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Lottery markets design, micro-structure, and macro-behavior: An ACE approach

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Abstract

An agent-based computational modeling of the lottery market is established in this paper to study the design issue, in terms of the lottery tax rate, as well as the emerging market behavior. By using genetic algorithms and fuzzy logic, lottery participants are modeled as autonomous agents who may endogenously adapt to exhibit behavioral properties consistent with well-noticed behavior of lottery markets. Three major findings are presented. First, as anticipated, a Laffer curve is found in this model; nonetheless, the Laffer curve has a flat top, which indicates the non-uniqueness of the optimal lottery tax rate. Second, conscious selection behavior is also observed, but it becomes weaker as time goes on. Third, for the halo effect, we observe exactly the opposite. Each of these three findings are then compared with available empirical results, and the mechanism of genetic algorithms is further examined in light of the anti-halo effect.

JEL classification: D78; D83; E60; H21

Keywords: Lottery; Agent-based computational modeling; Genetic algorithms; Sugeno fuzzy models; Laffer curve

1. Introduction

Agent-based computational modeling has become a very promising new research tool in economics. One of its main advantages is its encapsulation of the idea of *autonomous agents*. Through modern techniques of *agent engineering*, the researcher is now endowed with the rich expressive power of the *life* of agents. This rich expressive power not only helps us bridge the gap between the artificial world and the real world but also enables us to evaluate the consequences of some external interventions when the route from cause to effect becomes so complicated that it is hard to trace every step of it. Over the past decade, fresh and interesting insights have been brought to economic analysis in some active application areas of agent-based computational modeling, such as the artificial financial market (LeBaron, 2006). As an extension of studies in artificial stock markets, this paper addresses an agent-based model of lottery markets.

Like the artificial stock markets, the research paradigm based on the representative agent already existed in the study of the lottery markets before the launch of agent-based modeling, such as Morgan and Vasché (1979, 1982), Mikesell (1994), Mason et al. (1997), McConkey and Warren (1987), Walker (1998), and Purfield and Waldron (1999). These

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Table 1
The lottery tax rates

Nation	Official issuer	Tax rate (percent)	Commission rate (percent)	Net tax rate (percent)
Austria	Austrian Lotteries	54.6	9.30	45.3
Belgium	Lotterie Natlonale	48.4	6.60	41.8
Brazil	Caixa Econômica Federal Bank	68.4	8.20	60.2
Canada	Loto-Quebec	48.7	6.80	41.9
Canada	Ontario Lottery Corp.	51.2	7.40	43.8
France	La Francaise	42.3	5.00	37.3
Germany	Westdeutsche	53.0	8.30	44.7
Italy	Lottmatica S.P.A.	48.4	10.00	38.4
Italy	Sisal Sport Italia	65.4	7.90	57.5
Japan	Dai-Ichi Kangyo Bank	54.2	7.40	46.8
Spain	ONCE	50.4	16.50	33.9
Sweden	Svenska Spel	48.8	9.60	39.2
Taiwan	Taipei Bank	40.0	8.40	31.6
UK	U.K. National Lottery	53.4	5.10	48.3
USA	Ohio State	40.3	6.40	33.9
USA	Michigan State	45.4	7.00	38.4
USA	Georgia State	45.9	7.00	38.9
USA	Maryland State	46.1	5.70	40.4
USA	Illinois State	45.9	5.10	40.8
USA	Texas State	46.4	5.20	41.2
USA	New Jersey State	47.2	5.40	41.8
USA	California State	49.3	6.70	42.6
USA	New York State	49.4	6.00	43.4
USA	Florida State	50.0	5.60	44.4
USA	Pennsylvania State	49.1	4.70	44.4

Data sources: "U.S. Lotteries' Unaudited FY00 Sales by Game," La Fleurs Lottery World (http://www.lafleurs.com/); Taiwan Lotto (http://www.roclotto.com.tw/) (the data for Taiwan are from the year 2002, whereas the data for other markets are for the year 2000).

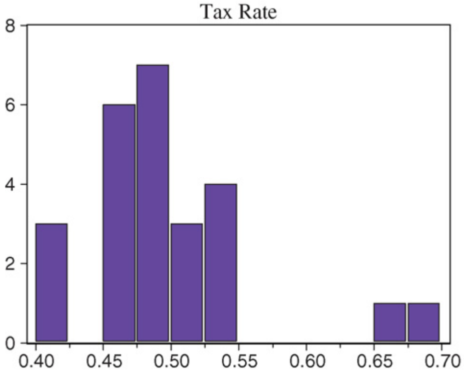
earlier studies treated the demand for lottery tickets as an individual rational choice problem and used demographic and socioeconomic data to estimate lottery demand. Nevertheless, our departure from the conventional research device to the agent-based modeling is motivated by the following two empirical observations.

First, Table 1 surveys *the lottery tax rates* of 25 lottery markets in the world. We see quite a wide distribution of the tax rate (the takeout rate).[1] From the lowest rate of 40 percent in Taiwan to the highest rate of 68.4 percent in Brazil, the difference is almost as high as 30 percent. Even in the U.S., there is a 10 percent gap from the lowest to the highest. The difference, which is also reflected in Fig. 1, brings us closer to *the design issue*. However, the tax rate is only one dimension of the complex lottery design. Starting from the numbers offered to be selected, the matching rules, to the money to be awarded for different prizes (such as the jackpot), one can face a great number of combinations (designs). Nevertheless, in the literature, we see that little effort has been made to evaluate the impact of different designs, such as their effects upon lottery revenue.[2]

Since the lottery revenue is a major source of funding for good causes, it is imperative to have a reason to explore an extensive class of "what-if" scenarios. In this paper, agent-based modeling, as an effective tool for dealing with "what-if" scenarios, is used to analyze the effect of the tax rate on tax revenue. More specifically, we are interested in knowing whether there is a *Laffer-curve phenomenon* in the lottery market. Stated slightly differently, is tax revenue globally sensitive to the tax rate? If so, what is the optimal tax rate? If not, within what range is it insensitive, and is it wide enough to justify the empirical range shown in Fig. 1?

[1] The tax rate here refers to the gross tax rate, including what is reserved for bookmakers' commission. It is called the takeout rate, to be distinguished from the net tax rate. In this paper, the two terms, the tax rate and the takeout rate, will be used interchangeably.

[2] The only studies known to us are Scoggins (1995), Hartley and Lanot (2003), and Paton et al. (2003). That the design of the United Kingdom National Lottery was not maximizing tax revenue was suggested by Hartley and Lanot. Interestingly enough, in October 2001, the U.K. government implemented a dramatic shift in the taxation of gambling that resulted in a substantial decline in taxes levied on U.K. bookmakers. An empirical study conducted by Paton et al. (2002) indicated that the tax reduction caused a one-third reduction in duty receipts.



	Tax Rate
Mean	49.7%
Median	48.8%
Maximum	68.7%
3rd Quartile	50.8%
1st Quartile	45.9%
Minimum	40.0%
Std. Dev.	0.064538
Skewness	1.358239
Kurtosis	5.332021

Fig. 1. The distribution of tax rates. *Data source*: see Table 1.

The other empirical observation that motivates an agent-based model of lottery markets is the *psychology* of the lottery market. Ordinary gamblers seem to be not so much concerned with the probabilistic calculation of winning odds; instead, they rely on *heuristic strategies* for handling the available information. Despite the fact that the series of the winning numbers is by all means generated by a random mechanism, they tend to believe that future predictions can be made on the basis of past history, and they tend to choose numbers in a non-random manner, called *conscious selection*. There are even professional people who make a living by detecting "patterns." Griffiths and Wood (2001) provide a splendid review of various heuristics and biases involved in the psychology of lottery, such as the hindsight bias, representation bias (gambler's fallacy), the availability bias, and so on. These heuristics and biases are, however, not easily captured by the standard rational analysis. Nonetheless, in agent-based modeling, agents can be *initialized* with various cognitive considerations: the description and design of agents is basically open-ended.

