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## **Abstract**

Because liability insurance involves the insured (the injurer), the victim, the insurer and the legal system, the moral hazard in liability insurance is much more complicated than other types of insurance. The soaring claim costs of liability insurance since 1980's have reduced the incentive of the insurer to supply liability insurance. Risk managers have recently tried to develop the alternative risk-transfer techniques for liability risks. However, this paper considers that liability insurance remains the best choice for financing legal risk once the moral hazard problem can be mitigated. To mitigate the moral hazard and supply crisis of liability insurance, this paper incorporates the concept of decoupling liability and proposes a remedy through a mixed system of private insurance and public fund.

Under this system, the expected total cost for the firm to engage in risky activity must be greater than expected loss payment of an accident so that the incentive of safety care can be maintained. The insurance premium is lowered so

#### 國立政治大學學報第八十二期

to be affordable by the insured. The monetary compensation paid by the insurer to the victim is reduced which mitigates social inflation of liability damages award. The public fund is collected from the penalty of firms that cause the accidents. Part of the fund is applied to risk management research so that the overall safety level of the society can be improved, which is non-monetary compensation to the consumers (victims). Part of the fund serves as a subsidy for insurer's solvency. The insurer's incentive to supply liability insurance is raised through a subsidy of buffer fund.

**Keywords:** liability insurance crisis, decoupling liability, mixed system, incentive of safety care, liability damages award.

## 責任保險危機之補救措施

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## 中文摘要

責任保險由於牽涉被保人(肇事人)、受害人與保險人三方當事人,以及社會之侵權法律制度,因此本身具有相當複雜之道德危險問題。自 1980 年代以來,保險人即使年年提高保費,仍然無法支應飛脹的理賠支出,使得保險人供給意願大為降低,此種情況在商業責任保險尤其明顯,常稱之為「責任保險危機」。近年來風險管理人員紛紛提出替代責任保險之方案,以解決侵權賠償之難題,但本文認為保險之風險理財功能,是其他方法所難以匹敵的,只要能夠降低其道德危險問題,保險仍是最佳之侵權責任賠償工具。因此本文納入「非對稱責任」(decoupling liability)之理念,經由模型分析而提出一項建議,採用民營責任保險與公共基金之合併制度,來減緩此種道德危險與保險供給困境。

在此制度下,廠商從事有風險之事業時,其預期總成本將高於意 外事故之預期損失賠償金額,使得廠商應有之安全謹慎動機得以維 持,不至於因責任保險之存在,而疏忽安全防護工作。另一方面,由 於受害人可直接獲自保險人之損失賠償金減少,因此保費可以降低, 使被保人(廠商)有能力購買保險,並且由於保險賠償金減少,可減輕

#### 國立政治大學學報第八十二期

受害人利用保險而哄抬索賠的現象。引起事故之廠商必須繳納罰款以成立公共基金,公共基金部分用以從事風險管理之研究,以提昇社會整體之生活安全水準,對於消費大眾(潛在受害人)是一種非貨幣性補償;部分基金則用以作為責任保險之安全準備金,減少保險人無力清償的機率,因此可提高保險人供給責任保險之誘因。

關鍵字:責任保險危機、非對稱責任、合併制度、安全防護誘因、責任賠償金。

#### 1. Introduction

Private liability insurance has been extensively applied to compensate the victim in an accident during the past decades. The advantage of using liability insurance to pay the damages is that it can reduce the judgement-proof problem (Shavell, 1986) so that the victim will not be left with nothing due to the bankruptcy of the injurer. As the public policy intends to provide more protection to the victims, liability insurance has even become compulsory in some fields, e.g., automobile liability and worker's compensation. Even if it is not compulsory in other areas such as commercial general liability, most of the firms incline to purchase insurance for financing the unexpected payments of injury compensation.

Although liability insurance is a convenient way to provide compensation to the injured victim, it also induces huge social cost due to moral hazard of insurance. The moral hazard in liability insurance is more complicated than that in life or property insurance because it may involve not only the insured (i.e., the potential injurer) but also the victim and her legal representatives (i.e., the third-party). Whether liability insurance will reduce the incentive of risk control by the insured is theoretically controversial as discussed in the studies by Shavell (1982) and Sarah (1991). On the other hand, the effect of legal intervention on loss settlements of liability insurance from the injured third-party side is empirically evident (see ISO survey, 1996). The damages of pain and suffering are difficult to measure objectively because they depend on the condition of the victim, the intervention of her attorney, and the judgement of court. The lengthy delays in the litigation process also contribute to the increase of claim costs for liability insurance. All these factors which are so-called social inflation (Long,1981) have dramatically increased the total payments of liability damages and discouraged the supply of liability insurance since 1980's.

The affordability and availability problem is most serious in commercial liability insurance as tort law moves to provide more protection to the consumers. For example,

product liability has changed from negligence rule to strict liability and the polluters of environment may be held jointly and severally liable (Abraham, 1997). The soaring costs of claim settlements in commercial liability insurance have driven the premiums too high to pay, or even driven the insurers to withdraw from the market which is so-called insurance crisis (Cummins and Danzon, 1997). Therefore liability insurance is not an easy way to transfer risk for the business firms any more.

The enormous cost of liability risk without insurance protection may depress the firms to engage in business or product innovation as indicated by Viscusi and Moore (1993). The public policy maker has been forced to develop other alternatives for the risk management of liability loss exposures since 1980, such as Product Liability Risk Retention Act and Superfund Act for environmental pollution liability. However, risk retention group may be inefficient due to the shortage of professional knowledge for handling loss settlements (Head, 1996), and more than one-third of total costs of Superfund are legal expenses spent for settling the disputes (Action and Dixson, 1992).

The purpose of this paper is to propose a remedy for commercial liability insurance so that the supply of insurance can be maintained with affordable premiums and reasonable claim costs. To achieve this objective, the problem of moral hazard in liability insurance must be mitigated. Polinsky and Che (1991) show that decoupling liability may be desirable because it maintains the level of safety care but reduces the incentive of litigation. Decoupling liability requests the defendant to pay the damages as high as possible so that the safety care can be maintained, but it allows the award to the plaintiff to be lowered so as to reduce the plaintiff's incentive to sue and save the litigation cost. In practice most of the liability costs for the firm have been pass through insurance. If the spirit of decoupling liability can be maintained in insurance system, together with its efficiency of risk-sharing function, then the problems of commercial liability insurance crisis may be mitigated.

The added value of this paper is that it incorporates the concept of decoupling liability

to develop a model for optimal liability insurance system, which has not been studied in the previous literature yet. With consideration of uncertainty in determination of liability, the model applies the mathematical approach of Polinsky and Shavell (1984) to discuss the tort liability of injury. Different from Polinsky and Shavell's paper which focuses on the punishment of the injurer, this paper emphasizes on the compensation for the victim. A mixed system of private insurance and a public fund is then proposed for the compensation of tort liability.

