Early surrender and the distribution of policy reserves

Chenghsien Tsai [a,*], Weiyu Kuo [b], Wei-Kuang Chen [c]

[a] *Department of Risk Management and Insurance, National Chengchi University, No. 64, Sect. 2, Chihnan Road, Taipei 11623, Taiwan*
[b] *Department of International Trade, National Chengchi University, Taipei 11623, Taiwan*
[c] *Department of Money and Banking, National Chengchi University, Taipei 11623, Taiwan*

Received October 2001; received in revised form July 2002; accepted 24 September 2002

Abstract

We extend the literature by incorporating early surrender into the distribution estimation for policy reserves. First, we employ the cointegrated vector autoregression technique to estimate an empirical relation between the lapse rate and interest rate. The tests indicate a significant cointegrated vector that implies a long-term relation between the lapse rate and interest rate. Based on the estimated error-correction model, we then simulate the policy reserve distribution with stochastic mortality, interest rate, and early surrender. We find that early surrender reduces the expected value as well as the risk for policy reserves due to surrenders in the low interest rate periods. Further analyses indicate that the early surrender effect depends on the sign and magnitude of the difference between the market interest rate and policy credit rate. When the credit rate is higher (lower) than the market interest rate, early surrender acts to decrease (increase) the mean reserve. This effect increases with the magnitude in the difference.

JEL classification: G22; C50

Subj. Class.: IM22; IB11

Keywords: Cointegration analysis; Early surrender; Lapse rate; Policy reserves

1. Introduction

Policy reserves have consistently been the largest liability item on the balance sheet of life insurers. It is usually determined by discounting the expected cash flow associated with issued policies into a single figure based on an appropriate discount rate that reflects the undiversifiable risk of the cash flow. This single discounted figure conceals the underlying uncertain nature of policy reserves that is critical to insurers' risk management. The uncertainty in the reserves arises from the uncertain cash flow and stochastic discount rate. The expected cash flows are uncertain because they are contingent upon factors like mortality, disability, and early surrender. Various macroeconomic events may also make the discount rate stochastic. Due to the immense size of the policy reserve, the uncertainty embedded in reserving could have significant impacts on the solvency of a life insurer. Therefore, it is important to quantify the uncertainty associated with the policy reserve.

* Corresponding author. Tel.: +886-2-2936-9647; fax: +886-2-2939-3864.
E-mail address: ctsai@nccu.edu.tw (C. Tsai).

PII: S0167-6687(02)00188-9

An essential way to quantify the uncertainty in policy reserves is to estimate the policy reserve distribution. This estimation requires explicit modeling on the stochastic cash flow and discount rate. The reserving methods described in Jordan (1967) and Bowers et al. (1986, 1997) consider the probabilistic future lifetime, but assume a deterministic discount rate. Incorporating the stochastic interest rate into reserving, Panjer and Bellhouse (1980), Bellhouse and Panjer (1981), Giaccotto (1986), Beekman and Fuelling (1990, 1991, 1993), and De Schepper and Goovaerts (1992) derive the first two moments or the distribution of policy reserves for one insurance policy under certain assumptions on the interest rate dynamics. In addition, Frees (1990), Parker (1994a–c, 1996, 1997) and Marceau and Gaillardetz (1999) extend the analyzed subject from a single policy to a pool of policies. The literature to date provides us with good understanding about the risk of policy reserves in an environment with stochastic mortality and interest rates.

We contribute to the literature by incorporating another risk factor, early surrender, into the policy reserve estimation. Most insurers include in their contracts a provision that grants the policyholder who elects to terminate the policy a right to a cash surrender value. The policyholder's option to demand the policy's cash value at any time before the termination of policy can have considerable impacts upon life insurers. First, the surrender option itself could be costly to the insurer. Albizzati and Geman (1994) and Grosen and Jorgensen (2000) demonstrate that the surrender option might comprise a substantial portion of the present value of all future premiums. It would account for more than 50% of the contract value if exercised optimally with changes in the interest rate. Second, early surrender could cause the cash flow of life insurance policies to be sensitive to the interest rate and significantly alter the risk characteristics of policies. An illustrative example is the disintermediation that occurred to US life insurers during the 1980s. This incident and several actuarial studies in the *Transactions of Society of Actuaries Reports* suggest that early surrender increases when the market interest rate rises. The surrender option thus might make the cash flow of life insurance policies sensitive to the interest rate. Babbel (1995) and Briys and de Varenne (1997) show that the dependence of a policy's cash flow on the interest rate is critical in measuring the interest rate risk of the policy. They demonstrate that misspecification of the dependence structure would cause large errors in the effective duration estimates and even greater errors in the convexity estimates. Overall, these findings suggest that considering early surrender is important to the risk management of policy reserves.