In this paper, our agents will be initialized with two heuristics and one psychological force. The first heuristic indicates agents' *portfolio strategies* (betting stake) based on their perception of the winning odds and yields the general observation that the agents' betting momentum increases with the jackpot prize. This heuristic, however, may have nothing to do with the sophisticated calculation of the winning probability. In reality, the grand prize is generally well publicized, which creates an additional excitement referred as to *lottomania* (Beenstock et al., 2000). Lottomania takes possession of the public and attracts their greater involvement.[3] The second heuristic reveals agents' *perception of the winning-numbers pattern*. No matter how fairly or how randomly the winning lottery numbers were generated, gamblers tend to believe that some sequences of numbers are less likely than others. For example, a sequence of consecutive numbers, say, 1, 2, . . ., 6, is considered more improbable than other sequences.

Finally, a psychological force added to the agents' initialization is a *feeling of regret*, which is known as *the aversion to regret* in the literature (Statman, 2002). Usually, when the mass media intensively reports the winners with their gigantic prizes, it may make those people who did not gamble feel regret: had they bet, the prize would have been theirs.[4] This psychological force referred as to the *regret effect* indicates the interdependence of the agents' utility functions.

The two above-mentioned heuristics and psychological characteristics are first randomly generated to initialize the agents' characteristics. These characteristics will evolve over time as agents are presumably utility-maximizers. As has been popularized in the literature on agent-based economic models, the evolution will be driven by *genetic algorithms*.

The remainder of this paper is organized as follows. Section 2 introduces an agent-based model of the lottery market. Section 3 describes in detail the use of the genetic algorithm. Section 4 outlines the experimental designs. The simulation results together with a discussion are given in Sections 5 and 6, respectively. Section 7 wraps up the paper with concluding remarks.

[3] In a way, this observation can be related to the *availability bias* as initially proposed by Kahneman and Tversky (1973).

[4] Similarly, if the winner is absent, those who did not gamble may now have a degree of comfort as "I knew it".

2. An agent-based model of the lottery market

2.1. The lottery market and its design

Typically, an agent-based model is comprised of two parts, namely, the *environment* and the *agent engineering*. The environment is characterized by a set of rules of the game, governing how agents are connected to the system and to other agents in the system. Here, we are concerned with a set of rules for a lottery game or design for a lottery game.[5] Generally speaking, a lottery game can be parameterized by two parameters (x, X). In an x/X lottery game, we required both a gambler and the lottery agency to pick x numbers out of a total of X numbers, and then different prizes are set for different numbers matched. Let y denote the numbers matched. Clearly, $y = 0, 1, \ldots, x$. Let S_y be the *prize pool* reserved for the winners who matched the y numbers. A special term is given to the largest pool, S_x, namely, the *Jackpot*.

Each prize pool, S_y, is to be shared by the number of players who match y numbers, say N_y. In the event that $N_y = 0$, S_y is added to the next draw. A particularly interesting case is $N_x = 0$. A common feature of lotteries is that if there are no winners in a given draw, the jackpot prize pool from that draw is added to the pool for the next draw, referred to as a *rollover*. Rollovers usually enhance the attractiveness of the next draw, called the *rollover draw*. The prize pool is defined by the *lottery tax rate*, τ, which is the proportion of sales that is not returned in the form of prizes. Thus, the overall prize pool is $(1 - \tau)S$, where S is sales revenue and $1 - \tau$ is also called the *pay-out rate*. The overall prize pool will then be distributed to each separate pool based on a distribution $(s_0, \ldots, s_x: \sum_0^x s_y = 1)$, that is, $S_y = s_y(1 - \tau)S$. It is anticipated that s_y will be increasing in y. To recap, a lottery game and its design can be represented by the following $x + 4$-tuple vector:

$$L = (x, X, \tau, s_0, \ldots, s_x). \tag{1}$$

One purpose of this agent-based simulation of the lottery market is to see how the changes in the design L can affect sales revenue and more importantly, tax revenue.[6] This brings us to another dimension of the lottery market, the *likely size of the market* can be determined by a series of economic and demographic factors. However, in this paper, we restrict our attention to only two factors, namely, *population size* and *income*. Both variables are treated as control variables in our agent-based lottery market. Let N denote the number of agents in the market, with each of them being indexed by ι ($\iota = 1, 2, \ldots, N$). For simplicity, one can assume that their income y_ι is exogenously given and fixed. In the simplest case, y_ι is further assumed to be identical among all agents, $y_\iota = \bar{y}, \forall \iota$. $(N \times \bar{y})$ gives us only an upper limit on the market size. Lottery draws take place at regular intervals, and at each draw, agents decide how many tickets to purchase. Therefore, the actual market size is determined by *agents' participation*, which is the aggregation of the behavior of individual agents.

In the literature, there are two approaches used to analyze agents' participation in the lottery markets. The first approach is to use the empirical data to model the principal features of the observed aggregate behavior.[7] The second approach is to start from a rational model of *representative agents*, and to then aggregate these representative agents.[8] The agent-based model is closer to the latter, while not using the devices of *rationality* and *homogeneity*. Agents are initially heterogeneous and boundedly rational, but they are autonomous and learning over time. Their details are left for the next subsection.

2.2. Agent engineering

What motivates agents to gamble, and how much to bet? We do not think that there is a unique answer or unique approach to this issue. Therefore, there are a number of possibilities in agent engineering. Nevertheless, a sound principle is to ground agent engineering with theoretical and empirical observations. By doing so, one can minimize the degree of arbitrariness. Our efforts in this agent-based model are in order to capture the following three "stylized

[5] We would like to draw readers' attention to Walker and Young (2001) for an excellent introduction to the design of a lottery game. They also stimulated discussion on the issue of an optimal design.

[6] This way of formulating the design issue is very similar to Walker and Young. However, the issue they addressed is different from ours. They addressed the effect of lottery design on lottery sales (lottery revenues), whereas we address the effect of lottery design on lottery tax revenues.

[7] Papers belonging to this category are Farrell and Walker (1999) and Farrell et al. (1999).

[8] The number of papers in this category is much smaller. Hartley and Lanot is the only one known to us.

facts" of the lottery market, namely,

- lottomania and the halo effect,
- conscious selection, and
- aversion to regret.

2.2.1. Lottomania and the halo effect

That the lottery participation level is positively related to the size of the jackpot prize seems to be one of the most important empirical observations. The phenomenon that sales following a rollover are higher than sales prior to the rollover is known in the industry as the *halo effect* (Walker and Young, 2001, Creigh-Tyte and Farrell, 2003). The halo effect is partially due to considerable media attention being paid to rollovers, which in turn creates a bout of *lottomania*. Therefore, we can start building our agents from a *participation function* that relates the participation level to the jackpot size,

$$\mu = \rho(J), \tag{2}$$

where ρ is the participation function, μ is a measure of the participation level, and J is the size of the jackpot. The exact functional form of ρ depends on the framework within which the problem is formulated. In the standard rational analysis, μ is related to J via a change in the *expected value* or, more generally, the *expected utility*, of the lottery ticket (Hartley and Lanot). However, here, we take a heuristic approach, and assume that gamblers base their decision on some heuristics rather than the possibly quite demanding work on expectations computation.[9]

Based on the heuristic approach, Eq. (2) can be approximated by a few simple *if-then* rules. For example, "if the jackpot is unusually high, then I will spend 10 percent of my income to buy lottery tickets," or "if there is no rollover, I will spend only a little." Notice that the antecedent or consequent of the rule contains the use of natural language that may not have concrete numerical meanings, such as the linguistic terms "high" and "only a little" in the above example. While natural language has its ambiguities, people seem to be able to reason effectively with added vague and uncertain information, and very often the decisions they make are the outcome of their approximate reasoning. Over the last four decades, we see the development of *fuzzy logic* as a formal approach to deal with these ambiguities. In this paper, we propose representing the function ρ by a set of *fuzzy if-then* rules that are manipulated by the standard mathematical operations of fuzzy sets as prescribed by fuzzy set theory.