The mixed system is consistent with the concept of decoupling liability. It suggests that the expected total cost for the firm to engage in risky activity must be greater than the expected loss payments of an accident so that the incentive of care can be maintained. The insurance premium is lowered after adopting the mixed system so to be affordable by the insured. The monetary compensation paid by the insurer to the victim is reduced to mitigate social inflation problem, while the existence of public fund can contribute non-monetary welfare (i.e., security) to the victim. Finally a subsidy of buffer fund maintains the insurer's incentive to supply liability insurance.

This paper is organized as follows. Section 2 describes the basic model. Section 3 derives the optimal insurance coverage for the case without decoupling liability. It serves as a benchmark to discuss the social inflation problem. Section 4 proposes a mixed system with decoupling liability. Section 5 extends the model to include heterogeneous insured. Finally, the concluding remarks are provided in section 6.

### 2. The Model

Based on the approaches of Polinsky and Shavell (1984) and Shavell (1985) to study the punishment of injury and the uncertainty in liability determination, this paper develops a

#### 國立政治大學學報第八十二期

model to discuss the roles of all three involved parties (the insured injurer, the victim, and the insurer) in the tort compensation. In the previous literature about tort liability, either the insurer (e.g., Png (1987)) or the victim (e.g., Sarath (1991)) has been neglected because most papers focus moral hazard problem only on the injurer (the insured). This paper considers the utilities of the three parties since in practice they are all important in the tort liability.

It is assumed that the firm is risk neutral and its purpose of engaging in business activity (e.g., manufacturing products or practicing medical operation) is to earn profits. Let

 $\pi$  = potential profits for the firm from the business,  $\pi \in [0,\Pi]$ ,

 $f(\pi)$  = probability density of profit, f > 0,

 $F(\pi)$  = cumulative distribution of profit.

The business activity has a potential to cause an accident to the consumer. In order to avoid the judgement-proof problem, the firm is required to purchases insurance to cover potential liability of the accident. The insurer will compensate the harm done by the insured (i.e., the firm) when the consumer is hurt in an accident. However, there is uncertainty in loss payment because natural factor or other parties may also contribute to the harm (Shavell, 1987; Che and Earnhart, 1997). The amount of recovery for the consumer may also be affected by litigation process (Sarath, 1991) because it is related to social inflation factor as well as the harm itself. Let

L = loss (harm) of an accident to the consumer and assumed to be a constant,

p = probability of an accident due to the risky business activity,

 $\overline{L} = pL =$ expected loss of an accident due to the risky business activity,

 $\alpha$  = probability (i.e. proportion) of loss contributed by factors other than the firm,

<sup>&</sup>lt;sup>1</sup> According to the practice in insurance market, liability insurance contract covers only the damages caused by the insured and excludes coverage for damages caused by other parties (Malecki and Flitner, 1998).

$$0 \leq \alpha < 1$$

I = liability insurance coverage purchased by the firm,<sup>2</sup>

Q = claim settlement paid by the insurer.

The expected profit  $E(\pi)$  from the business activity is assumed greater than the expected loss  $\overline{L}$ , otherwise it is not socially beneficial for the firm to enter the business. The probability of a loss really results from the firm is  $p(1-\alpha)$ . The insurance premium for the firm will be  $p(1-\alpha)I$  in case it intends to take the risky business activity and purchase insurance. It is assumed that the firm is rational and will engage in the business only if the profit is greater than the premium cost. Therefore the expected utility for the firms to enter the business is

$$U = \int_{\pi_0}^{\Pi} \pi f(\pi) d\pi - p(1-\alpha) I \{ 1 - F[p(1-\alpha)I] \}$$
 (1)<sup>5</sup>

 $\pi_0$  in equation (1) is the minimum profit for the firm to engage in the business which is equal to  $p(1-\alpha)I$  in this basic model. Since the probability for the firm to engage in the

$${}^{5}U = \int_{\pi_{0}}^{\Pi} \{\pi - p(1-\alpha)I\} f(\pi) d\pi = \int_{\pi_{0}}^{\Pi} f(\pi) d\pi - p(1-\alpha)I \{1 - F[p(1-\alpha)I]\}$$

<sup>&</sup>lt;sup>2</sup> For liability insurance, the loss payment is unknown ex ante and the insured only chooses coverage limit at the beginning of contract.

<sup>&</sup>lt;sup>3</sup> Shavell (1985) and Che and Earnhart (1997) assume the two events of harm caused by the defendant (the firm) and by the nature (other factor) are mutually exclusive. This assumption is not required here because this paper integrates uncertainty of liability on the amount of loss payment. The proportions of damages involved natural factor or social inflation are all included in the parameter  $\alpha$ . For example, a defective stove made by the firm causes a fire, the windy weather (natural factor) accelerates the spread of fire, and the court awards generous damages (social inflation).

<sup>&</sup>lt;sup>4</sup> Since this paper is to discuss the problem of liability insurance crisis, the firms running business without insurance are not included.

business activity is  $\{1 - F[p(1-\alpha)I]\}^6$ , the insurer will receive the total premiums  $\{1 - F[p(1-\alpha)I]\}p(1-\alpha)I$  from those firms that take the risky business activity and purchase insurance coverage for the potential liability risk. The insurer expects to incur claim cost Q. The expected utility for the insurer to supply the liability insurance is<sup>7</sup>

$$R = \{1 - F[p(1 - \alpha)I]\} p(1 - \alpha)I - \{1 - F[p(1 - \alpha)I]\} p(1 - \alpha)Q$$
(2)

In case the firm takes the risky activity, the consumer may suffer from the harm with probability  $\{1 - F[p(1-\alpha)I]\}p$ . The expected loss to be encountered by the consumer is equal to  $\{1 - F[p(1-\alpha)I]\}p$  when the firm takes the risky activity. However, she can get compensation from the insurer for the injury done by the insured firm. Thus expected utility of the consumer from this business activity is<sup>8</sup>

$$V = -\overline{L}\{1 - F[p(1 - \alpha)I]\} + \{1 - F[p(1 - \alpha)I]\}p(1 - \alpha)Q$$
(3)

The social planner must consider the total welfare of those three parties when setting up a risk management program for liability risk. Let the total welfare W = U+R+V. The optimal insurance coverage to maximize the social welfare is solved from the following equation.

$$\max_{I} W = \int_{\pi_0}^{\Pi} n f(\pi) d\pi - \overline{L} \{ 1 - F[p(1 - \alpha)I] \}$$
(4)

<sup>&</sup>lt;sup>6</sup> Probability =  $\int_{\pi_0}^{\Pi} f(\pi) d\pi = 1 - F[p(1-\alpha)I]$ . It can also be interpreted as the number of firms to engage in the

business when the population size of firms is normalized to one (Polinsky and Shavell, 1984).