To integrate the interest-rate-sensitive surrender behavior of policyholders into the estimation for the policy reserve distribution, we first employ the cointegrated vector autoregression (VAR) model developed by Engle and Granger (1987) to construct an empirical model for the relation between the lapse rate and interest rate. Cointegration modeling is designed to identify potential long-term relations between variables of interest. It could render a more comprehensive specification between the interest rate and lapse rate. Based on the estimated lapse rate model, we then simulate the distribution of policy reserves for a pool of level-premium endowment policies with cash value schedules fixed at policy inception in an environment with stochastic mortality, interest rate, and early surrender.[1]

Our empirical analyses indicate that there is a cointegrated vector between the lapse rate and interest rate. The interest rate may therefore affect the lapse rate through a long-term mechanism. We also find that the interest rate can be used to explain the short-term dynamics of the lapse rate. The dependence of the lapse rate on the interest rate is thus confirmed.

Our simulation results show that the mortality risk is unimportant while the interest rate risk is substantial. The results further show that early surrender decreases the mean, the standard deviation, and the 95th percentile of

[1] The endowment policy analyzed in this study has a feature not commonly seen in the literature: level premiums. Most previous studies considered single-premium contracts only, leaving the extension to annual premium policies implicit. If the cash flow were independent of the interest rate, the extension would not be a problem (Parker, 1996, 1997). If the lapse rate is a function of the interest rate, generalizing the cases from single-premium contracts to those for level-premium policies is no longer straightforward. The results for single-premium cases may not hold for level-premium ones because the cash outflow resulting from early surrenders and cash inflows determined by the number of people left in the pool are contingent upon the interest rate. Since most policies are sold with level premiums, our analyses may have more practical implications.

the policy reserve distribution. In other words, the surrender option actually benefits life insurers. The beneficial effect comes from surrenders that occur during low interest rate periods because these policyholders relinquish the valuable fixed credit rate provision.[2] Moreover, we find that the effect of early surrender depends on both the sign and magnitude of the difference between the policy credit rate and market interest rate. Specifically, when the credit rate is higher (lower) than the market interest rate, early surrender decreases (increases) the mean reserve; the larger the difference, the greater the effect.

The remainder of this paper is organized as follows. Section 2 estimates an empirical lapse rate model through the cointegration method. Section 3 consists of risk analyses on the mortality, interest rate, and early surrender using the Monte Carlo simulations and robustness checks. Section 4 contains concluding remarks and suggestions for future research.

2. An empirical lapse rate model

Few empirical studies have looked into the relation between the lapse rate and interest rate. Outreville (1990) find a rather weak connection between them using data from the United States and Canada. Stronger connections are documented in more recent actuarial studies in the *Transactions of Society of Actuaries Reports*, e.g., Cox et al. (1992) and the Annuity Persistency Study in the 1995–1996 reports (pp. 559–638). They find that the lapse rate increases with the spread between the policy credit rate and market interest rate. The inconsistent findings among previous studies are probably due to the differences in sampling periods and methods. While the sample periods for the actuarial studies are the late 1980s and 1990s, the sample in Outreville (1990) covers up to 1979 only and misses the wide swing in the interest rate during the 1980s and 1990s. Outreville performs ordinary least squares (OLS) analysis with the Cochrane–Orcutt adjustment for the first-order serial correction of residuals. The analyses in the actuarial reports are performed based on univariate analysis without control variables, however. Since the evidence is relatively scarce and inconclusive, we re-examine the relation between the lapse rate and interest rate based on more comprehensive data and methods.