We proceed as follows. First, let J_{t_r} be the jackpot prize updated on the rth day of the tth issue, where $r = 1, 2, \ldots, w$. w denotes the gap between two draws. If we suppose that the lottery draw takes place weekly, then $w = 7$. Furthermore, let the set $\{J\}_{t_r}$ be the time series of the jackpot prize up to the time of t_r. Second, given the historical data, the *attractiveness* of the lottery game can be measured by how unusual the J_{t_r} is as compared to $\{J\}_{t_{(r-1)}}$, if $r>1$, or $\{J\}_{(t-i)_w}$, if $r=1$. The agent will then act upon the degree of attraction. For example, if the jackpot is *huge*, the agent may react more energetically by betting greatly. Alternatively, if the jackpot is perceived as *low*, the agent may be not interested in spending a penny.

Technically, each agent gambles with his/her own *fuzzy rule-based system*, which comprises a number of fuzzy if-then rules. Each fuzzy if-then rule within the system can be represented as follows:

$$\text{If } J_{t_r} \text{ is } A_i, \text{ then } a_i. \tag{3}$$

The A_i $(i=1,\ldots,k)$ are *fuzzy sets* representing k different states of the jackpot prize. For example, consider the case $k=4$. Then $A_1,\ldots,A_4$ can denote the following four linguistic descriptions of the size of the jackpot: "low," "medium," "high" and "huge." a_i is the level at which the agent decides to participate given that the current state is A_i. The participation level can be measured by the *proportion of income* that agents would spend to purchase lottery tickets. Call the vector $\vec{a}(= (a_1, \ldots, a_k))$ the *participation vector*. Then different heuristics can be captured by different as. For example, $\vec{a} = (0.1\,\text{percent}, 1\,\text{percent}, 5\,\text{percent}, 10\,\text{percent})$ characterizes the agent whose betting stake is increasing

[9] Many details can complicate the computation of the expected value. First of all, the expected value depends on the expected number of winning gamblers: the higher the expected number of winners, the lower the share of the jackpot for each winner. On the other hand, the expected number of winners depends positively on the participation level, by which the size of the jackpot is also positively affected. This circular phenomenon applies to other non-fixed prize pools. Second, the expected value can differ among different agents, given their conscious-selection behavior.

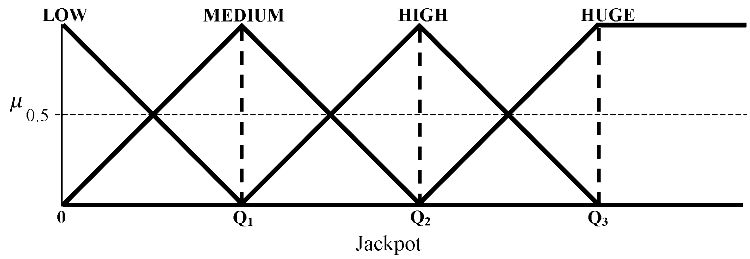


Fig. 2. The membership function.

with the size of the jackpot prize. On the other hand, $\vec{a} =$ (0.1 percent, 0.1 percent, 0.1 percent, 0.1 percent) indicates that the agent's betting stake is independent of the size of J.

Based on our description above, only the input set, A_i, of (3) is fuzzy, and the output set, a_i, is a crisp numerical value. This type of fuzzy rule is known as the Sugeno style of fuzzy rules, as distinguished from the Mamdani style of fuzzy rules, in which the input and output sets are both fuzzy. Fuzzy sets are distinct from the classical sets (crisp sets) in the sense that the *membership* in the latter is *all or nothing*, whereas that in the former is a matter of *degree* (*more or less*). The degree is mathematically characterized by a *membership function*.

There is a wealth of membership functions. We, however, see little guidance as to the selection of them.[10] Therefore, before more research is done on this area, we have to accept some degree of arbitrariness. For simplicity, we choose the frequently used *triangular-shaped fuzzy membership function* specifically shown in Fig. 2. In Fig. 2, the domain of J is partitioned into four overlapping intervals by a sequence of base points Q_0, Q_1, Q_2, Q_3: $[Q_0, Q_1)$, (Q_0, Q_2), (Q_1, Q_3), and (Q_2, ∞). Let us denote them by $I_1, \ldots, I_4$, respectively. For each fuzzy set A_i, $\mu_{A_i}(J) > 0$ if $J \in I_i$; otherwise $\mu_{A_i}(J) = 0$, where $\mu_{A_i}(J)$ is the membership function, $\mu_{A_i} : R^+ \to [0, 1]$. However, unlike the usual fuzzy membership functions, the base points upon which the membership functions are defined are *not fixed*. This is because all the linguistic terms have no absolute meaning. What is perceived as *high* or *low* by agents will depend on what has happened before. It is the *frequency* that determines how we describe the event perceived so *huge* should refer to some events that happen more infrequently than what the term *medium* may refer to. This justifies the use of *sample statistics* as the base points, such as, *quartiles*, and Q_1, Q_2, and Q_3 are the first, second and third quartiles of the sample $\{J\}_{t_r}/0$. The sample quartiles may converge if $\{J\}_{t_r}$ turns out to follow a stationary distribution; otherwise, they will change over time.[11]

The implementation of the fuzzy rules (3) proceeds as follows. For each period of time t_r, the agents observe the time series of the jackpot up to the beginning (the first second) of t_r, $\{J\}_{t_r}$. All Q statistics can be determined accordingly, as with the membership function $\mu_a(J)/J$. Given J_{t_r} (i.e. the jackpot at the beginning of this period), the agent can then figure out the membership degree of each possible state (each fuzzy set), for example, $\mu_{A_i}(J_{t_r})$ $(i = 1, \ldots, k)$. In the Sugeno fuzzy model, each corresponding rule is activated to a degree $\mu_{A_i}(J_{t_r})$, and the output is a weighted average of all consequent actions a_i, weighted by the membership degree.

$$\alpha_{t_r} = \alpha_{t_r}(J_{t_r}) = \sum_{i=1}^{k} \mu_{A_i}(J_{t_r}) a_i. \tag{4}$$

The agents' involvement in the lottery is defined by the fuzzy if-then rules (3) associated with the participation vectors $\vec{a}$. Adaptive behavior can be characterized by changes made in $\vec{a}$. In Section 3, we shall show how $\vec{a}$ can be encoded as a bit string and evolved via genetic algorithms.

2.2.2. *Conscious selection*

To take conscious selection into account, let $\vec{b}$ be an X-dimensional vector whose entities take either "0" or "1." Consider a number z, where $1 \le z \le X$. If "0" appears in the respective zth dimension, the number z will not be consciously selected by the agent, while "1" indicates the opposite. Therefore, $\vec{b}$ shows a list of numbers that may

[10] There are some limited experimental studies conducted by psycholinguists.

[11] A similar way of using sample statistics to determine the base points of membership functions can also be found in Draeseke and Giles (2002).

be consciously selected by the agent. If $\vec{b}$ has exactly x 1s, then one and only one combination is defined and the agent would select only that combination while purchasing the lottery ticket (s). If $\vec{b}$ has more than x 1s, then many more combinations can be defined. The agent will then randomly select from these combinations while purchasing the ticket(s). Finally, if $\vec{b}$ has less than x 1s, then those designated numbers will appear in each ticket bought by the agent, whereas the rest will be randomly selected from the non-designated numbers.

The agent's betting heuristic, h, is fully characterized by the vector:

$$h = (\vec{a}, \vec{b}).$$

To make it apparent that the h are different over time (evolving) and are different over space (heterogeneity), we shall denote the heuristic used by agent ι at time t by $h_{\iota,t}$. In Section 3 we will detail the implementation of the evolution of $h_{\iota,t}$ via genetic algorithms.