<sup>&</sup>lt;sup>7</sup> For liability insurance, the insured chooses the coverage limit I and the insurer pays the claim up to maximum limit

<sup>&</sup>lt;sup>8</sup> The consumer's utility for enjoyment of business activity (e.g., new product) is offset by the price she paid, so only potential harm appears in the utility function.

## 3. Optimal Insurance Coverage Without Decoupling Liability

If liability is not decoupled as the traditional case, the optimal insurance coverage I\* can be solved as equation (5) (see appendix 1 for proof).

$$\underline{I}^* = \overline{L}/p(1-\alpha) \tag{5}$$

Since the expected loss  $\overline{L}=pL$ , the optimal private insurance coverage  $I^*$  can be revised as

$$J^* = L/(1-\alpha) \tag{6}$$

If there is no social inflation or ambiguity in causation and the firm is only source for the accident and harm (i.e.,  $\alpha = 0$ ), the firm should buy insurance equivalent to the potential harm, that is,  $I^* = L$ . The insurance premium cost for the firm is equal to expected loss  $\overline{L} = (pL)$ . The insurance premium incomes received by the insurer just balanced off by the loss settlements since all the harm is attributed to the insured. The consumer is compensated exactly for the harm done by the firm. That is, Q = I = L.

When natural factor or social inflation also contributes to the loss in addition to the firm, there is uncertainty in liability and the parameter  $\alpha$  is positive but less than one. In such case the optimal insurance coverage  $I^* = L/(1-\alpha) > L$ . The insurance coverage purchased by the firm is greater than the loss L. However, the premium cost for the firm remains the same as before, i.e.,  $\overline{L}$ . This result implies no incentive for the firm to conduct risk management. Since the proportion of loss caused by other factors is not controllable by the firm, these

<sup>&</sup>lt;sup>9</sup> If  $\alpha = 1$ , the loss is caused completely by other factors than the firm. Since insurance is meaningless in such case,  $\alpha$  is assumed less than one in this paper.

factors will not induce the firm to be more careful.

The compensation paid to the consumer will depend on legal system and insurance market. When strict liability is applied or the damage awarded by the court is inflated by social factors, the consumer usually can receive more compensation. In addition, the market competition in insurance industry also has an impact on the indemnity since most of claims are settled by the insurer. When the market is very competitive such that the expected profit is zero, the insurer spend all the premium income on loss settlement. Therefore the compensation Q in equations (2) and (3) is equal to I. The consumer receives full recovery, L, from the insurer even if the proportion of loss attributed to the insured is only  $(1-\alpha)L$ .

This result implies the consumer "earns" a payments  $\alpha$  L which attributes to social inflation. Without insurance market, the consumer can only request  $(1-\alpha)L$  from the firm through litigation. The existence of insurance will encourage the consumer to pursue more loss payment. Based on the survey of ISO (1996), more than 90% of commercial general liability insurance claimants file suit. This unanticipated claim cost,  $\alpha$  L, reduces the insurer's profit for current year and in turn will raise premium rates for the following year. Thus the incentives for the insurer to supply insurance and the firm to engage in business are depressed.

<sup>&</sup>lt;sup>10</sup> For example, average loss payment of claims for product liability insurance (\$324,000) is much higher than that of other liability insurance (\$217,000) based on the survey of ISO (1996). Product liability usually applies strict liability instead of negligence rule, and it is competitive because it is one of the major businesses for the insurer.

According to the survey of ISO(1996), more than 90% of commercial general liability insurance claims are settled by the insurer before conclusion of court verdict.

Usually the insurer is assumed to have deeper pocket than the firm. The consumer may get nothing when she sues the firm, so it is less possible for her to sue the firm for  $\alpha L$  when the firm is not insured. Although theoretically the insurer may defend the payment  $\alpha L$  through litigation, the litigation cost may be higher than the loss payment. Therefore, the insurer usually does not prefer to defense. In practice, about 90% of claims are settled before trial (ISO, 1996).

## 4. Optimal Insurance Coverage With Decoupling Liability

In order to mitigate the problem of social inflation in liability insurance, this paper proposes that the social planner charges a penalty ex post to the firm for those portions of ambiguous harm in case of an accident. The penalty is collected into a public fund and is not covered by insurance. Part of the fund is applied to support risk management research, and the remainder serves as a buffer fund for the insurer to set up the contingency reserve. This will provide incentive for the insurer to supply liability insurance since the buffer fund reduces risk of insolvency.<sup>13</sup> Therefore, the public fund can provide a non-monetary utility (i.e., security) to the consumer even though the fund is not directly distributed to the consumer.<sup>14</sup> Let

- z = target amount of the public fund,
- $\kappa$  = parameter of penalty % charged to the firm for the ambiguous harm,  $0 < \kappa \le 1$ ,
- $\theta$  = parameter of buffer fund % provided to the insurer,  $0 < \theta < \kappa$ ,
- Y(z) = non-monetary benefit function of the consumer due to the existence of the fund,  $Y' \ge 0$ .

The utility functions for the firm, the insurer, and the consumer are revised as

$$U = \int_{\pi_0}^{\Pi} f(\pi) d\pi - p(1-\alpha) I \{1 - F[p(1-\alpha)I + p\alpha\kappa z]\}$$

$$- p\alpha\kappa z \{1 - F[p(1-\alpha)I + p\alpha\kappa z]\},$$
(7)

<sup>&</sup>lt;sup>13</sup> According to statutory accounting principles of insurance, contingency reserve is treated as an item of equity to prevent unanticipated catastrophic loss. It is not an amount directly used for paying claim cost of the insurer.

<sup>14</sup> For example, the consumer can avoid judgement-proof problem with liability insurance and can enjoy better quality of life through risk management research.

$$R = \{1 - F[p(1-\alpha)I + p\alpha\kappa z]\}p(1-\alpha)(I-Q) + p\alpha\theta z\{1 - F[p(1-\alpha)I + p\alpha\kappa z]\}, \tag{8}$$

$$V = -\overline{L}\{1 - F[p(1-\alpha)I + p\alpha\kappa z]\} + \{1 - F[p(1-\alpha)I + p\alpha\kappa z]\}p(1-\alpha)Q + Y(z). \tag{9}$$

The probability for the firm to engage in the business activity is affected by the penalty charge. The minimum profit  $\pi_0$  in equation (7) is also changed to  $\{p(1-\alpha)I + p\alpha\kappa z\}$ . The insurer receives the premiums from the firm and expects to incur claim cost Q. In addition, the insurer can obtain a buffer fund subsidy from the public fund for supplying liability insurance. The consumer's expected utility is modified with the revised probability and non-monetary benefit of public fund. Then the total welfare becomes

$$W = \int_{\pi_0}^{\Pi} f(\pi) d\pi + Y(z) - \overline{L} \{1 - F[p(1 - \alpha)I + p\alpha kz]\}$$

$$- p\alpha z(k - \theta) \{1 - F[p(1 - \alpha)I + p\alpha kz]\}.$$
(10)