2.1. Data

We acquire the lapse rate data from the *Life Insurance Fact Book*, an annual statistical report from the American Council of Life Insurance (ACLI, 1997).[3] Our sample consists of the annual voluntary termination rates for all ordinary life insurance policies in force from 1959 to 1995.[4] Compared with the data utilized by previous studies, our data have more sample observations, span a longer period, and cover the highly volatile interest rate periods in the 1980s and early 1990s. We collect the 1-year treasury rate from the US Financial Database maintained by the Ministry of Education in Taiwan.[5] The time series of the lapse rate, interest rate, and their first-order differences are depicted in Fig. 1.

---

[2] The policy credit rate represents the discount rate used to calculate the premium and cash value. Since the analyzed endowment policy has cash values fixed at policy inception, it has a fixed policy credit rate.

[3] The data in the *Fact Book* are derived from the annual statements filed by life insurance companies with the National Association of Insurance Commissioners, ACLIs surveys, and/or external sources such as government agencies and trade associations.

[4] The usage of the adjective “voluntary” by ACLI is probably to distinguish policy terminations caused by lapses from terminations caused by the death of the insured that is beyond the control of either the policyholder or the insured, even though some lapses could be against the policyholder’s will.

[5] Because 1-year treasury bill rates are recorded on a monthly basis in the database, we transform the monthly interest rates into annual rates using the following compounding method:

$$\text{annual interest rate} = (1 + \tfrac{1}{12}m_1)(1 + \tfrac{1}{12}m_2)\cdots(1 + \tfrac{1}{12}m_{12}) - 1,$$

where m_i denotes the interest rate in month i, $i = 1, 2, \ldots, 12$.

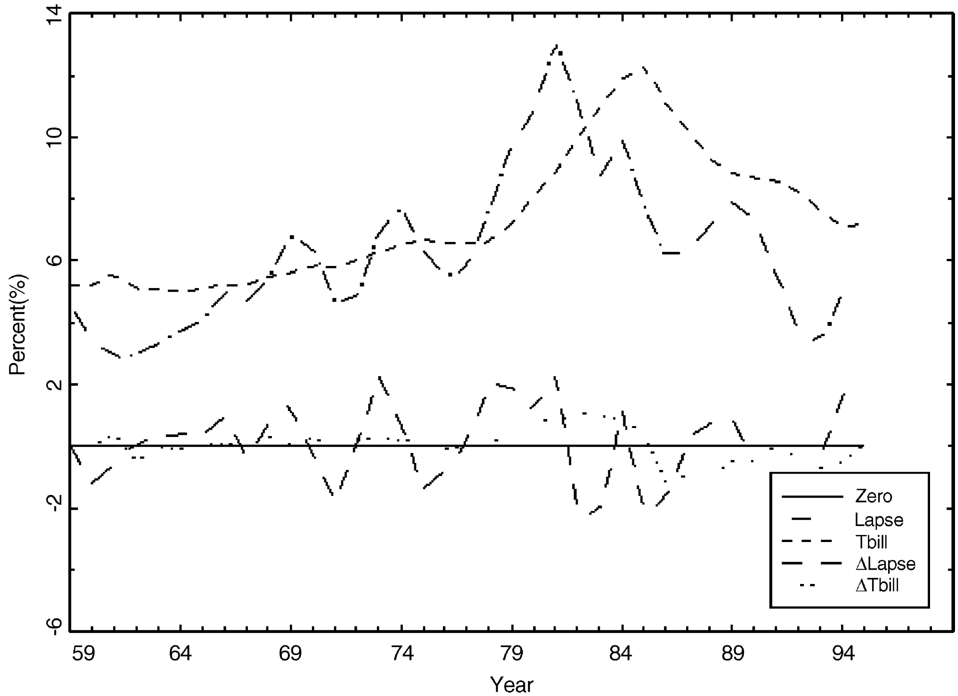

Fig. 1. Time series of the lapse rate, interest rate, and their first-order differences.