2.2.3. Aversion to regret

Regret is the pain we feel when we find, too late, that a different choice would have led to a better outcome. In the case of the lottery market, regret simply means that the utility of not gambling depends on whether there are winners. If nobody wins, that would make those who do not gamble feel no regret; however, if someone does win, they may feel regret because it could have been a certain person's had he given it a try. Lottery promoters capitalize on the aversion to regret when they encourage lottery buyers to keep on buying. If regret does play an important role, then the agent's utility function is no longer independent.

For simplicity, let us assume that agent ι has a simple one-period linear utility function of consumption:

$$u(c) = c, \tag{5}$$

with the budget constraint:

$$c \leq e - \alpha(\vec{a})e + \pi, \tag{6}$$

where e is his initial income, α is the proportion of his income spent on the lottery, and π is the lottery prize. For those agents whose α is zero, their utility depends on whether there is a jackpot winner. The utility function (5) has to be modified as follows:

$$u(c) = \begin{cases} (1-\theta)c, & \text{if } \alpha = 0 \text{ and } Nx = 0, \\ c, & \text{otherwise.} \end{cases} \tag{7}$$

The θ in the utility function (7) measures how regretful the non-gambler would be if the jackpot were drawn.[12] On the other hand, opposite to regret, the non-gamblers may also derive pleasure from gamblers' misfortunes, in particular when the jackpot is not drawn ($N_x = 0$). As a result, the utility function (7) can be extended as follows:

$$u(c) = \begin{cases} (1-\theta)c, & \text{if } \alpha = 0 \text{ and } Nx > 0, \\ (1+\theta)c, & \text{if } \alpha = 0 \text{ and } Nx = 0, \\ c, & \text{otherwise.} \end{cases} \tag{8}$$

Obviously, the larger the θ, the less independent will agent ι's utility be. While we can treat θ as an exogenous variable, from the viewpoint of psychology, it would be interesting to see how θ is determined endogenously. In this way, θ is treated as a personal *trait* that indicates how agents experience things and how they feel about them.

To sum up, agents in our artificial lottery markets are fully characterized by the vector:

$$(h_{\iota,t}, \theta_{\iota,t}) = (\vec{a}_{\iota,t}, \vec{b}_{\iota,t}, \theta_{\iota,t}), \tag{9}$$

where $\theta_{\iota,t}$ is the preference parameter of agent ι at time period t. The vector $(h_{\iota,t}, \theta_{\iota,t})$ will be encoded as a bit string, and then genetic algorithms will be applied to evolve a population of $(h_{\iota,t}, \theta_{\iota,t})$, which is detailed in the next section.

[12] Certainly, regret may work in the reverse direction as well. Nevertheless, since in general mass media will only give a large coverage to the jackpot winners and are not interested at all in anything happening to the non-gamblers, that asymmetric coverage makes the regret that works in the reverse direction rather negligible, and hence it is assumed away in this paper.

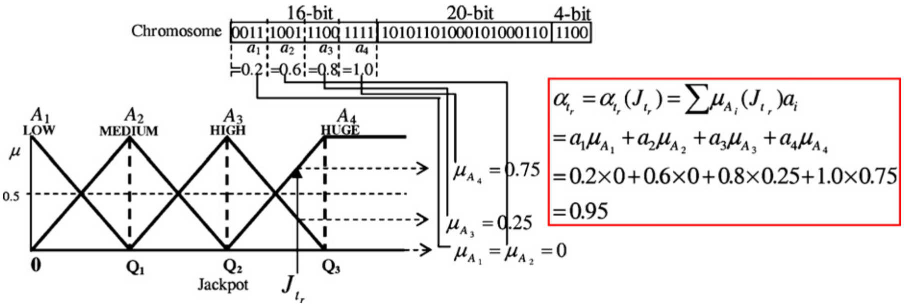


Fig. 3. Betting heuristics based on the Sugeno fuzzy inference system.

3. Genetic algorithms

We choose GA in this paper primarily based on the following two considerations. First, the choice of learning model crucially depends on how the problem (environment) is presented to decision makers. Since in this paper the strategy space presented to our agent, as in Eq. (9), is large and complex, it makes handling this situation by using GA rather than other alternatives easier. Second, GA constitutes one of the most important classes of learning models in the ACE literature. We want to give it a further exploitation and document some useful evidence to enrich our understanding of it. Therefore, we choose GA not because it is the best learning model, which is still unknown to us, but because we are satisfied with using it as a starting point and as a benchmark.[13]

3.1. Representation

In our model, binary coding is applied to the vector $(\vec{a}_{\iota,t}, \vec{b}_{\iota,t}, \theta_{\iota,t})$, which fully characterizes an individual ι at time t.[14] First, let us start with $\vec{a}$, the participation vector that is a k-dimensional vector, $(a_1, \ldots, a_k)$, where a_i $(1 \le i \le k)$ lies between 0 and 1. Each a_i is first coded by a binary string with length l_a. The decoding is performed in the following way:

$$a = \frac{\sum_{i=1}^{l_a} c_i 2^{i-1}}{2^{l_a} - 1}, \tag{10}$$

where c_i is the cth bit counted from the right. Thus, totally, a is coded by k, l_a bits, exemplified as follows.

$$(\overbrace{\underbrace{01..0}_{l_a \quad bits} \mid \underbrace{10...1}_{l_a \quad bits} \mid \underbrace{11...1}_{l_a \quad bits}}^{k \quad sub-strings}).$$

Fig. 3 illustrates *a fuzzy inference system* (3) with $k = 4$ and the corresponding binary string of $\vec{a}$ (with $l_a = 4$) decoded as $\vec{a} = (0.2, 0.6, 0.8, 1.0)$. The input J is perceived by the agent, and the membership degree of each fuzzy set is calculated as follows: $[\mu_{A1}(J), \ldots, \mu_{A4}(J)] = [0, 0, 0.25, 0.75]$. Thus, by Eq. (4), the agent will invest $\alpha = \sum_{i=1}^{4} \mu_{A4}(J) = a_i = 0.95$ of his income to purchase the lottery tickets.

---

[13] How to model learning of agents has been an intensive issue over the last decade. In the context of agent-based economic modeling, this research subject was recently reviewed by Brenner (2006) and Duffy (2006). Various learning models have been proposed, ranging from near zero-intelligent models to highly sophisticated ones, from non-conscious learning to conscious learning, from individual to social learning, and from statistically based learning to psychologically based and to biologically based learning. Given these large varieties, the parsimony principle, also known as the KISS principle, has been suggested as a guideline for model selection. Nonetheless, since ACE models can have different purposes, such as replicating stylized facts, generating scenarios and facilitating thought experiments, exactly how the KISS principle should be implemented and which learning model should be selected remain unclear. Even though we focus exclusively on empirical validity, the problem is still present. See Chen and Tai (2006) for an in-depth discussion of this issue.

[14] Binary coding has a problem known as the *Hamming cliff*. Basically, the Hamming distance defined in the space of binary strings is not geometrically equivalent to the metric usually used in the Euclidean space. Therefore, a small mutation (perturbation) to a binary string may result in a large change in its real number counterpart. This may make the binary-coding representation inherently more volatile than the real-coding one. We, however, have found that these additional volatilities can help the market avoid the fast convergence to a perfectly idle market.

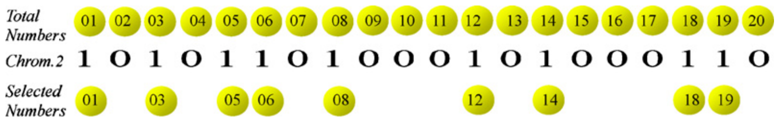


Fig. 4. An example of agents' picking numbers.

Second, it is straightforward to code the $\vec{b}$, the number-picking vector. As we mentioned in Section 2.2.2, it is simply an X-bit string. An example of the case $X = 20$ is shown in Fig. 4.

Finally, the regret parameter θ, which also lies between 0 and 1, can be encoded in a fashion similar to Eq. (10) by an l_θ-string of bits. Therefore, the full characterization is encoded by a string with a total of $k\,l_a + 20 + l_\theta$ bits.