The maximization of social welfare is through the choice of insurance coverage, I, and target amount of public fund, z. Thus, the optimal insurance coverage to maximize the total welfare is (see appendix 2 for proof)

$$I^{**} = (L - \alpha \theta z)/(1 - \alpha)$$
(11)

Compared with equation (6), now the optimal insurance coverage purchased by the firm is reduced to reflect the subsidy to the insurer. The insurance premium paid by the firm is equal to  $p(L-\alpha\theta z)$  which is affected by the proportion of ambiguous loss  $\alpha$ . If  $\alpha$  is small which implies most of the harm is caused by the firm, the premium cost will be higher, and

vice versa. Therefore, the premium cost is more responsive to causality of the loss. The expected total cost for the firm to engage in the business activity is increased by the amount  $p\alpha z(\kappa - \theta)^{15}$  though the insurance premium cost is reduced. The penalty charge has an impact on the incentive of care level because it is related to the probability of an accident.<sup>16</sup>

Suppose the insurance market is competitive, the insurer spends out all the premium incomes on claim cost. Then Q is equal to  $I^{**}$ . The expected compensation for the consumer is  $p(L-\alpha\theta z)$  which is less than the expected loss pL. Since the consumer cannot expect to be fully compensated from the insurer, the problem of social inflation is mitigated. The incentive for the insurer to supply liability insurance is raised because expected utility is increased by the subsidy of buffer fund. 18

The optimal target amount of public fund z\* is (see appendix 2 for proof)

$$z' = \left\{ F^{-1}(x) - \overline{L} \right\} / p\alpha(\kappa - \theta), \tag{12}$$

where  $x = 1 - Y'/p\alpha(\kappa - \theta)$ , and  $Z' < \{\Pi - \overline{L}\}/p\alpha(\kappa - \theta)$ . Equation (12) indicates that the optimal target amount of public fund must be related to marginal benefit of public fund for the consumer as well as the expected loss and the net penalty percentage  $(\kappa - \theta)$ . The target amount can be reduced if marginal non-monetary benefit of public fund to the consumer is larger. When the expected loss or the net penalty percentage increases, the

<sup>&</sup>lt;sup>15</sup> In the previous section, the expected cost is  $p(1-\alpha)I^* = pL$  when the firm engages in the business. Here the total expected cost includes insurance premium and penalty charge and is equal to

 $p(1-\alpha)I^{**} + p\alpha\kappa z = pL + p\alpha(\kappa - \theta)z.$ 

<sup>&</sup>lt;sup>16</sup> Insurance premium is a sunk cost paid ex ante by the firm, so it affects the incentive of engaging in business. The penalty is charged ex post only if the accident occurs. It has an impact on the safety care level of the firm.

<sup>17</sup> The subsidy of buffer fund is an earmarked item for insurer's equity and cannot be applied to pay claim cost directly.

The insurer's expected utility is zero in the previous section, but here it is equal to  $p\alpha\theta z \left\{1 - F\left[p(1-\alpha)I + p\alpha\kappa z\right]\right\}$  which is positive.

<sup>&</sup>lt;sup>19</sup> For example, let the value of security be equivalent to 100G. It needs \$100 of fund if each dollar of fund induces 1G of security, and it only needs \$50 of fund if each dollar of fund can contribute 2G of security. In addition, the

target amount must be reduced so that the incentive for the firm to engage in the business will not be depressed by a huge total cost of premiums and penalty charge. Finally, the target amount can be raised as the maximum profit of business activity is increased.

## 5. Optimal Insurance Coverage for the Heterogeneous Insured

In this section the model is extended to consider heterogeneous risk among the insured. Suppose the firms spend different safety care levels when they engage in the business activity. Let the probability of an accident from low-risk and high-risk firms be  $p_1$  and  $p_2$  respectively, and the corresponding loss be  $L_1$  and  $L_2$ . If the industrial society consists of  $\lambda$  percent low-risk firms and  $(1-\lambda)$  percent high-risk firm, then the model is revised as follows.

$$\max_{\text{II,I2,z}} W =$$

$$\lambda \left\{ \int_{\pi_{1}}^{\Pi} f(\pi) d\pi - \overline{L}_{1} \left\{ 1 - F[p_{1}(1 - \alpha)I_{1} + p_{1}\alpha\kappa z] \right\} - (\kappa - \theta) p_{1}\alpha z \left\{ 1 - F[p_{1}(1 - \alpha)I_{1} + p_{1}\alpha\kappa z] \right\} \right\}$$

$$+ (1 - \lambda) \left\{ \int_{\pi_{2}}^{\Pi} \pi f(\pi) d\pi - \overline{L}_{2} \left\{ 1 - F[p_{2}(1 - \alpha)I_{2} + p_{2}\alpha\kappa z] \right\} \right\}$$

$$- (\kappa - \theta) p_{2} \alpha z \left\{ 1 - F[p_{2}(1 - \alpha)I_{2} + p_{2}\alpha\kappa z] \right\} + Y(z)$$

$$(13)$$

In equation (13),  $\pi_1 = p_1(1-\alpha)I_1 + p_1\alpha\kappa z$  and  $\pi_2 = p_2(1-\alpha)I_2 + p_2\alpha\kappa z$ . If liability is

consumer will receive less monetary compensation from the insurer when z is larger. The consumer must face a trade-off between non-monetary benefit and monetary compensation. Thus, this result is not inconsistent with the assumption  $Y'(z) \ge 0$ .

not decoupled and insurance market is competitive, the relationship of optimal insurance coverage and the risk levels of the firms are shown as the following equations (see appendix 3 for proof).

$$I_1^* = \overline{L}_1/p_1(1-\alpha) = L_1/(1-\alpha) \tag{14}$$

$$I_2^* = \overline{L}_2 / p_2 (1 - \alpha) = L_2 / (1 - \alpha) \tag{15}$$

Equations (14) and (15) show that the optimal insurance coverage is again greater than the loss if  $\alpha > 0$ . The coverage for each firm is directly related to the potential loss it causes respectively. In case the loss amounts are equal for both low-risk and high-risk firms (i.e.,  $L_1 = L_2 = L$ ), <sup>20</sup> the optimal insurance coverage will be the same. However, the premium costs are different for each firm to reflect its care level, which are  $p_1L$  for the low-risk and  $p_2L$  for the high-risk. Therefore, it is not necessary for social planner to set up higher requirement of insurance coverage for the high-risk firm unless its severity of harm is higher (i.e.,  $L_2 > L_1$ ).