2.2. Estimation of the cointegrated VAR model

Cointegration analysis generally involves four steps: (1) ensuring that the individual variables are $I(1)$ processes, (2) determining the order of the VAR model, (3) conducting cointegration tests to determine the rank of the cointegrated system, and (4) estimating the error-correction model. We adopt the augmented Dickey–Fuller (ADF) unit root test to examine whether a unit root exists in the lapse rate and interest rate, respectively. Since the ADF test depends upon the assumption made about the underlying process and the estimated regression, we conduct the test based on three different assumptions that correspond to cases 1, 2, and 4 in Hamilton (1994, p. 502). We perform the ADF test on both levels and the first-order differences in the lapse rate and interest rate to verify that both series are $I(1)$ processes rather than processes with higher-order integration, that is, $I(j)$, $j > 1$.[6]

Table 1 reports the results from the unit root test. The ADF statistics for the level of both series are not significant at the 5% significance level, implying that the null hypothesis of a unit root cannot be rejected. The corresponding statistics for the first-order differences are significant at the 1% significance level, which suggests rejection of the null hypothesis. Based on these results, we conclude that the lapse rate and interest rate follow non-stationary $I(1)$ processes individually.

The order of the VAR model must be determined after the unit root test. The Akaike information criterion (AIC) is used for this purpose. The optimal model indicated by AIC is VAR(3). According to this VAR(3) specification we conduct two cointegration tests developed by Johansen (1991), the maximal eigenvalue test and the trace test, to determine the number of cointegration vectors. The results are reported in Table 2. Both tests indicate a cointegrating

[6] If this is the case, the ADF test will reject the null hypothesis of a unit root in these differenced series at conventional significance levels but fail to do so for the level series.

Table 1
Unit root test for the lapse rate and interest rate

Variable	ADF test[a]		
	H_{20}	H_{30}	H_{40}
L_t	–	−1.708	−2.804
I_t	–	−2.481	−2.386
ΔL_t	−3.008[b,**]	–	–
ΔI_t	−5.116[**]	–	–

[a] The regressions used to test the null hypothesis for a unit root include neither an intercept nor a time trend in H_{20}, include an intercept in H_{30}, and include both an intercept and a time trend in H_{40}, respectively. The optimal lags in the regressions are selected based on AIC to include enough lags of lagged variables to eliminate autocorrelations in the residuals.

[b] The critical values of the ADF tests are based on MacKinnon (1991).

** Significant at the 1% level.

vector between the lapse rate and interest rate. In other words, there exists a long-term equilibrium relation between the lapse rate and interest rate.

We then use the maximum likelihood method to estimate an error-correction model with one cointegrating vector between the lapse rate and interest rate as follows:

$$\begin{bmatrix} \Delta L_t \\ \Delta I_t \end{bmatrix} = \begin{bmatrix} -0.243^{***}\,(-5.193) \\ -0.199\,(-0.890) \end{bmatrix} \begin{bmatrix} 1 & -1.053^{***}\,(-9.819) & -0.008\,(-1.148) \end{bmatrix} \begin{bmatrix} L_{t-1} \\ I_{t-1} \\ 1 \end{bmatrix}$$
$$+ \begin{bmatrix} 0.240\,(1.650) & -0.046\,(-0.881) \\ -0.146\,(-0.210) & 0.149\,(0.597) \end{bmatrix} \begin{bmatrix} \Delta L_{t-1} \\ \Delta I_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} -0.012\,(-0.094) & -0.151^{***}\,(-2.934) \\ -0.642\,(-1.037) & -0.514^{*}\,(-2.085) \end{bmatrix} \begin{bmatrix} \Delta L_{t-2} \\ \Delta I_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^L \\ \varepsilon_t^I \end{bmatrix}, \quad (1)$$

where $\mathbf{E} = \begin{bmatrix} \varepsilon_t^L & \varepsilon_t^I \end{bmatrix}' \sim N(0, \hat{\boldsymbol{\Sigma}})$ and

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} 1.67 \times 10^{-4} & 8.09 \times 10^{-6} \\ 8.09 \times 10^{-6} & 7.28 \times 10^{-6} \end{bmatrix}.$$

Table 2
The maximal eigenvalue test and trace test from Johansen (1991)[a]

Null	Alternative	The maximal eigenvalue test		
		$\hat{\lambda}_{max}$	95% Critical value	90% Critical value
$r = 0$	$r = 1$	22.42*	15.87	13.81
$r \leq 1$	$r = 2$	3.81	9.16	7.53
		The trace test		
		$\hat{\lambda}_{trace}$	95% Critical value	90% Critical value
$r = 0$	$r \geq 1$	26.23*	20.18	17.88
$r \leq 1$	$r \geq 2$	3.81	9.16	7.53

[a] The tests are performed based on $\Delta y_t = \mu + \gamma y_{t-1} + C \Delta y_{t-1} + \varepsilon_t$, where C is a 2×2 matrix polynomial in the lag operator, Δ is the first-order difference operator, μ is an intercept vector, γ a 2×2 constant matrix, and ε_t a white noise error term vector. We use Microfit 4.0 to obtain relevant statistics.

