine the *best* linear structure, i.e., the best ARMA (*AutoRegressive and Moving Average*), of the time series in question.[5] Once the linear signals are filtered, any signals left must be *nonlinear*. The next step is then to apply the BDS test ((Brock et al. (1996))), one of the most frequently used tests for nonlinearity, to the residual series, i.e., the series after PSC filtering. The null hypothesis of the BDS test is that the series in question is identically and independently distributed. If we fail to reject the null hypothesis, then the series is said to be independent. Since the BDS test statistic under a large sample follows a standardized normal distribution, it is quite easy to have an eyeball check on the results.

5.2 Results: Price and Market Efficiency

The two-stage econometric procedure as outlined above gives three statistics for each market experiment, namely, PSC, R^2, and BDS. They are all shown in Table 2. The (p,q) under the column "PSC" is the orders p and q selected based on the Rissanen's PSC criterion. The column "R^2" reports the coefficient of determination derived by running the PSC-selected $ARMA(p,q)$ regression. The column "BDS" gives the BDS test statistic. If the return series is independent, there shall be completely no structure, be it linear or nonlinear, found in the series. In our statistical language, p, q, and R^2 should all be zero; the BDS test should be as low as possible, but not greater than a critical value, say, 1.96.[6] Using these reference numbers, we can have a quick grasp of our simulation results.

First of all, we would like to draw readers' attention to the experiment (M_2, C). As detailed in the previous section, the market in the experiment has three characteristics, large number of traders, large population size, and individual learning. Based on the reference numbers given above, one can easily see that the return series generated by this experiment are all independent. The only exception is Run 2, which has a linear structure being detected ($p = 1$). But the associated R^2 is so small (0.009) that one can hardly be interested in this linear signal. So, one can generally conclude that markets

[5] The ARMA process is a canonical linear time series model. Given a time series $\{X_t\}$, we say that $\{X_t\}$ follows a AR(p) process if

$$X_t = a_0 + \Sigma_{i=1}^{p} a_i X_{t-i} + \varepsilon_t, \tag{29}$$

and $\{X_t\}$ follows a MA(q) process if

$$X_t = b_0 + \Sigma_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t, \tag{30}$$

where ε_t is a Gaussian white noise, i.e.,

$$\varepsilon_t \sim N(0, \sigma^2), \quad \text{and} \quad E(\varepsilon_s, \varepsilon_t) = 0 \quad \text{if} \quad s \neq t. \tag{31}$$

$\{X_t\}$ is said to follow a ARMA(p,q) process, if

$$X_t = c_0 + \Sigma_{i=1}^{p} a_i X_{t-i} + \Sigma_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t \tag{32}$$

[6] The exact number depends on the chosen significance level of the test.

Table 2: PSC Filtering and BDS Test

Market Size: M_1									
	Market A			Market B			Market C		
Run	PSC	R^2	BDS	PSC	R^2	BDS	PSC	R^2	BDS
1	(2,2)	0.104	7.860	(2,3)	0.057	5.479	(2,3)	0.045	6.349
2	(3,3)	0.089	7.749	(0,2)	0.049	7.104	(3,0)	0.046	6.561
3	(2,2)	0.072	6.845	(2,2)	0.047	6.369	(0,0)	None	4.948
4	(1,2)	0.086	8.462	(2,2)	0.066	6.988	(2,2)	0.085	8.153
5	(3,3)	0.082	7.750	(0,2)	0.029	6.802	(4,5)	0.086	6.274
6	(2,2)	0.072	6.909	(0,2)	0.033	7.052	(2,2)	0.056	6.099
7	(2,3)	0.102	6.958	(1,2)	0.091	10.831	(0,2)	0.047	7.726
8	(1,0)	0.058	6.890	(1,0)	0.051	7.012	(3,4)	0.085	6.413
9	(4,5)	0.133	7.431	(0,2)	0.061	7.777	(1,2)	0.044	6.073
10	(0,3)	0.058	6.956	(2,2)	0.057	5.727	(0,2)	0.031	5.833
Average		0.086	7.381		0.054	7.114		0.058	6.443