When liability is decoupled and a public fund is set up, the optimal insurance coverages for the firms are (see appendix 4 for proof)

$$I_1^* = (L_1 - \alpha \theta z)/(1 - \alpha), \tag{16}$$

$$I_2^* = (L_2 - \alpha \theta z)/(1 - \alpha). \tag{17}$$

Again the insurance requirements are lower than those in the case without decoupling liability. Whether the coverage amounts are equal or not must depend on the severity of

<sup>&</sup>lt;sup>20</sup> It is not unusual to have the same loss, for example, both result in death although the probability to cause an accident differs.

harm by the respective firms as discussed in the above case. The optimal target amount z is obtained by solving the following first-order condition (see appendix 4 for proof),

$$\lambda(\kappa - \theta) p_1 \alpha \left\{ 1 - F \left[ p_1 (1 - \alpha) I_1^* + p_1 \alpha \kappa_Z^* \right] \right\}$$

$$+ (1 - \lambda)(\kappa - \theta) p_2 \alpha \left\{ 1 - F \left[ p_2 (1 - \alpha) I_2^* + p_2 \alpha \kappa_Z^* \right] \right\} = Y'.$$

$$(18)$$

The exact value of z\* cannot be solved directly from equation (18). However, it is approximately approach to

$$\overset{\vee}{\mathbf{z}} = \left\{ F^{-1}(\mathbf{x}) - \overline{L}^{@} \right\} / p^{@} \alpha \left( \kappa - \theta \right). \tag{19}$$

where 
$$x = 1 - Y'/p^m \alpha(\kappa - \theta)$$
,  $p^m = \lambda p_1 + (1 - \lambda)p_2$ ,  $a = \lambda p_1/p^m$ ,  $p^@ = a p_1 + (1 - a)p_2$  and

$$\overline{L}^{@} = a\overline{L}_1 + (1-a)\overline{L}_2$$
. Again,  $z < \{\Pi - \overline{L}^{@}\}/p^{@}\alpha(\kappa - \theta)$  because  $F^1(x)$  is less than  $\Pi$ . The

interpretation of equation (19) is similar to that of equation (12). The optimal target amount of public fund depends on marginal benefit of public fund for the consumer as well as the expected loss and the net penalty percentage  $(\kappa - \theta)$ ; however, the expected loss and accident probability are replaced with weighted average of those values of low-risk and high-risk firms.

## 6. Concluding Remarks

Liability insurance is a convenient way to provide compensation to the injured victim, but its moral hazard has discouraged the supply of liability insurance since 1980's. The huge cost of liability risk without insurance protection may depress the firms to engage in business

or product innovation. To mitigate the moral hazard and supply crisis of liability insurance, this paper develops a model with consideration of all three parties involved in the compensation for tort damages. It incorporates the concept of decoupling liability into insurance system to maintain the safety care incentive of the insured injurer and control the soaring compensation requested by the victim.

This paper proposes a remedy for liability insurance crisis through a mixed system of private insurance and public fund. Under this system, the expected total cost for the firm to engage in risky activity must be greater than expected loss payment of an accident so that the incentive of safety care can be maintained. The insurance premium is lowered so to be affordable by the insured. The monetary compensation paid by the insurer to the victim is reduced which mitigates social inflation of liability damages award. The insurer's incentive to supply liability insurance is raised through a subsidy of buffer fund.

Once the problem of moral hazard can be mitigated, insurance should remain one of the most important techniques for risk management because its efficiency in risk sharing cannot be easily outperformed by other methods. Private insurance is more effective for risk management in the situation where the injurer of an accident is easier to identify, such as product liability or malpractice liability. In other cases of long-term hazards such as environmental risk, it is difficult to identify the injurer and thus the functions of private insurance system may not work well. This paper focuses on the former which traditional commercial liability insurance targets on. The remedy for long-term hazards with non-identifiable injurer will be studied in the future research.

Additionally, a few remarks regarding the results of this paper that should be addressed are provided as follows.

## (1) Integration of incentives for several parities.

The mixed system is consistent with the concept of decoupling liability. It suggests the

#### 國立政治大學學報第八十二期

expected total cost for the firm to engage in risky activity must be greater than the expected loss payments so that the incentive of care can be maintained. However, the insurance premium is lowered so to be affordable by the insured, which implies the incentive of engaging in business will not be depressed. The monetary compensation paid by the insurer to the victim is reduced to mitigate social inflation problem, and the insurer's incentive to supply liability insurance is raised through a subsidy of buffer fund.

## (2) Public fund vs. public insurance.

Different from the studies by Besley (1989), Selden (1993), and Bolmqvist and Johansson (1997), the public fund in this paper is not a public insurance. The public fund here is not to pay indemnity to the victim directly, but provides welfare to her through supporting risk management research and reducing insolvency of insurer. With introduction of this public fund, the supply of insurance can be maintained with affordable premiums and reasonable claim costs.

## (3)Focus of moral hazard.

The concept of expected loss in this paper is consistent with insurance or actuarial literature for premium rating, which separates expected loss into frequency and severity and emphasizes on the latter. Most of economic literature about moral hazard of insurance, e.g., Shavell (1982), treats loss severity as fixed and focuses on the relationship between care level of the insured and accident frequency. Although moral hazard of the insured may be also a problem in liability insurance, the social inflation of loss settlements for the third-party is more critical to the liability insurance crisis in practice. Therefore, this paper focuses on loss severity and the third-party problem; however, it does not imply liability insurance without moral hazard from the insured.

## (4) Uncertainty in determination of liability.

When considering uncertainty in determination of liability, this paper assumes the accident is initially caused by the insured firm because insurance covers loss only if the insured is liable. Thus the injurer is identifiable in this paper, but the severity of loss payment may be exaggerated by natural factor or social inflation which involves uncertainty. The analysis in this study is more suitable for traditional liability insurance such as product liability and malpractice liability, the victim at least has target injurer to sue, for example, the manufacturer or the surgeon of medical operation, even if other (natural or social) factors may complicate the harm. For long-term hazards such as environmental risk, it is difficult to identify the injurer and private insurance probably is not a good approach to manage risk. The remedy for liability of long-term hazards with ambiguous injurer will be studied in the future research.

## Appendix 1

Proof of equation (5).

Since W = 
$$\int_{\pi_0}^{\Pi} \pi f(\pi) d\pi - \overline{L} \{1 - F[p(1 - \alpha)I]\}$$
$$= \int_{0}^{\Pi} \pi f(\pi) d\pi - \int_{0}^{\pi_0} \pi f(\pi) - \overline{L} \{1 - F[p(1 - \alpha)I]\},$$

the first order condition

$$\frac{\partial W}{\partial I} = -p(1-\alpha)[p(1-\alpha)I] f[p(1-\alpha)I] + \overline{L} p(1-\alpha)\{f[p(1-\alpha)I]\} = 0.$$

$$\Rightarrow p(1-\alpha) f[p(1-\alpha)I] \{\overline{L} - p(1-\alpha)I\} = 0. \tag{A1.1}$$

Because  $f[p(1-\alpha)I]$  and  $p(1-\alpha)$  are assumed greater than zero,  $\overline{L}-p(1-\alpha)I=0$ .