* Significant at the 5% level.

Table 3
Error-correction models for the lapse rate and interest rate[a]

Variable	ΔL_t			ΔI_t		
	Coefficient	*t*-Value	*p*-Value	Coefficient	*t*-Value	*p*-Value
ΔL_{t-1}	0.240	1.650	0.110	−0.146	−0.210	0.836
ΔI_{t-1}	−0.046	−0.881	0.385	0.149	0.597	0.555
ΔL_{t-2}	−0.012	−0.094	0.926	−0.642	−1.037	0.308
ΔI_{t-2}	−0.151	−2.934	0.006	−0.514	−2.085	0.046
ECM_{t-1}	−0.243	−5.193	0.000	−0.199	−0.890	0.381
R^2	0.799			0.252		
DW[b]	1.577			1.985		
SCorr[c]	$\chi^2(1) = 1.736[0.188]$			$\chi^2(1) = 0.013[0.909]$		
Hetero[d]	$\chi^2(1) = 0.005[0.944]$			$\chi^2(1) = 0.701[0.402]$		
Normal[e]	$\chi^2(2) = 0.980[0.612]$			$\chi^2(2) = 0.225[0.893]$		
FF[f]	$\chi^2(1) = 0.764[0.382]$			$\chi^2(1) = 0.117[0.732]$		

$\text{ECM}_{t-1} = L_{t-1} - 1.053 I_{t-1} - 0.008$

[a] These error-correction models are estimated by Microfit 4.0.
[b] Durbin–Watson test.
[c] Lagrange multiplier test for residual serial correlation.
[d] Based on the regression of squared residuals on squared fitted values.
[e] Based on the test on the skewness and kurtosis of residuals.
[f] Ramsey's RESET test using the square of fitted values.

The specifications for the individual variables in the error-correction VAR model as well as the results of their misspecification tests are presented in Table 3. The results including the high coefficient of determination for the lapse rate equation, the normality test, heteroscedasticity test, and the test for serial correlations in residuals suggest that the estimated models for the lapse rate and interest rate are generally well specified. Therefore, the estimated error-correction model has reasonably good capability in interpreting the relation between the lapse rate and interest rate.

2.3. *Discussions about estimation results*

The lapse rate equation in the vector error-correction system (1) suggests that changes in the lapse rate result from two sources: changes in the lagged variables $[\, \Delta L_{t-1} \quad \Delta I_{t-1} \,]$ and the levels of the lagged variables $[\, L_{t-1} \quad I_{t-1} \,]$. The impact from the lagged variable levels can be represented by the cointegration vector $\text{ECM}_t = L_t - 1.053 I_t - 0.008$. In other words, the variation in the lapse rate could result from changes in the lagged variables and/or a non-zero cointegrating vector.

Since the cointegration vector found in system (1) implies a long-term relation between the lapse rate and interest rate that can be expressed as $L_t = 0.008 + 1.053 I_t$, a non-zero cointegration vector represents the influence from a long-term force. Any deviation from the long-term equilibrium relation will cause the lapse rate to change. The impact of the deviation will accumulate because it is the levels of the variables that cause the change. Besides, the adjustment toward the long-term relation is only partial as indicated by the coefficients for the cointegration vectors that have absolute values smaller than 1, namely, −0.243 and −0.199, respectively. Since a non-zero cointegration vector has enduring effects, it also represents the influence in the long run.

The estimated error-correction model also reveals interesting short-term lapse rate dynamics. The significant negative coefficient of ΔL_{t-2} in the lapse rate equation suggests that changes in the interest rate two periods ago would cause the present lapse rate to adjust downwards. This downward adjustment would be countered by the first-order serial correlation of the lapse rate, i.e., 0.240, which is barely significant at the 10% significance level though.