Market Size: M_2									
	Market A			Market B			Market C		
Run	PSC	R^2	BDS	PSC	R^2	BDS	PSC	R^2	BDS
1	(1,0)	0.012	3.947	(1,0)	0.006	0.385	(0,0)	None	-0.009
2	(1,0)	0.011	3.096	(1,0)	0.007	1.479	(1,0)	0.009	0.788
3	(1,0)	0.018	2.971	(0,0)	None	0.801	(0,0)	None	0.455
4	(1,0)	0.015	2.076	(1,0)	0.007	0.373	(0,0)	None	-0.163
5	(1,0)	0.009	3.452	(0,0)	None	0.319	(0,0)	None	0.591
6	(1,0)	0.011	3.221	(1,0)	0.005	2.107	(0,0)	None	0.180
7	(1,0)	0.029	2.635	(0,0)	None	0.003	(0,0)	None	0.678
8	(2,3)	0.027	3.529	(1,0)	0.009	1.292	(0,0)	None	1.565
9	(1,0)	0.011	3.866	(1,0)	0.010	0.945	(0,0)	None	1.154
10	(0,2)	0.025	3.944	(1,0)	0.013	0.034	(0,0)	None	1.318
Average		0.0168	3.274		0.008	0.774		0.009	0.656

The BDS test statistic is asymptotically normal with mean 0 and standard deviation 1. The significance level of the test is set at 0.95. In BDS test, the distance parameter (standard deviations) is set to be 1, and the embedding dimension is set to be 5.

with large number of participants, each learn with the evolution of his own large number of forecasting models, is highly efficient in the sense that the efficient market hypothesis is sustained (return series is unpredictable).

From (M_2, C), we have three directions to move. We would first like to know what will happen when *the number of forecasting models* assigned to each trader is reduced. To answer this question, we move from (M_2, C) to (M_2, B). In contrast, markets under (M_2, B) has a stronger linear structure: the $AR(1)$ signal is detected in seven out of the ten markets, while their associated R^2s are generally very low. Furthermore, in one case (Run 6), nonlinear structure is also found (BDS =2.107). All these evidences suggest that *the number of forecasting models can have a positive effect on the market efficiency*. The larger the number, the higher the efficiency.

The second direction to move is to consider the effect of *the number of market participants* (*market size*), i.e., to move from (M_2, C) to (M_1, C). From PSC, we can see that the linear structure is even stronger than the case (M_2, C). Linear signals are pervasively found in all markets with Run 3 as the only exception. Some of the linear signal are even highly structured, say, Run 5. Turn to BDS, none of these markets fail to reject the null hypothesis. As a result, all markets are not efficient. The striking fact is that in experiment (M_2, C), we have all market efficient, but now all inefficient in (M_1, C). The only change is the number of participants. Therefore, *the market size can have a dramatic impact on the market efficiency*. The more the participants, the higher the efficiency.

The third direction to move is to examine the effect of *social learning*. Here, we move from (M_2, C) to (M_2, A). From all the three statistics, one can easily see that all markets under (M_2, A) are relatively inefficient. Hence, the social learning scheme represented by SGP with the business school can have a negative impact upon market efficiency.

5.3 Results: Trading Volume and Market Diversity

In addition to price dynamics, another important observation in stock market is trading volume. The trading volume serves as an important indicator of *the diversity of traders' expectations*. Consider an extreme case where all traders hold the same expectation (forecast). With the same expectation, traders always stand in the same position, either all to buy or all to sell at any given price. In this case, no trade can possibly happen in the market, and this is corresponding to the famous *no-trade theorem* in neo-classical economics ((Tirole (1982))). On the other hand, if traders' expectations are diversified (heterogeneous), then optimistic traders can easily be matched to pessimistic traders at many different prices, and one can expect a large trading volume appearing in the market.

To have a slice of the idea that the trading volume is very different among the experiments, Figures 1-3 plots the time series of the trading volume observed in one typical run of the three market experiments (M_1, A), (M_1, B), and (M_1, C). By presenting these figures together, one can immediately see that the market (M_1, A) is rather quiet as opposed to the other two. Its trading volume is almost nil in many of the trading days. Therefore, except for a few cases, traders in market (M_1, A) share very similar forecasts.

Of course, three individual cases may tell us little what may cause the difference in the diversity of traders' expectations. To be more systematic, Table 3 gives us two summary statistics of all experiments. One is the average daily trading volume (the "Mean" column), and the other is the volatility of the daily trading volume (the "Std. Dev." column). Based on this table, one can well articulate the main factors which contribute to the diversity of traders' expectations, and there are two factors, *learning styles* and *market size*.