3.2. Evolutionary cycle

Genetic algorithms start with an initialization of a population of chromosomes (binary strings), called Generation 1 (GEN 1). The number of chromosomes or the *population size*, denoted by *Pop*, is fixed during the whole evolution. Then a fitness criterion (fitness function) is used to evaluate the performance of each chromosome. Based on the performance evaluation, the next generation of chromosomes shall be genetically produced by the incumbent. The genetic production starts from the selection of a *mating pool*. There are several different selection schemes in GA. However, to have a better focus, only *tournament selection* will be tried in this paper. Nevertheless, according to the progress we make, the other selection scheme will be included at a later stage.

By tournament selection, each individual in the mating pool is determined as follows. We first randomly select φ random chromosomes without replacement and then take the best *two* of them. The parameter φ is known as the *tournament size*, and it is also the mating-pool size. Two genetic alterations, crossover and mutation, are operated on them to produce two offspring.

The crossover (the point crossover) cuts each parent chromosome into κ pieces. Since each chromosome represents altogether three different aspects of agents' behavior, the crossover operator is made in a *pair-by-pair* manner (i.e. by restricting the exchange only to the paired characteristic, called the *paired crossover*).

The second genetic alteration is *mutation*. After the crossover, each bit of the resultant chromosome has a chance of being flipped from "0" to "1" or "1" to "0." The offspring will, after the mutation process, then *replace* the old generation. Based on the parameter η, the agents belonging to the top $1-\eta$ percent will remain, and the agents belonging to the bottom η percent will be replaced by offspring.

4. Experimental designs

The agent-based lottery market as introduced in Sections 2 and 3 is summarized by two sets of parameters, the one associated with the *market*, and the other associated with the *agents*. Parameters associated with the market are encapsulated into the vector M:

$$M = (x, X, \tau, s_0, \ldots, s_x, w, N, \bar{y}).$$

This paper studies the possible relationship between the lottery tax rate and the tax revenue by hypothesizing the existence of a Laffer curve, and hence an optimal interior τ. To do so, different values of τ ranging from 0 to 90 percent are attempted in this paper. The rest of the market parameters are treated as *constants* throughout the entire simulation, and they are listed in Table 2.

The set of prize ratios, $s_0, \ldots, s_5$, are chosen to be consistent with the Taiwan Lotto. Similarly, the drawing period for each issue (w) is also motivated by it, assuming that each period is equivalent to 1 day, and there are two issues per week. The most intriguing part, however, is the setting of x/X, which must be determined simultaneously with N and y. To run the simulation in a reasonably fast way, N can only be set as a number such as between 5000 and 10,000, which can hardly match the population size of a real country. This forces us to modify x/X in a such way that it can be

Table 2
Experimental design

Market parameters	
Pick x from X (x/X)	5/16
Lottery tax rate (τ) (percent)	0, 10, . . ., 90
$s_0, s_1, \ldots, s_5$ (percent)	0, 0, 35, 15, 12, 38
Drawing periods (w)	3
Number of agents (N)	5000
Income (y)	200
GA parameters	
Number of fuzzy states (k)	4
Number of bits (l_a, l_θ)	4,4
Periods (generations) (T)	500
Crossover rate (P_c) (percent)	90
Mutation rate (P_m) (percent)	0.1
Tournament size (φ)	200
Generation gap (η)	100

comparable to a real market, say, Taiwan. This makes us consider a rather smaller X, only 16. A game of 5/16 is then matched to a market size of $N \times y = 5000 \times 200$.[15]

The second set of parameters is concerned with the control parameters of the genetic algorithm:

$$A = (k, l_a, l_\theta, T, P_c, P_m, \varphi, \eta).$$

To have a focus and make our presentation easier, all these parameters are also fixed during the entire simulation, as shown in Table 2.

Models built upon genetic algorithms are stochastic models in the sense that even the same fixed design may come up with different results. Therefore, to enhance the validity of what we may conclude from the simulation, multiple runs of the same design are inevitable. Each set of parameters is run 25 times.

5. Experimental results

5.1. The take-out rate and tax revenue

As shown in Table 2, each run lasted 500 periods (i.e. 500 draws). Tax collected from each game is indexed by R_t (the tax revenue from the tth issue of the tickets). A time series $\{Rt\}_{t=1}^{500}$ is observed after each run. To make sense of the results, we further normalize the revenue series by dividing R_t by the total income $N \times \bar{y}$ and call this new series, $\{r_t\}_{t=1}^{500}$, the *normalized tax revenue* series. Notice that normalized tax revenue can be interpreted as an *effective tax rate*. To avoid the possible initialization biases, we took away the first 100 periods of the data and calculated the *mean* for the rest of the sample (i.e. $\{r_t\}_{t=101}^{500}$). Let us denote it by $\bar{r}$. Since we have 25 runs for each single lottery tax rate, we therefore report the median of $\bar{r}$ over these 25 runs, and the results are shown in Fig. 5.

The figure shows that the (normalized) tax revenue first increases with the lottery tax rate τ, and then decreases with it. The highest tax revenue appears at $\tau = 40$ percent with an f of 10.5 percent. In addition to the median, it is also interesting to notice the change in the uncertainty of tax revenue under different tax rates. This is reflected by the associated box–whisker plot also shown in Fig. 5. The box in the middle of the plot covers 50 percent of the simulated tax revenue. The longer the box, the more uncertain the tax revenue. From Fig. 5, the tax revenue is relatively low and stable when the tax rate comes to its two extremes ($\tau = 10$ and 90 percent). However, the box starts to inflate when the tax rate is away from the two extremes, which signifies the growing uncertainty in tax revenue. The degree of uncertainty is further compounded by the enlarging whiskers that extend the box to the frontier of the sample distribution.

Another way of describing what is found in Fig. 5 is that the *elasticity* of changes in tax revenues with respect to changes in the tax rate is *unstable*. Tax revenue can be *statistically insensitive* to a range of the tax rate, say, from

15 These figures are derived from the figures offered by Walker and Young.

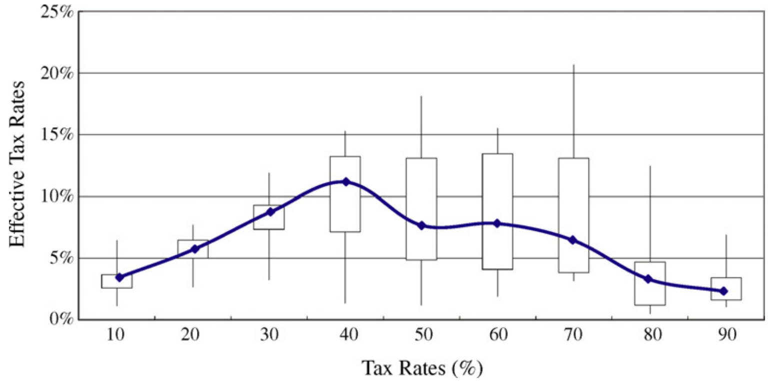


Fig. 5. Tax revenue curve and the associated box–whisker plot.

$\tau = 0.3$ to 0.7.[16] This *statistically flat* Laffer curve makes the determination of the optimum tax rate τ^* less certain. The flat, however, fits roughly well with the range as shown in Fig. 1, namely, from the lowest 40 to 68.7 percent. Therefore, this ACE lottery model provides a possible explanation for the coexistence of different lottery tax rates as we summarize in Fig. 1.