The only significant coefficient in the interest rate equation is for ΔI_{t-2}. This suggests that the interest rate is not influenced by the lapse rate. The insignificant coefficients of ΔL on ΔI reinforce our confidence in our error-correction model because the lapse rate intuitively should not play an important role in the working of the economic system.

3. Monte Carlo simulation

Our simulations consist of three risk layers with one on top of another. In the first layer, we consider the mortality risk resulting from random survivorship. We adopt the probabilistic interpretation of the life table to estimate the risk of random survivorship. The second layer considers the interest rate risk due to the randomness of the interest rate. We employ the interest rate equation in the estimated error-correction model to assess the interest rate risk. Early surrender is incorporated into the simulations on top of random mortality and interest rate in the final layer based on the estimated cointegration system. The effect from early surrender must be analyzed after the interest rate risk because it is derived from the stochastic interest rate. The layer-adding analyses enable us to examine the marginal as well as the combined effects of various risk factors.

3.1. Simulation setting

Consider a group of N *life-aged-a* policyholders. Assume that these policyholders have two contingencies: death and surrender. For each of these policyholders, the termination probabilities during the age interval of x and $x+1$ because of death and surrender are specified by $q_x^{(\mathrm{m})}$ and $q_x^{(\mathrm{l})}$, respectively,[7] where x is a positive integer and $x \geq a$. In addition, let $C^{(\tau)}(x)$ denote the cohort's number of survivors at age x out of the original N lives and $D_x^{(i)}$ denote the number of lives who leave the group between ages x and $x+1$ for contingency i, where $i = \mathrm{m}, \mathrm{l}$, or τ.[8]

Let the T-year endowment policies issued to the policyholders have face amount of F dollars payable at the end of the death or the Tth year and annual premium P dollars receivable at the beginning of the surviving years. If policyholders surrender their policies during the age interval of x and $x+1$, they receive an amount, S_x, at the end of that year. We assume that[9]

$$S_x = \left(0.8 + 0.2\left(\frac{x-a+1}{T}\right)\right) {}_{x-a+1}V_a, \tag{2}$$

where ${}_{x-a+1}V_a$ is the policy reserve calculated with random future lifetime and deterministic interest rates as in Bowers et al. (1986, 1997) and $x < a+T$. Let L be the present value of the cash flows generated by this portfolio. We then have

$$L = \sum_{x=a}^{a+T-1} [(FD_x^{(\mathrm{m})} + S_x D_x^{(\mathrm{l})}) v_{x-a+1}] + FC^{(\tau)}(a+T)v_T - \left[\sum_{x=a}^{a+T-1} PC^{(\tau)}(x) v_{x-a}\right], \tag{3}$$

where

$$v_{x-a} = \begin{cases} 1 & \text{if } x = a, \\ \dfrac{1}{(1+r_1)(1+r_2)\cdots(1+r_{x-a})} & \text{if } a < x < a+T, \end{cases}$$

and r_{x-a} is the market interest rate prevailing over policy year $x-a$ for $a < x < a+T$.

[7] The superscript m indicates the mortality contingency and l the lapse contingency.
[8] The superscript τ refers to all contingencies. Notice that $C^{(\tau)}(a) \equiv N$.
[9] Although this specific formula comes from the Model Provisions of Life Insurance Policies in Taiwan, it possesses the general property of surrender charges: high at the beginning and decline as policies mature.