First, *learning styles*. To see how learning styles can change the diversity of traders' expectations, let us compare the experiment of social learning ((M_1, A)) to the experiment of individual learning ((M_1, B), (M_1, C)). The ten markets in the experiment (M_1, A) are all very quiet. On average, the mean trading volume over these ten markets is only 0.15 units, whereas the same figure in the other two experiments are 15 ($(M_1, B$) and 14 ($(M_1, C$) units respectively, i.e. almost 100 times higher. This sharp contrast also appears in the comparison between the experiment (M_2, A) and the experiments (M_2, B) and (M_2, C). For the former, the mean trading volume is only 1 unit, but for the latter, the figure jumps to 81 and 79.

These figures clearly show that social learning and individual learning can have dramatic impact upon the diversity on traders' expectations. This result may not come to us as a surprise. If traders learn and adapt via *a pool of common knowledge* (business school, library,...), then the diversity of them is constrained by the diversity of the pool. Since in the business school direct imitation (reproduction) is feasible, competitive forecasting models can be disseminated to a large number of faculty members, which reduces the diversity of the pool, and the diversity of traders' expectations. On the other hand, traders who learn and adapt on their own would not allow other traders to imitate their best forecasting models kept as a *business secret*. Therefore, dissemination of knowledge is more

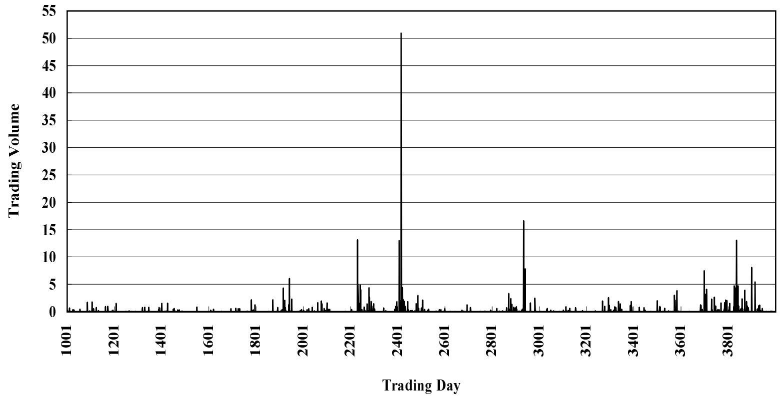


Figure 1 : Time Series Plot of the Trading Volume (Market A, First Run)

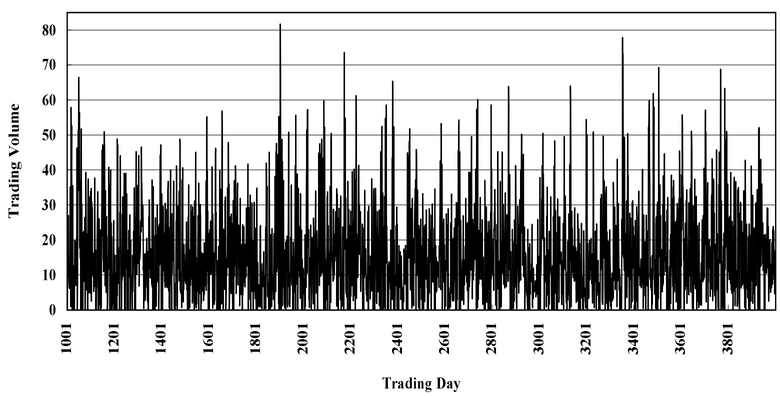


Figure 2 : Time Series Plot of the Trading Volume (Market B, First Run)

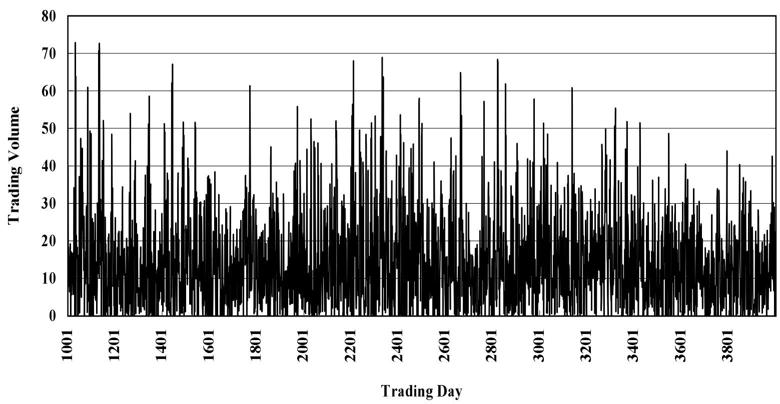


Figure 3 : Time Series Plot of the Trading Volume (Market C, Sixth Run)

difficult to proceed in the context of individual learning, and hence it tends to maintain a greater diversity of traders' expectation.