Thus, 
$$I^* = \overline{L}/p(1-\alpha)$$
. (A1.2)

## Appendix 2

## (A) Proof of equation (11).

According to equation (10) in the text,

$$W = \int_{\pi_0}^{\Pi} \pi f(\pi) d\pi + Y(z) - \overline{L} \{1 - F[p(1 - \alpha)I + p\alpha \kappa z]\} - (\kappa - \theta) p\alpha z \{1 - F[p(1 - \alpha)I + p\alpha \kappa z]\}$$

$$= \int_{0}^{\Pi} \pi f(\pi) d\pi - \int_{0}^{\pi_0} \pi f(\pi) d\pi + Y(z)$$

$$- \overline{L} \{1 - F[p(1 - \alpha)I + p\alpha \kappa z]\} - (\kappa - \theta) p\alpha z \{1 - F[p(1 - \alpha)I + p\alpha \kappa z]\},$$

so the first order condition

$$\begin{split} \partial W/\partial I &= -\big[p(1-\alpha)\,I + p\alpha\kappa z\big]\,f\big[p(1-\alpha)\,I + p\alpha\kappa z\big]p\big(1-\alpha\big) \\ &+ \overline{L}\big\{f\big[p(1-\alpha)\,I + p\alpha\kappa z\big]p\big(1-\alpha\big)\big\} + \big(\kappa - \theta\big)p\alpha z\,f\big[p(1-\alpha)\,I + p\alpha\kappa z\big]p\big(1-\alpha\big) = 0\,. \end{split}$$

$$\Rightarrow p(1-\alpha) f[p(1-\alpha)I + p\alpha\kappa z] \{ \overline{L} - p\alpha\theta z - p(1-\alpha)I \} = 0$$
(A2.1)

Because  $p(1-\alpha)$  and  $f[p(1-\alpha)I + p\alpha\kappa z]$  are assumed

$$>0, \{\overline{L} - p\alpha\theta z - p(1-\alpha)I\} = 0.$$
 Thus,  $I^{**} = (L - \alpha\theta z)/(1-\alpha).$  (A2.2)

## (B) Proof of equation (12).

The first order condition

$$\partial W/\partial z = - \left[ p \big( \mathbf{l} - \alpha \big) \, I + p \alpha \kappa \mathbf{z} \right] \, f \big[ p \big( \mathbf{l} - \alpha \big) \, I + p \alpha \kappa \mathbf{z} \big] \, p \alpha \kappa + Y' + \overline{L} \big\{ f \big[ p \big( \mathbf{l} - \alpha \big) \, I + p \alpha \kappa \mathbf{z} \big] \, p \alpha \kappa \big\}$$

#### 國立政治大學學報第八十二期

$$-(\kappa - \theta)p\alpha \left\{1 - F[p(1 - \alpha)I + p\alpha\kappa z]\right\} + (\kappa - \theta)p\alpha z f[p(1 - \alpha)I + p\alpha\kappa z]p\alpha\kappa$$

$$= 0.$$

$$\Rightarrow p\alpha\kappa f[p(1-\alpha)I + p\alpha\kappa z]\{\overline{L} - p\alpha\theta z - p(1-\alpha)I\} + Y'$$

$$= (k-\theta)p\alpha\{1 - F[p(1-\alpha)I + p\alpha\kappa z]\}$$
(A2.3)

Since 
$$I^{**} = (L - \alpha \theta z)/(1 - \alpha)$$
 according to (A2.2), the left hand-side of equation (A2.3) is equal to Y'. Thus,  $(\kappa - \theta)p\alpha \left\{1 - F\left[p(1 - \alpha)I^{**} + p\alpha \kappa z\right]\right\} = Y'$ . (A2.4)

If  $\alpha = 0, Y' = 0$ . It implies no marginal benefit to the consumer from the public fund when all the loss can be attributed to the firm. In such case z can be any number including zero. Therefore, it is not necessary to set up the public fund if there is no social inflation for the loss. The private insurance system itself is efficient for risk management because optimal insurance coverage  $I^{**}$  equals the loss amount L and premium equals the expected loss.

If  $\alpha \neq 0$  and Y' = 0,  $\{1 - F[p(1 - \alpha)I^* + p\alpha\kappa z]\} = 0$  since  $(\kappa - \theta) > 0$ . If  $\{1 - F[p(1 - \alpha)I^* + p\alpha\kappa z]\} = 0$ , it implies no any firm will engage in the business activity. In such case there is no harm and the public fund is not needed.

If 
$$\alpha \neq 0$$
 and  $Y' \neq 0$ ,  $\{1 - F[p(1-\alpha)I^{**} + p\alpha\kappa z]\} = Y'/p\alpha(\kappa - \theta)$ .  

$$\Rightarrow F[p(1-\alpha)I^{**} + p\alpha\kappa z] = 1 - Y'/p\alpha(\kappa - \theta).$$

$$\Rightarrow p(1-\alpha)I^{**} + p\alpha\kappa z = F^{-1}[1 - Y'/p\alpha(\kappa - \theta)].$$
(A2.5)

Since 
$$I^{**} = (L - \alpha \theta z)/(1 - \alpha)$$
,  $z^* = \left\{F^{-1}(x) - \overline{L}\right\}/p\alpha(\kappa - \theta)$ ,  
where  $x = 1 - Y'/p\alpha(\kappa - \theta)$ .

Because F is a probability distribution function with p.d.f f(.) > 0,  $0 < F \le 1$  and  $F(\Pi) = 1$ . Since Y' > 0, F[.] < 1. Thus  $p(1-\alpha)I^{\bullet \bullet} + p\alpha\kappa z = F^{-1}(x) < \Pi$ . (It is assumed  $Y'/p\alpha(\kappa-\theta) < 1$  such that F will not be negative.)

## Appendix 3

Proof of equations (14) and (15).