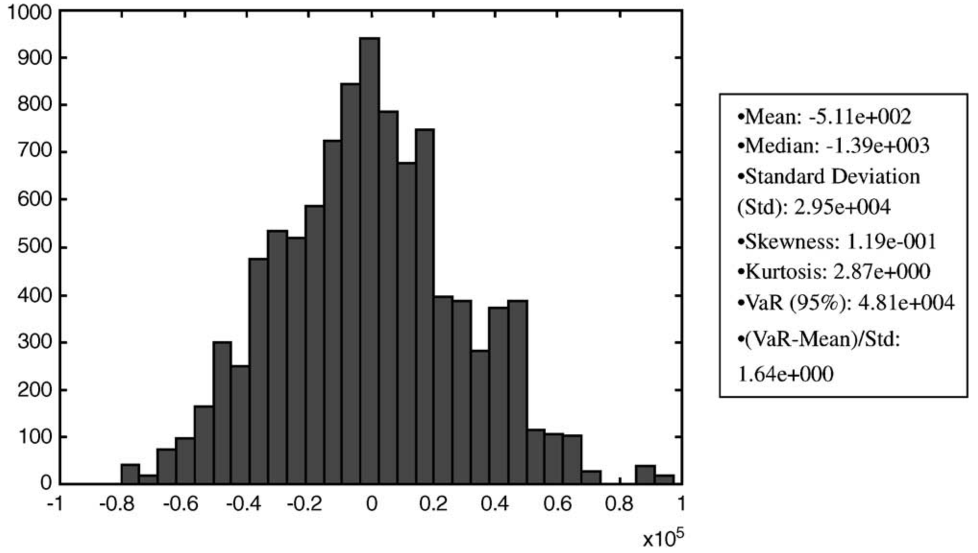

Fig. 2. One decrement (mortality) with constant interest rate.

The random variable L represents the present value of the insurers' liabilities associated with a pool of policies. The statistical properties of L are critical to the risk management of life insurance companies and are of great concerns to actuaries, insurance regulators, and other stakeholders. Our goal is to simulate the distribution of L.

In the following, we specify that $N = 100{,}000$, $T = 20$, $F = 1000$, $a = 30$, the discount rate used to calculate P and ${}_{x-a+1}V_a$ is 6%,[10] and $q_x^{(\mathrm{m})}$ is distributed as the 1980 CSO male mortality table. Here P is $ 27.133 according to the equivalence principle. We do not consider dividends, expenses, loadings, taxes, or new business in the simulations.

3.2. Mortality risk

The focus in this subsection is the risk arising from random survivorship exclusively. More specifically, we assume that the market interest rate is fixed at 6% and there are no early surrenders. In addition, we assume that $D_x^{(\mathrm{m})}$ has a binomial distribution with parameters $(C^{(\tau)}(x), q_x^{(\mathrm{m})})$, which is justifiable if the deaths among policyholders are mutually independent. We simulate 10,000 samples of $D_x^{(\mathrm{m})}$ for $30 \leq x < 50$ to obtain the distribution of L. The distribution is shown in Fig. 2.

It is interesting to notice that the mortality risk is actually trivial according to our simulations. The expected value of the distribution is close to zero and the standard deviation is only about 1% of the annual premiums. The 95th percentile of this distribution (denoted as VaR (95%)) is less than 2% of annual premiums, which implies that the insurer could keep the premium-surplus ratio as high as 50 for an insolvency probability of 5%. The insignificance of the mortality risk is mainly due to the assumption of independence among policyholders' deaths and the large pool size. We experiment with different pool sizes and confirm that a pool with n times of the policy number (and thus n times of premium income) has a standard deviation about $\sqrt{n}$ times. Since the premium income increases faster than the standard deviation as the pool size increases, the risk measured by the ratio of the standard deviation to premium income decreases with the pool size. When the pool is sufficiently large, the risk of the pool relative to the premiums diminishes.

[10] Six percent is about the average for the sampled interest rates and the average of the interest rates simulated later in this study.

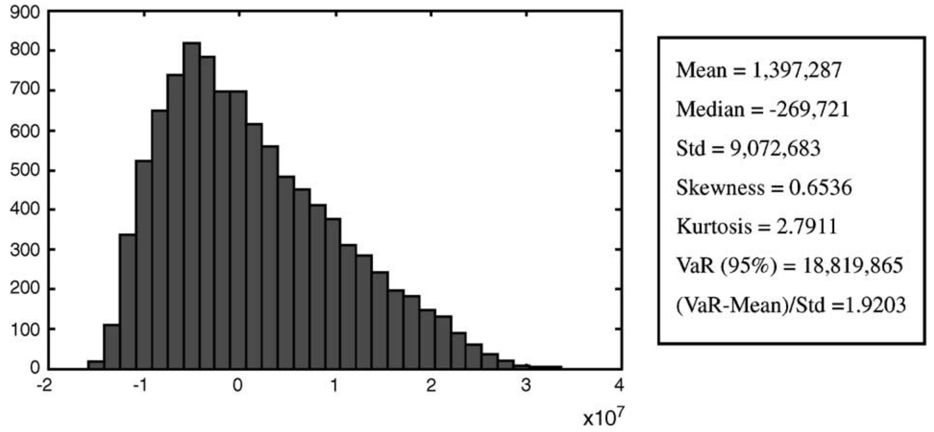

Fig. 3. One decrement (mortality) with stochastic interest rate.