Second, *market size*. To see the effect of market size on market diversity, the experiment (M_1, A) is compared to the experiment (M_2, A), i.e., the one with one hundred traders to the one with five hundred traders. From Table 3, one can see that when the number of traders increases by 5 times, the mean daily trading volume also increase about 9 to 10 times, from 0.15 units to 1 units. But, caution should be exercised on interpreting these numbers. The thing is that when market size increases by 5 times, the total number of shares also increases by that amount (so that share per capita remains unchanged). Therefore, it would not surprise us if trading volume *only* increases in proportion to the increase in market size. However, here we see that the trading volume increases by more than 5 times, and also double that figure; therefore, the effect of market size on market diversity is not superficial. One can confirm this finding by the other two sets of experiments under individual learning, i.e., $\{(M_1, B), (M_2, B)\}$, and $\{(M_1, C), (M_2, C)\}$. By comparing the same figure pairwisely, one can see once again that when market size increases by five times, the trading volume increases by more than five times.

There is a simple explanation for the impact of market size upon market diversity. In the case of social learning, even though there is only a single pool of common knowledge, a larger number of traders (students) implies a larger sampling of the pool, and hence a greater diversity. In the case of individual learning, since there is no channel for direct dissemination of knowledge, a larger number of traders implies more secrets kept by each individuals, and hence also a greater diversity.

But, this explanation is only one-way and may oversimplify the situation. The markets which we study are typical examples of *co-evolving* systems. What has not been mentioned in the explanation above is the *interaction* among different components of the market. For example, in the case of social learning, a greater diversity of traders' expectations may result in more complex aggregate (price) phenomena, which in turn also nurse the diversity of the business school. And a pool with a greater diversity may make traders' expectations even more diversified. This *reinforcing mechanism* can go on and on, which eventually increases the trading volume by far more than 5 times.

As we have discussed earlier, in the context of individual learning, there is another key parameter which has an effect on market efficiency, i.e., *population size*. However, it is interesting to note that population size does not have an positive effect on trading volume. If we compare the experiment (M_1, B) to the experiment (M_1, C), we see the mean daily trading volume decreases from 15 units to 14 units when population size increases from 10 to 25. A similar result is also observed in the other pair of experiments, $\{(M_2, B), (M_2, C)\}$.

Why does population size have a *negative effect* on the diversity of traders' expectations? The reason is also intuitive. In the case of social learning, each trader can base their final decision only on *one model* no matter how many models, be it 10 or 25, they are able to process. As a result, the diversity of traders' expectations is not directly affected by population size. Nonetheless, a larger population size means to equip each trader with a better capability to process information. Hence, if there is a best model at a point in time, the chance of discovering this model improves for all traders. Even though they do not come up with the same model, it is still likely that they find similar ones. That explains why the diversity of traders' expectations and the trading volume may actually drop with the presence of a larger population size.

Table 3: Trading Volume

Market Size: M_1						
	Market A		Market B		Market C	
Run	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
1	0.150	1.384	16.216	13.034	15.189	12.805
2	0.277	1.328	17.665	14.517	15.363	13.067
3	0.410	1.998	15.373	13.785	15.613	13.035
4	0.150	0.928	16.713	13.898	15.843	13.323
5	0.049	0.285	15.122	13.405	13.702	12.439
6	0.080	0.480	14.342	12.345	14.613	12.380
7	0.044	0.328	13.955	11.980	12.892	12.168
8	0.172	1.242	15.174	13.116	12.841	11.371
9	0.059	0.329	14.365	12.535	14.820	12.369
10	0.105	1.134	14.153	12.447	14.838	12.503
Average	0.150	0.944	15.308	13.106	14.571	12.546
Market Size: M_2						
	Market A		Market B		Market C	
Run	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
1	0.728	5.172	73.330	47.525	71.622	49.105
2	0.935	4.042	82.657	53.155	82.150	53.602
3	0.718	7.004	84.541	50.787	82.813	51.563
4	0.778	5.195	72.865	50.061	76.502	49.392
5	1.112	8.127	95.009	60.465	86.131	57.656
6	1.319	6.277	85.942	52.902	79.125	52.582
7	0.409	1.807	74.807	48.631	90.215	55.193
8	0.877	7.791	78.962	50.327	70.133	45.129
9	1.944	10.873	81.123	53.567	78.114	49.935
10	1.453	7.110	83.132	51.648	78.295	52.167
Average	1.0273	6.340	81.237	51.907	79.510	51.632