The statistically flat Laffer curve can be further pursued by testing whether the product of the lottery sales (S) and lottery tax rate (τ) is a constant over a range. This can be done by running a simple regression,

$$\ln^{S} = \beta_0 + \beta_1 \ln^{\tau} + \varepsilon. \quad (11)$$

If the constancy relation holds, then β_1 should be close to minus one. By pooling all runs with τ from 0.3 to 0.7, it is found that the null hypothesis that $\beta_1 = -1$ cannot be rejected.[17] The result, therefore, suggests that setting any τ between 0.3 and 0.7 is at about the right level. This finding can also be related to the empirical study on the elasticity of the demand for lottery tickets. After adjusting the bias of conscious selection, Farrell et al. (2000) found that the demand elasticity for the U.K. lottery ticket is also close to one.[18]

5.2. Rollovers and sales

It is generally assumed that the large size of the rollovers will enhance the attractiveness of the lottery game. The statistics also tell us that the mean sales conditional upon the rollover draw is normally *higher* than that of the regular draw. For example, based on the time series data of the U.K. lottery from 19 November 1994 to 5 March 2003, a total of 751 draws, the average sales are 56.0 million pounds over the rollover draws, whereas they are 41.4 million over the regular draws. Nevertheless, exceptions exist. Among a total of 112 rollover draws for the U.K. lottery, it happened on 25 occasions that sales actually *fell*.

To have a general picture of the empirical relationship between rollovers and sales, Table 3 summarizes some basic statistics of seven lottery markets, namely, the U.K., Taiwan, South Africa, Ireland, Switzerland, Japan, and Turkey. We first conduct a statistical test for the significance of the difference between the sales in the rollover draw and the sales in the regular draw. The t-test statistics are shown in the second column. Below each test statistic is the corresponding p value. Second, for the rollover draws, we further regress sales against the jackpot size as

$$S_{t,\text{rollover}} = \alpha_0 + \alpha_1 J_{t-1} + \varepsilon_t. \quad (12)$$

[16] Using ANOVA, we test whether the effective tax rate is the same under different intervals of τ. The test statistics of the interval [0.3,0.7] and [0.4,0.7] have a p-value of 0.6402 and 0.6847. Further extensions of the intervals to either 0.2 or 0.8 all result in a p-value close to 0.

[17] There are a total of 125 observations (5 τs each with 25 runs) involved in this regression. $\hat{\beta}_1 = -1.217$. The Wald test of the null that $\beta_1 = -1$ has a F value of 1.36, and the associated p-value is 0.2446.

[18] However, whether Laffer curve has a flat top is difficult to address empirically because the lottery administration cannot fine-tune the tax rate too frequently. As a result, it is difficult to get enough observations to support an estimation. *This limitation shows the potential value of agent-based modeling in policy analysis*.

Table 3
Rollovers and sales: statistics from the real data

Nation/period (periods)	t-Statistic (p-value)	α_1 (p-value)	$\bar{R}^2$	Anomalies (percent)
United Kingdom (November 19, 1994 to March 8, 2003)	4.2231 (0.0000)	1.9418 (0.0000)	0.2126	22.32
Taiwan (January 22, 2002 to May 23, 2003)	3.4578 (0.0013)	10.1766 (0.0000)	0.9133	0.00
South Africa (March 11, 2000 to June 4, 2003)	3.6959 (0.0001)	1.3307 (0.0000)	0.2052	27.78
Ireland (January 2, 2002 to June 28, 2003)	5.9171 (0.0000)	0.7451 (0.0000)	0.6371	1.04
Switzerland (January 1, 2003 to June 28, 2003)	5.7976 (0.0006)	9.3881 (0.0015)	0.9227	0.00
Japan (July 4, 2002 to June 26, 2003)	5.9727 (0.0000)	1.1291 (0.0000)	0.9460	6.67
Turkey (June 20, 2001 to July 2, 2003)	0.5384 (0.3067)	12.6762 (0.0713)	0.6196	0.00

where "J_{t-1}" is the jackpot size rolled in from the $t-1$th issue. Eq. (12) is only applied to the sales in the rollover samples, $S_{t,\text{rollover}}$. Sales in the regular draw are not taken into account since the jackpot size must start from 0 for all the regular draws. The values of the coefficient α_1 and $\bar{R}^2$ are reported in Columns 3 and 4. Finally, as mentioned earlier, it is surprising to see that sales may fall in some rollover draws. To acknowledge the occurrence of this anomalous relationship, the fifth column gives the percentage of the rollover draws whose sales actually declined rather than rose. We consider this statistic important because it hints that the underlying agents' behavior connecting rollovers to sales may be more complicated than one may hypothesize from a simple linear regression.

Table 3 shows quite consistent patterns for the seven lottery markets. First, the *halo effect* is evident in all markets. This is reflected by the significantly positive t-statistic (the second column) that means that sales in the rollover draw are significantly greater than those in the regular draw. Second, as we expect, the jackpot size significantly prompts sales. Its positive effect on sales is statistically significant in all markets. The only question is whether its explanatory power is good enough. In some markets, $\bar{R}^2$ is surprisingly high, reaching up to 90 percent, whereas in the other two countries, it is only 20 percent. However, what should not be hidden from this general expected result is the existence of anomalies. The anomalous relationship between rollovers and sales is prevalent in two of the three markets. In South Africa, sales fell in 28 percent of the rollover draws, whereas in the U.K. they declined in 22 percent of them. What may cause these anomalies is an issue that we would like to pursue in this line of study.

Based on these references, it is interesting to see whether similar patterns hold for our artificial lottery markets. Therefore, we look at the same statistics over the simulated data. What we do here is *pool* together all the simulated data under the same tax rate and obtain the statistics based on the tax rate. The results are shown in Table 4. Marked contrasts between Tables 3 and 4 are observed. First, *the halo effect disappears*. More than that, all t statistics now become significantly *negative*. We now have the opposite of the halo effect, namely, the *anti-halo effect*. Second, the effect of the jackpot size is by and large positive, which is consistent with what was observed in the real data. However, its explanatory power diminishes very quickly with the increase in the takeout rate. Given the above result, it is not surprising to see that "anomalies" now become normal. For all takeout rates, sales declined in more than 50 percent of the rollover draws.

The disappearance of the halo effect and the appearance of the anti-halo effect is certainly astonishing. This is even more so because our agent engineering is based upon the consideration of the halo effect (see Section 2.2.1). However,

Table 4
Rollovers and sales: statistics from the simulated data

Tax rates	t-Statistic (p-value)	α_1 (p-value)	$\bar{R}^2$	Anomalies (percent)
0	−19.3379 (0.0000)	0.6014 (0.0000)	0.1352	49.14
0.1	−23.0334 (0.0000)	0.6438 (0.0000)	0.2996	58.67
0.2	−66.1523 (0.0000)	0.5583 (0.0000)	0.0645	63.63
0.3	−99.0913 (0.0000)	0.1093 (0.0240)	0.0042	63.50
0.4	−117.1700 (0.0000)	0.2144 (0.0000)	0.0148	62.09
0.5	−100.7600 (0.0000)	0.1563 (0.0000)	0.0093	63.36
0.6	−87.8737 (0.0000)	0.0322 (0.4165)	−0.0001	61.58
0.7	−82.4286 (0.0000)	0.1121 (0.0462)	0.0010	60.52
0.8	−49.3922 (0.0000)	0.0899 (0.0840)	0.0004	57.47
0.9	−44.7909 (0.0000)	−0.2789 (0.0130)	0.0010	56.34

to compare what we have from the real data with what we have from the artificial data provides us with a chance to reflect upon something that we may take for granted. In particular, what is the essence of the phenomenon of the halo effect? Why did the agent-based system built upon GA fail to deliver this feature? Also, given the halo effect, why are there so many exceptions (about 20–30 percent in real markets)? Why is the agent-based model particularly good at producing these "anomalies"? These are the questions to be addressed in Section 6.

5.3. Conscious selection

Hard empirical statistics on conscious-selection behavior are not available yet in the real market, and few empirical studies have estimated that participants do not choose their numbers randomly (Farrell et al., 2000; Wang and Lin, 2006). In our simulation, the numbers favored by each agent are observable. The vector $\vec{b}$, as detailed in Section 2.2.2, shows the numbers picked or excluded by the agents. This profile provides us with the chance to observe the behavior of conscious selection. In particular, it enables us to address the question of whether the agent essentially believes that winning numbers are randomly selected.