Because z = 0,

$$W = \lambda \left\{ \int_{\pi_{1}}^{\Pi} \pi f(\pi) d\pi - \overline{L}_{1} \left\{ 1 - F \left[ p_{1} (1 - \alpha) I_{1} \right] \right\} \right\} + \left( 1 - \lambda \right) \left\{ \int_{\pi_{2}}^{\Pi} \pi f(\pi) d\pi - \overline{L}_{2} \left\{ 1 - F \left[ p_{2} (1 - \alpha) I_{2} \right] \right\} \right\}$$

$$= \lambda \left\{ \int_{0}^{\Pi} \pi f(\pi) d\pi - \int_{0}^{\pi_{1}} \pi f(\pi) d\pi - \overline{L}_{1} \left\{ 1 - F \left[ p_{1} (1 - \alpha) I_{1} \right] \right\} \right\}$$

$$+ \left( 1 - \lambda \right) \left\{ \int_{0}^{\Pi} \pi f(\pi) d\pi - \int_{0}^{\pi_{2}} \pi f(\pi) d\pi - \overline{L}_{2} \left\{ 1 - F \left[ p_{2} (1 - \alpha) I_{2} \right] \right\} \right\}.$$

The first order condition

$$\partial W/\partial I_1 = -\lambda p_1 (1-\alpha) [p_1 (1-\alpha) I_1] f[p_1 (1-\alpha) I_1] + \lambda \overline{L}_1 p_1 (1-\alpha) \{f[p_1 (1-\alpha) I_1]\} = 0$$

$$\Rightarrow \lambda p_1 (1-\alpha) f[p_1 (1-\alpha) I_1] \{\overline{L}_1 - p_1 (1-\alpha) I_1\} = 0$$
(A3.1)

Because  $\lambda$ ,  $p_1(1-\alpha)$  and  $f[p_1(1-\alpha)I_1]$  are assumed >0,

$$\left\{\overline{L}_1 - p_1 \left(1 - \alpha\right) I_1\right\} = 0$$
.

Thus, 
$$I_1^* = \overline{L}_1 / p_1(1-\alpha) = L_1 / (1-\alpha)$$
 since  $\overline{L}_1 = p_1 L_1$ .

By the same token for the high-risk firm,

$$\partial W/\partial I_2 = -(1-\lambda)p_2(1-\alpha)[p_2(1-\alpha)I_2] f[p_2(1-\alpha)I_2] + (1-\lambda)\overline{L}_2 p_2(1-\alpha)\{f[p_2(1-\alpha)I_2]\} = 0.$$

$$\Rightarrow (1-\lambda)p_2(1-\alpha)f[p_2(1-\alpha)I_2]\{\overline{L}_2 - p_2(1-\alpha)I_2\} = 0$$
 (A3.2)

Because 
$$1-\lambda$$
,  $p_2(1-\alpha)$  and  $f[p_2(1-\alpha)I_2]$  are assumed >0, 
$$\left\{\overline{L}_2-p_2(1-\alpha)I_2\right\}=0$$
.

Thus, 
$$I_2^* = \overline{L}_2 / p_2 (1-\alpha) = L_2 / (1-\alpha)$$
 since  $\overline{L}_2 = p_2 L_2$ .

## Appendix 4.

(A)Proof of equations (16) and (17).

$$W = \lambda \left\{ \int_{\pi_{1}}^{\Pi} \pi f(\pi) d\pi - \overline{L}_{1} \left\{ 1 - F\left[p_{1}(1 - \alpha)I_{1} + p_{1}\alpha\kappa z\right] \right\} - (\kappa - \theta)p_{1}\alpha z \left\{ 1 - F\left[p_{1}(1 - \alpha)I_{1} + p_{1}\alpha\kappa z\right] \right\} \right\}$$

$$+(1-\lambda)\left\{\int_{\pi_{2}}^{\Pi}\pi f(\pi)d\pi - \overline{L}_{2}\left\{1 - F\left[p_{2}(1-\alpha)I_{2} + p_{2}\alpha\kappa z\right]\right\} - (\kappa - \theta)p_{2}\alpha z\left\{1 - F\left[p_{2}(1-\alpha)I_{2} + p_{2}\alpha k z\right]\right\}\right\} + Y(z)$$

$$\begin{split} \partial W/\partial I_1 &= - \big[ p_1 \big( \mathbf{1} - \alpha \big) \, I_1 + p_1 \alpha \kappa \mathbf{z} \big] \, f \big[ p_1 \big( \mathbf{1} - \alpha \big) \, I_1 + p_1 \alpha k \mathbf{z} \big] \, p_1 \big( \mathbf{1} - \alpha \big) \\ &\quad + \overline{L}_1 \big\{ f \big[ p_1 \big( \mathbf{1} - \alpha \big) \, I_1 + p_1 \alpha \kappa \mathbf{z} \big] \, p_1 \big( \mathbf{1} - \alpha \big) \big\} + \big( \kappa - \theta \big) p_1 \alpha \mathbf{z} \, f \big[ p_1 \big( \mathbf{1} - \alpha \big) \, I_1 + p_1 \alpha \kappa \mathbf{z} \big] \, p_1 \big( \mathbf{1} - \alpha \big) \\ &= 0. \end{split}$$

$$\Rightarrow p_1(1-\alpha)f[p_1(1-\alpha)I_1+p_1\alpha\kappa z]\{\overline{L}_1-p_1\alpha\theta z-p_1(1-\alpha)I_1\}=0$$
(A4.1)

Because  $p_1(1-\alpha)$  and  $f[p_1(1-\alpha)I_1 + p_1\alpha\kappa z]$  are assumed

$$>0$$
,  $\{\overline{L}_1-p_1\alpha\theta z-p_1(1-\alpha)I_1\}=0$ .

Thus, 
$$I_1^* = (L_1 - \alpha \theta_z)/(1 - \alpha)$$
.

By the same token for the high-risk firm,

$$\begin{split} &\partial W/\partial I_{2} = - \big[ p_{2} \big( \mathbf{1} - \alpha \big) I_{2} + p_{2} \alpha \kappa \mathbf{z} \big] \, f \big[ p_{2} \big( \mathbf{1} - \alpha \big) I_{2} + p_{2} \alpha \kappa \mathbf{z} \big] p_{2} \big( \mathbf{1} - \alpha \big) \\ &+ \overline{L}_{2} \big\{ f \big[ p_{2} \big( \mathbf{1} - \alpha \big) I_{2} + p_{2} \alpha \kappa \mathbf{z} \big] p_{2} \big( \mathbf{1} - \alpha \big) \big\} + \big( \kappa - \theta \big) p_{2} \alpha \mathbf{z} \, f \big[ p_{2} \big( \mathbf{1} - \alpha \big) I_{2} + p_{2} \alpha \kappa \mathbf{z} \big] \, p_{2} \big( \mathbf{1} - \alpha \big) \\ &= 0. \end{split}$$

$$\Rightarrow p_{2}(1-\alpha)f[p_{2}(1-\alpha)I_{2}+p_{2}\alpha\kappa z]\{\overline{L}_{2}-p_{2}\alpha\theta z-p_{2}(1-\alpha)I_{2}\}=0$$
Because  $p_{2}(1-\alpha)$  and  $f[p_{2}(1-\alpha)I_{2}+p_{2}\alpha\kappa z]$  are assumed  $>0$ ,
$$\{\overline{L}_{2}-p_{2}\alpha\theta z-p_{2}(1-\alpha)I_{2}\}=0. \text{ Thus, } I_{2}^{*}=(L_{2}-\alpha\theta z)/(1-\alpha).$$