5.4 Results: Portfolio and Market Diversity

In the previous section, market diversity is studied from the perspective of the trading volume. There is another way to observe market diversity, i.e, *trader's portfolios*. A portfolio is the distribution of traders' wealth into *money* and *shares of the stock*. If any point in time traders' are homogeneous in their portfolio decision, then they must hold the same shares of stock, and the variance (diversity) of shares hold among all trader is *zero*. Therefore, *the variance of shares hold* provides us another measure to examine market diversity.

Corresponding to Figures 1-3, Figures 4-6 present the time series plots of the variance of shares hold in the three different market experiments. A large degree of homogeneity of portfolios is observed in the experiment (M_1, A). In many of the trading days, the variance is almost zero, which suggests that traders hold the same amount shares. But (M_1, B) and (M_1, C) give quite different pictures.

Table 4 gives the mean variance of all experiments. We shall not give detail accounts of the results here, because the findings are the same as those in Table 3. Briefly speaking,

Table 4: The Mean Variance of Stock Shares between Traders

Run	Market Size: M_1			Market Size: M_2		
	Market A	Market B	Market C	Market A	Market B	Market C
1	0.004	1.743	1.652	0.003	1.733	1.722
2	0.008	2.034	1.757	0.004	1.961	1.937
3	0.012	1.837	1.768	0.005	1.916	1.870
4	0.007	1.957	1.780	0.005	1.732	1.866
5	0.001	1.690	1.601	0.005	2.299	2.047
6	0.002	1.556	1.688	0.006	1.967	1.966
7	0.001	1.584	1.451	0.002	1.782	2.110
8	0.006	1.700	1.433	0.007	1.826	1.612
9	0.001	1.642	1.618	0.017	1.982	1.794
10	0.003	1.576	1.679	0.010	1.825	1.918
Average	0.005	1.732	1.643	0.006	1.902	1.884

a greater diversity of portfolios is observed under a larger market size and individual learning. However, a larger population size slightly reduce the diversity.

5.5 Market Diversity and Market Efficiency

The experimental results obtained above can be summarized as two effects on market efficiency (price predictability), namely, the *size effect* and the *learning effect*. The size effect says that the market will become efficient when the number of traders (market size) and/or the number of models (GP trees) processed by each traders (population size) increases. The learning effect says that the price will become more efficient if traders' adaptive behavior become more independent and private. Coming to market diversity, we observe very similar effects except population size: market diversity does not go up with population size.

These findings motivate us to search for a linkage between *market diversity* and *market efficiency*. A suggested argument goes like follows: *a larger market size, and a more isolated learning style will increase the diversity of traders' expectations, which in turn make the market become more active (high trading volumes), and hence more efficient (less predictable).* While the statement is somewhat plausible, we cannot give it a formal proof. In fact, in complex adaptive systems, the route from cause to effect are sometimes so complicated that no one can follow every step of it. Nevertheless, since the argument is empirically relevant, it can be taken as a hypothesis to test with real data.

Consider the Taiwan stock market as an example. A large proportion of market participants are individual investors who has little control of the market. Based on our "theorem", a large market size implies efficiency, Taiwan stock market should be efficient. However, empirical evidences have shown that this market is not that efficient ((Chen and Tan (1996))). What is missing here? By our "theorem", the answer rests on *social learning*. Actually, in this market, individual traders usually consult the professionals from companies, institutions and even mass medium., which is exactly a kind of social learning. Also, take the United States as another example. In the U.S. stock market, a