This can be done by asking each agent the following question: Does the agent believe that each number is equally likely (or unlikely) to be picked by the lottery administration? If the agent believes that winning numbers are randomly generated, then all combinations are available for him to select. Therefore, simply by counting how many combinations are excluded by the agent or how many combinations are effectively available to the agent, one can develop a metric to measure how far the agent is away from the belief of a fair game. Let d be the metric, and

$$d = \begin{cases} \begin{pmatrix} X-z \\ x-z \end{pmatrix} \Big/ \begin{pmatrix} X \\ x \end{pmatrix} & \text{if } z \le x, \\ \begin{pmatrix} z \\ x \end{pmatrix} \Big/ \begin{pmatrix} X \\ x \end{pmatrix}, & \text{if } z > x, \end{cases} \tag{13}$$

where z is the number of 1s appearing in $\vec{b}$.

When the agent believes that the game is fair and treats all the numbers equally, then $z = X$ (or 0), and the measure d achieves its maximum $d_{\max}$:

$$d_{\max} = \frac{\begin{pmatrix} X \\ x \end{pmatrix}}{\begin{pmatrix} X \\ x \end{pmatrix}} = 1. \tag{14}$$

On the other hand, if the agent has exactly x numbers in his mind, then the game for him is completely deterministic, and d gets to its minimum $d_{\min}$:

$$d_{\max} = \frac{1}{\begin{pmatrix} X \\ x \end{pmatrix}} \approx 0. \tag{15}$$

Thus, simply by watching how close d is to 1 or 0, one can have an idea of how far from or close to a fair-game believer the agent is. A time series display of the metric will shed light on how well the behavior of conscious selection is developed.

Fig. 6 displays the evolution of the metric $\vec{d}$ at a highly aggregated level. What is shown on the x-axis is time. An observation is taken for every 20 periods. For each sampling period, we pool together the $\vec{d}$ of all 5000 agents over 25 runs under all the tax rates, so each d shown here is the average of $5000 \times 25 \times 10$ individuals' d. The time series plot of d basically shows a monotone increasing behavior that characterizes the gradual convergence to the belief in a fair

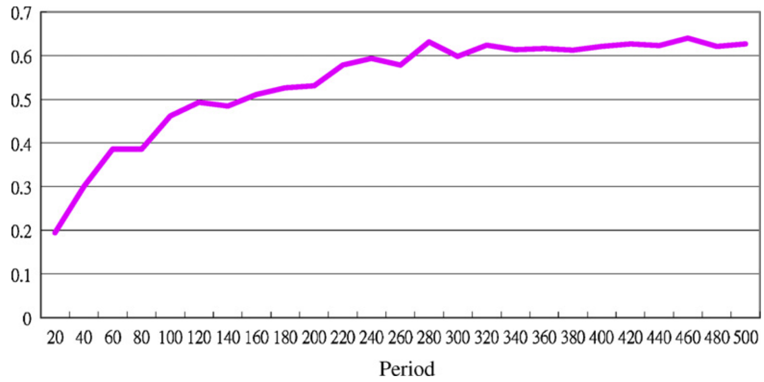


Fig. 6. The measure of the belief in a fair game.

game, which can be related to some empirical findings.[19] However, it does not converge enough to 1. Instead, it seems to settle around the level of 0.6, which is approximately equivalent to a z of 14. Therefore, a degree of conscious-selection behavior is weakly observed.

6. Discussion: What does the GA learning mean?

Given the simulation results displayed above, it is high time to pose a very fundamental question: *What does the GA learning mean?* This is a generic question shared by all kinds of agent-based simulations using the GA. To answer this question, we first have to notice that a possible optimal solution for all our agents in the lottery market is to take the *zero function* when the jackpot size is not high enough, for example,

$$\mu^* = \rho^*(J) = 0, \tag{16}$$

if J is not large enough. The solution is best in the sense that it maximizes the *risk-neutral expected utility* as specified in Eqs. (5)–(8). The second thing to notice is that the fundamental work GA did in a social learning framework is simply to *propagate* those well-performed strategies based on the fitness function supplied by the user. If the fitness function is in line with the utility function, then it is natural to ask whether the agents eventually find the optimal solution (16). In terms of the discretized version of ρ (i.e. the participation vector $\vec{a}$), the optimal solution is

$$\vec{a}^* = (a_1^*, a_2^*, a_3^*, a_4^*) = (0, 0, 0, 0). \tag{17}$$

To distinguish this type of agent from other types, we shall call agents with solution (17) the *standard neo-classical agents*.[20] Our first question is then to ask whether the solution (17) was propagated well enough to the entire market.

It is useful to look at the percentage of agents whose participation is in line with (16). If we let N_t^* be the number of simple neo-classical agents in the market, then the statistic $N_t^*/5000$ measures the density of neo-classical agents in the market. Since we have 25 simulations for each τ, each of which lasts for 500 issues, what is drawn in Fig. 7 is the box-and-whisker plot of $f_{500}^* = N_{500}^*/5000$. As before, the dots of the medians are connected in a line.

Fig. 7 basically indicates the difficulty associated with propagating behavior (17). The percentage f_{500}^* is almost down to nil for most simulations when τ is less than 40 percent. While a further increase in τ does facilitate the propagation of the survival of the neo-classical agents, their influence is still confined to a rather limited extent. When the take-out gets to its maximum ($\tau = 80$ and 90 percent), they start to become a large group (one fifth to one third) among the surviving agents. This result drives us to inquire what limits the survivability and propagation of the neo-

[19] For example, by estimating a generalized rollover probability function, Wang and Lin indicate that the lotto players initially pick numbers by way of conscious selection and later change their behavior to random selection.

[20] They are called “simple” neo-classical agents because their utility function is simply linear, and under the framework of expected-utility maximization, these agents will not gamble. Having said that, we notice that there is a long history in the literature trying to make expected utility theory able to accommodate gambling behavior (Hartley and Farrell, 2002). These variations, which will lead to a more sophisticated design of the utility function, are not considered in this paper.

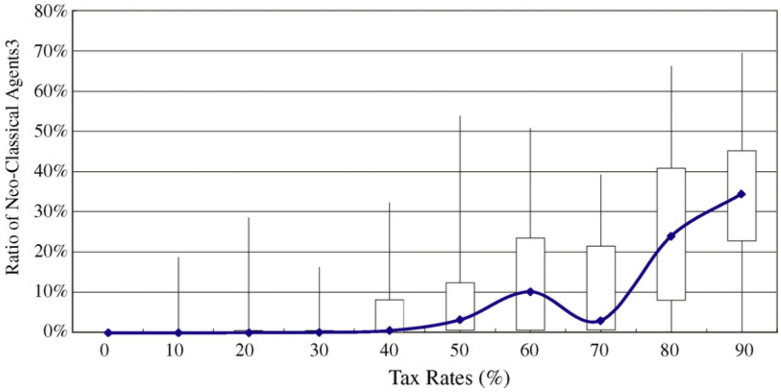


Fig. 7. The survival rate of the neo-classical agents.

classical agents, or to put the question in the context of GA, why did the *presumably* well-performing strategy (17) fail to dominate?

The property that prevents the behavior of the bounded-rational agents from converging to that of rational agents has been demonstrated in many ACE studies using GA. One key contributing factor to the divergence comes exactly from a life of bounded-rational agents (i.e. the *time-horizon* upon which agents react). In GA, this time-horizon is put into effect through the evaluation cycle. If the cycle is short, then the respective time-horizon is also short. The shorter the time-horizon, the more myopic the agents tend to be. Short time-horizons cause a problem well noticed by Lettau (1997), which is to be restated as follows. Agents in a setting in which the evaluation time-horizon is only *one period* are searching for

$$\vec{a}^{\dagger} = \arg\{\max_{a}\ U\}, \tag{18}$$

and Lettau has shown the non-equivalence between

$$E(\vec{a}^{\dagger}) = E(\arg\{\max_{a} U\}) \tag{19}$$

and

$$\vec{a}^{*} = \arg\{\max_{a}\ E(U)\}. \tag{20}$$

Lettau discussed the non-equivalence between the two. That discussion basically applies to this paper.