## (B)Proof for equations (18) and (19).

$$\begin{split} &\partial W/\partial z = \lambda \left\{ - \left[ p_1 (1-\alpha) I_1 + p_1 \alpha \kappa \ z \right] f \left[ p_1 (1-\alpha) I_1 + p_1 \alpha \kappa \ z \right] p_1 \alpha \kappa + \overline{L}_1 \left\{ f \left[ p_1 (1-\alpha) I_1 + p_1 \alpha \kappa \ z \right] \right\} p_1 \alpha \kappa \right. \\ & - \left( \kappa - \theta \right) p_1 \alpha \left\{ 1 - F \left[ p_1 (1-\alpha) I_1 + p_1 \alpha \kappa \ z \right] \right\} + \left( \kappa - \theta \right) p_1 \alpha \ z f \left[ p_1 (1-\alpha) I_1 + p_1 \alpha \kappa \ z \right] p_1 \alpha \kappa \right\} \\ & + \left( 1 - \lambda \right) \left\{ - \left[ p_2 (1-\alpha) I_2 + p_2 \alpha \kappa \ z \right] f \left[ p_2 (1-\alpha) I_2 + p_2 \alpha \kappa \ z \right] p_2 \alpha \kappa + \overline{L}_2 \left\{ f \left[ p_2 (1-\alpha) I_2 + p_2 \alpha \kappa \ z \right] \right\} p_2 \alpha \kappa \right. \\ & - \left( \kappa - \theta \right) p_2 \alpha \left\{ 1 - F \left[ p_2 (1-\alpha) I_2 + p_2 \alpha \kappa \ z \right] + \left( \kappa - \theta \right) p_2 \alpha \ z f \left[ p_2 (1-\alpha) I_2 + p_2 \alpha \kappa \ z \right] p_2 \alpha \kappa \right\} + Y' \\ & = 0. \end{split}$$

$$\Rightarrow Y' + \lambda f[p_{1}(1-\alpha)I_{1} + p_{1}\alpha\kappa z]p_{1}\alpha\kappa \{\overline{L}_{1} - p_{1}\alpha\theta z - p_{1}(1-\alpha)I_{1}\}$$

$$+ (1-\lambda)f[p_{2}(1-\alpha)I_{2} + p_{2}\alpha\kappa z]p_{2}\alpha\kappa \{\overline{L}_{2} - p_{2}\alpha\theta z - p_{2}(1-\alpha)I_{2}\}$$

$$= \lambda (\kappa - \theta)p_{1}\alpha \{1 - F[p_{1}(1-\alpha)I_{1} + p_{1}\alpha\kappa z]\} + (1-\lambda)(\kappa - \theta)p_{2}\alpha \{1 - F[p_{2}(1-\alpha)I_{2} + p_{2}\alpha\kappa z]\}$$
(A4.3)

Since  $I_1^* = (L_1 - \alpha\theta z)/(1-\alpha)$  and  $I_2^* = (L_2 - \alpha\theta z)/(1-\alpha)$ , the left hand-side of (A4.3) is equal to Y'. Thus, the first order condition is

$$Y' = \lambda (\kappa - \theta) p_{1} \alpha \left\{ 1 - F \left[ p_{1} (1 - \alpha) I_{1}^{*} + p_{1} \alpha \kappa z \right] \right\}$$

$$+ (1 - \lambda) (\kappa - \theta) p_{2} \alpha \left\{ 1 - F \left[ p_{2} (1 - \alpha) I_{2}^{*} + p_{2} \alpha \kappa z \right] \right\}$$
(A4.4)

If  $\alpha=0$ , Y'=0, or if  $\alpha>0$  and Y'=0, it is not necessary to set up public fund for the

same reasons as indicated in appendix 2.

If  $\alpha \neq 0$  and  $Y' \neq 0$ , equation (A4.4) is rearranged as

$$\lambda p_{1}F[p_{1}(1-\alpha)I_{1}^{*}+p_{1}\alpha \kappa z]+(1-\lambda)p_{2}F[p_{2}(1-\alpha)I_{2}^{*}+p_{2}\alpha \kappa z]$$

$$=\lambda p_{1}+(1-\lambda)p_{2}-Y'/\alpha(\kappa-\theta)$$
(A4.5)

Because the exact value  $z^*$  cannot be found directly, its approximate value  $z^*$  is derived by the following procedures. Let  $p^m = \lambda p_1 + (1 - \lambda) p_2$ ,  $a = \lambda p_1 / p^m$ ,  $F_1 = F \Big[ p_1 (1 - \alpha) I_1^* + p_1 \alpha \kappa z \Big]$ ,  $F_2 = F \Big[ p_2 (1 - \alpha) I_2^* + p_2 \alpha \kappa z \Big]$  and  $x = 1 - Y' / p^m \alpha (\kappa - \theta)$ . Then equation (A4.5) becomes

$$aF_1 + (1-a)F_2 = x.$$
 (A4.6)

The linear combination of  $F_1$  and  $F_2$  is at the point of A on figure A4.1. The optimal value  $z^*$  is supposed to be solved from point B, but now we can only find its approximate value  $z^*$  from point C which is equal to the inverse function, the  $F^{-1}(x)$ .

That is, 
$$a \{ p_1 (1-\alpha) I_1^* + p_1 \alpha \kappa z \} + (1-a) \{ p_2 (1-\alpha) I_2^* + p_2 \alpha \kappa z \} = F^{-1}(x).$$
 (A4.7)

Since 
$$I_1^* = (L_1 - \alpha\theta z)/(1-\alpha)$$
 and  $I_2^* = (L_2 - \alpha\theta z)/(1-\alpha)$ ,

$$\dot{z} = \left\{ F^{-1}(x) - \overline{L}^{@} \right\} / p^{@} \alpha (\kappa - \theta). \tag{A4.8}$$

where 
$$\overline{L}^@=a\,\overline{L}_1+\big(1-a\big)\overline{L}_2$$
 , and  $p^@=a\,p_1+\big(1-a\big)p_2$  .

( It is assumed  $Y'/p^m\alpha(\kappa - \theta) < 1$  such that F will not be negative.)

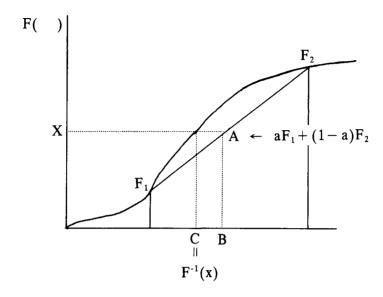


figure A4.1.

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#### 國立政治大學學報第八十二期

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