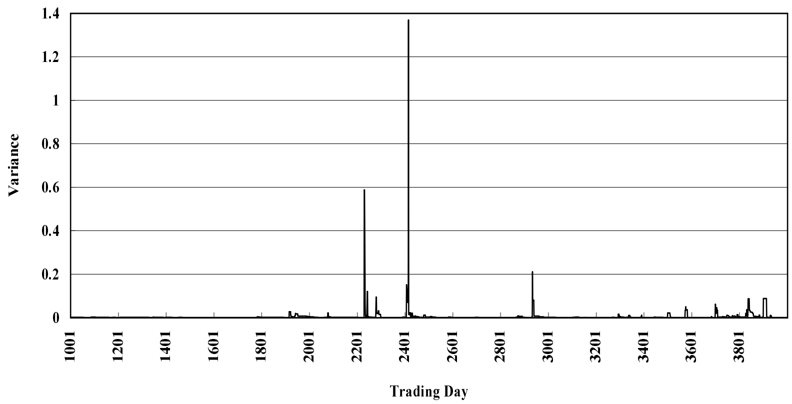


Figure 4 : The Variance of Stock Shares between Traders (Market A, First Run)

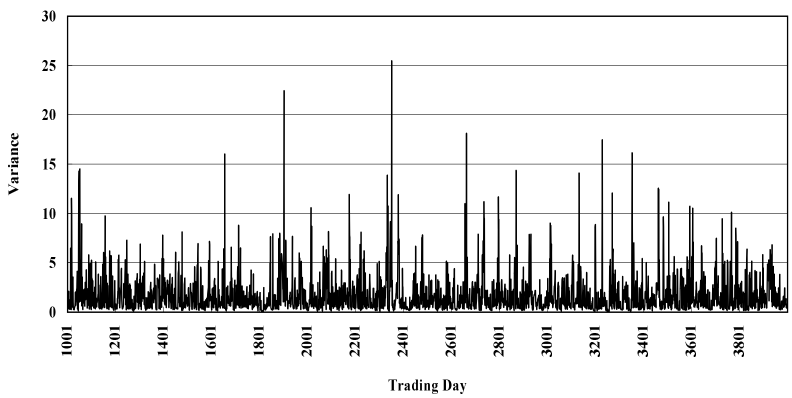


Figure 5 : The Variance of Stock Shares between Traders (Market B, First Run)

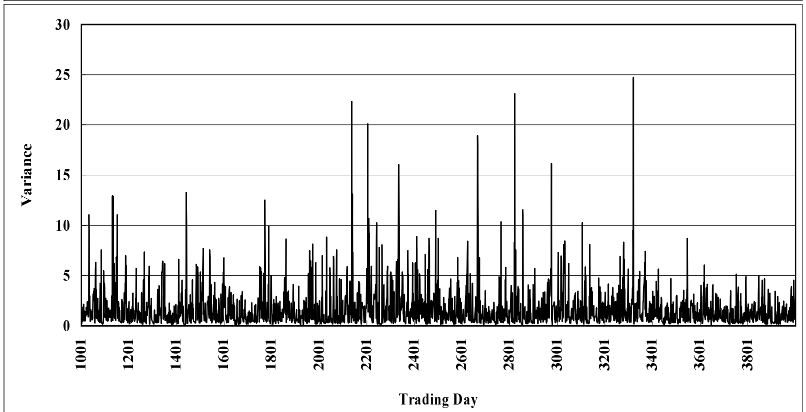


Figure 6 : The Variance of Stock Shares between Traders (Market C, Sixth Run)

large proportion of market participants are *institutional investors*, each have their own research department. This is more similar *individual learning*. By our "theorem", this market should be very efficient. Empirical studies have shown that it is indeed the case ((Chen and Tan (1996))).

6 Concluding Remarks

In this paper, the relation between market diversity and market efficiency is investigated. The simulation results reveal that the important driving force which makes market efficiently is market diversity. The increase of market size contributes to the market efficiency by means of introducing greater diversity into the market. Moreover, individual learning further reinforces the effect, whereas population size plays a mixing role. These three factors co-influence the market dynamics.

Of course, there exists some other determinants for market efficiency. For example, the degree of traders' *prudence*. This behavioral parameter concerns the number of periods (time horizon) which the traders look back while making their forecasts. A prudent trader cares long-term profits, which, in our framework, corresponds to a long evaluation cycle (n_2). Also, in a more flexible design, traders should be able to adaptively switch between individual learning and social learning, rather than getting stuck to only one learning style. The effects of these determinants are left for further research.