For convenience, agents in the society can be decomposed into two groups: *gamblers* (performing agents) and *non-gamblers* (non-performing agents). Given the design of the lottery game, most gamblers will fail with a lower utility as opposed to the non-gamblers. Since the fundamental work that the GA does is *propagate* those well-performing strategies, the strategies used by these failing non-gamblers have no influence in shaping the forthcoming behavior. Nonetheless, a minority of gamblers, in particular those gamblers with aggressive participation who are lucky enough to become the winners, gain utility that is significantly higher than that of non-gamblers. These gamblers alone are persuasive enough to invite many followers to increase their participation. This explains why *gamblers are good at propagating, even though most of them will fail*.

Non-gamblers can still exert some degree of influence on those losers, but their effect will be limited by the influence of the gigantic winners. However, if the gigantic winners do not show up (rollovers), neo-classical agents will then have a better chance to fight back. Therefore, *the frequency of rollovers matters*. The more frequently the games roll over, the more likely it is that the neoclassical agents will survive and propagate. To see this relationship, Fig. 8 depicts the percentage of rollover draws, also called the *rollover ratio*. Here, we see that the rollover ratio roughly increases with the take-out rate. When τ comes near to the maximum, it reaches about 60–70 percent, which means that most of the time the gigantic winners do not exist. As a result, neo-classical agents face much weaker survival pressure and can better propagate to a large proportion of market participants.

The explanation above also indicates the existence of an *asymmetric* account between the minority of the winning gamblers and the majority of the losing gamblers. The learning mechanism driven by the standard GA makes agents

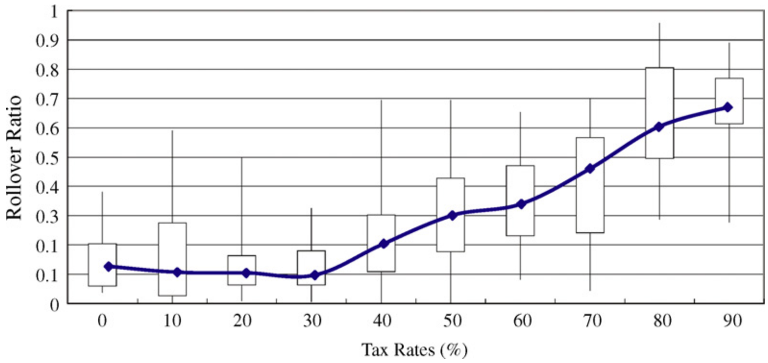


Fig. 8. The rollover ratio.

care very little about the losers, regardless of the strategies they used, even though they may have used the same aggressive strategies as the winners. As a result, by not taking into account the losers' strategies, agents tend to pass a *biased* or *over-optimistic* judgment on those aggressive gambling strategies.

Lettau noticed the influence of the stochastic fitness and showed that if the evaluation time-horizon can be enlarged, then the behavior biases will vanish.[21] However, the case of the lottery presents an extreme situation: the reward for gamblers ranges from zero to a gigantically high amount. Hence, a few more iterations will not help to average out that windfall reward. As a result, *an essentially "hocus-pocus strategy"* (*i.e. a strategy that works purely by luck*) *can still survive*.

The analysis above helps us to see how GA is related to bounded rationality and why behavior biases can sustain under the GA mechanism. First, humans are *bounded rational* in the sense that they learn in a *biased* way by only drawing attention to the winners. Second, there is a *small probability* of having an *extremely high reward* from following that "hocus-pocus strategy." Notice that the probability can be extremely small. What matters here is that based on *the law of large numbers*, it will almost surely happen, at least for a few people, if there are enough followers. By these two conditions, the biases behavior will not die away and may self-form into a wave of propagation. The case of the lottery fits these two conditions nicely.

Now, we come to the puzzle: Why do our simulated data rollovers affect sales *negatively*, which contradicts the most noticeable stylized fact of the lottery market? Again, this can be accounted for by the way GA operates in this paper. Remember that our evaluation time-horizon is shortened to a single-period draw. Given the circumstances, this is what will happen. Suppose there was one and only one jackpot winner in the last issue (hence a regular draw for this issue). Let us trace his possible influential power in a framework of *tournament selection*. Since on each single draw we have a chance of 1/5000 to pick this jackpot winner, the chance of including at least one copy of the jackpot winner in a tournament is approximately 0.04 ($g^{200\times(1/5000)}$) if the tournament size is 200. There are 5000 tournaments (1 for each individual), so on average 200 individuals have the jackpot winner in their tournament and hence are under his influence. Since the jackpot winner tends to have a more aggressive participation $\vec{a}$, the aggressive strategy is, therefore, propagated to a large group of gamblers. This causes a rise of sales during the regular draw.

On the other hand, the absence of the jackpot winner in the previous issue (the rollover draw for the current issue) hampers the propagation of aggressive strategies. Instead, the conservative strategies that lead to a low participation level dominate. Sales, therefore, fall in the rollover draw. Furthermore, when the rollover draw extends, the jackpot prize accumulates. Therefore, a large jackpot prize, as a result of non-interrupting rollovers, also has an adverse effect on sales. This explains the significantly negative t statistics and low $\bar{R}^2s$ in Table 4.

[21] Lettau showed that

$$\lim E(\vec{a}^{\dagger}) = \vec{a}^{*}, \tag{21}$$

where the limit is taken over the number of periods for per evaluation.

7. Conclusions

7.1. Remarks on the findings

This paper introduces an agent-based computational model of lottery markets. In this model, the agents' decisions on lottery participation are not based on sophisticated calculations of the winning odds but simply on heuristics. The heuristics considered in this paper capture the two empirical phenomena known as the halo effect and the conscious selection of numbers. In addition, the empirical observation referred to as aversion to regret motivates an interdependent utility function of agents. The Sugeno style fuzzy if-then rules are used to formalize agents' heuristics. Both the heuristics and preferences are evolving over time via the canonical genetic algorithm.

Coming out of this simple agent-based lottery market are the three major findings, namely, a Laffer curve with a flat top, the conscious selection behavior, and the anti-halo effect. Each of these findings has its empirical counterpart. First, while the appearance of Laffer curve is totally expected, the flat top is not well anticipated. This possibility is also neglected in the empirical literature, even though a unitary demand elasticity of lottery tickets was found in Farrell et al. (2000). The Laffer curve with a flat top provides a possible explanation for the wide difference of the lottery tax rates observed in different markets (Fig. 1) since it indicates the possibility that the optimum lottery tax rate may not be unique.

Second, our agent-based model also supports the conscious selection behavior, which is consistent with the empirical finding of Farrell et al. (2000). What the ACE model may tell us more about is the dynamics of this conscious selection behavior. The learning process driven by the GA does not eliminate this behavior bias; nonetheless, the degree of conscious selection, as we can easily measure in the ACE model, becomes weaker, which was also found true in some empirical studies (Wang and Lin, 2006).

Third is the anti-halo effect. This is the only finding that does not go well with the empirical fact. As discussed lengthily in Section 6, social learning driven by the GA works against the halo effect. Therefore, it seems that individual learning model, such as reinforcement learning or belief learning, or a hybrid learning model needs to be incorporated to remedy this problem.

7.2. Directions for further study

This specific agent-based lottery market exemplifies what agent-based models can do and may do. One specific feature is that it provides a great flexibility to generate many aspects of the lottery market simultaneously. As a first step, this paper certainly does not exhaust all possible features. Variations or extensions of this model can be developed depending on the questions that we are pursuing. Some issues left for further exploration are listed below.

In addition to the lottery tax rate, other important issues include the distribution of the prizes, which could be more intriguing if we extend the current model by varying the risk attitude or utility function, potentially making agents sensitive not only to the mean of the prize distribution, but also to the change in high-order moments such as the variance, skewness, and so on.[22]

Having said that, we may integrate this study with more elements from behavioral economics, such as prospect theory, in a further study.

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[22] Walker and Young is probably the only paper that studies the effect of the distribution of the prize on lottery sales.

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