Acknowledgements

Research support from NSC grants No. 89-2415-H-214-002 is gratefully acknowledged. This paper was revised from its original version by taking the advice and comments of four anonymous referees. The authors are very grateful for their painstaking review of this paper. Of course, all remaining errors are the authors' sole responsibility.

References

M. Aoki. Arrregate Dynamics and Agent Interactions in Economic Model: A Stochastic View. Clocluding lecture given at the *Fourth Workshop on Economics with Heterogeneous Interacting Agents* (**WEHIA'99**), Genoa, Italy, June 4-5, 1999.

J. Arifovic, J. Bullard and J. Duffy. The Transition from Stagnation to Growth: An Adaptive Learning Approach. *Journal of Economic Growth*, Vol. 2, pp. 185-209, 1997.

W. B. Arthur, J. H. Holland, B. LeBaron, R. Palmer, and P. Taylor. Asset pricing Under Endogenous Expectations in an Artificial Stock Market. In *The Economy as an Evolving Complex Systwm*, Vol. II, W. B. Arthur, S. Durlauf, and D. Lanl (eds.), Santa Fe Institute Studies in the Scieuces of Complexity, Proceedings Volume XXVII, Reading, MA: Addison-Wesley. pp. 15-44, 1997.

W. Brock, D. Dechert, J. Scheinkman and B. LeBaron. Test for Independence Based on the Correlation Dimension. *Econometric Reviews*, Vol. 15, pp. 197-235, 1996.

S.-H. Chen, C.-W. Tan. Measuring Randomness by Rissanen's Stochastic Complexity: Applications to the Financial Data. In *Information, Statistics and Induction in Science*, D. L. Dowe, K. B. Korb and J. J. Oliver (eds.), Singapore:World Scientific, pp.200-211, 1996.

S.-H. Chen, T. Lux and M. Marchesi. Testing for Non-Linear Structure in an Artificial Financial Market. *Journal of Economic Behavior and Organization*, Vol. 46, Issue 3, pp. 327-342, 2001.

S.-H. Chen and C.-H. Yeh. Evolving Traders and the Business School with Genetic Programming: A New Architecture of the Agent-Based Artificial Stock Market. *Journal of Economic Dynamics and Control*, Vol. 25, Issue 3-4, pp. 363-393, 2001.

S.-H. Chen, C.-H. Yeh, and C.-C. Liao. On AIE-ASM:Software to Simulate Artificial Stock Markets with Genetic Programming. In *Evolutionary Computation in Economics and Finance*, S.-H. Chen (ed.), Heidelberg:Physica-Verlag. pp. 107-122, 2001.

W. J. Den Haan. The Importance of the Number of Different Agents in a Heterogeneous Asset-Pricing Model. *Journal of Economic Dynamic and Control*, Vol. 25, Issue 5, pp. 721-746, 2001.

E. Egenter, T. Lux and D. Stauffer. Finite-Size Effects in Monte Carlo Simulations of Two Stock Market Models. *Physica A*, Vol. 268, pp 250 - 256, 1999.

S. J. Grossman and J. Stiglitz. On the Impossibility of Informationally Efficiency Markets. *American Economic Review*, 70, pp. 393-408, 1980.

P. Harrald. Economics and Evolution. The panel paper given at the *Seventh International Conference on Evolutionary Programming*, March 25-27, San Diego, U.S.A., 1998.

J. Koza. *Genetic Programming: On the Programming of Computers by Means of Natural Selection*. The MIT Press, 1992.

R. G. Palmer, W. B. Arthur, J. H. Holland, B. LeBaron, and P. Tayler. Artificial Economic Life: A Simple Model of a Stockmarket. *Physica D*, 75, pp. 264-274, 1994.

J. Tirole. On the Possibility of Speculation under Rational Expectations. Econometrica, 50, pp. 1163-1181, 1982.

N. Vriend. An Illustration of the Essential Difference between Individual and Social Learning, and Its Consequence for Computational Analysis. *Journal of Economic Dynamics and Control*, Vol. 24, Issue 1, pp. 1-19, 2000.

N. Vriend. On Two Types of GA-Learning. In *Evolutionary Computation in Economics and Finance*, S.-H. Chen (ed.), Heidelberg:Physica-Verlag. pp. 233-243, 2001.