3.3. Interest rate risk

In this section we include another risk factor: the interest rate. The fundamental problem with the stochastic interest rate lies in the fact that, as opposed to the mortality risk, it is not possible to diversify the interest rate risk by selling a large number of policies because every policy is subject to the same or highly correlated interest rates. The interest rate risk is therefore expected to be more imperative than the mortality risk.

We utilize the estimated error-correction model to simulate 10,000 sample paths for the 1-year *T*-bill rate for 20 years.[11] Combining these interest rate paths with the previous simulated $D_x^{(m)}$, we obtain the distribution of L under the consideration of stochastic interest rate as well as random survivorship. The results are depicted in Fig. 3.

As we can see from Fig. 3, the interest rate risk is momentous. The mean reserve that is supposed to be zero and is indeed close to zero with the presence of the mortality risk now increases to $ 1,397,287 that is about 50% of the annual premiums. The mean is positive due to the convexity of the present value function with respect to the interest rate. In particular, the decrease in the present value due to an increase in the interest rate is smaller than the increase in the present value for an equivalent amount of decrease in the interest rate.[12] The enormous figure for the mean reserve results from the long-term nature of life insurance policies that aggravates the convexity effect. The positive and large mean implies severe under-pricing of insurance policies. Life insurance policies sold within a stochastic interest rate environment but priced based on a deterministic interest rate would therefore result in serious under-estimation for the contract value.

Two risk measures also indicate the severity of the interest rate risk. The $ 9,072,683 standard error is more than three times of the annual premiums. The 95th percentile of the distribution is almost seven times of the annual premiums, which means that the insurer must keep tremendous amount of surplus to maintain an acceptable level of solvency probability. In sum, the large mean, the large standard error, and the large 95th percentile of the policy reserve distribution all suggest that the interest rate risk is substantial.

---

[11] When simulating the interest rate, we set the coefficients associated with the lapse rate in the interest rate equation to zeros because these coefficients are not significantly different from zero. For the lagged variables, we assume that the market interest rates at time $t-1$ and $t-2$ are 6% that is equal to the policy credit rate.

[12] We check that the positive mean is not from declines in the average interest rate. The mean of the simulated interest rates indeed rises slightly from 6 to 6.3%. The small increase results from the non-negative interest rate constraint imposed on the simulations.

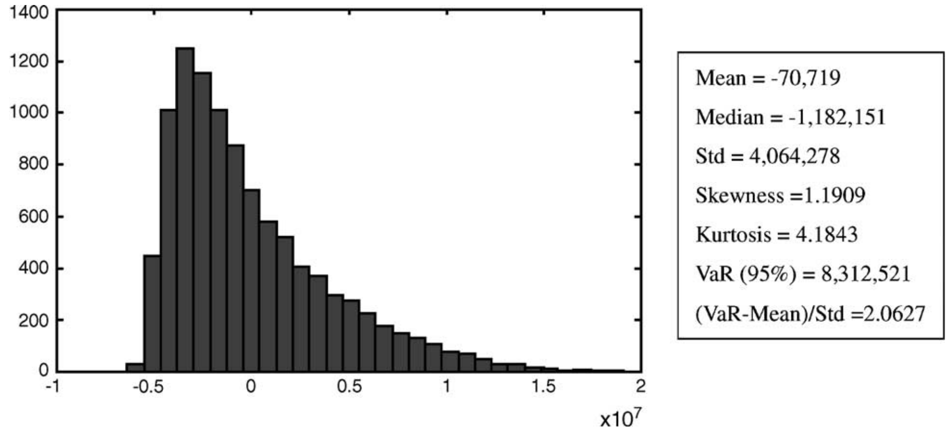

Fig. 4. Two decrements (mortality and surrender) with stochastic interest rate.

3.4. *Early surrender*

To evaluate the impact of early surrender, we first simulate 10,000 sample paths for the lapse rate based on our empirical error-correction model.[13] The simulated lapse rates along with the simulated mortality rates are then used to simulate 10,000 samples of $D_x^{(\mathrm{m})}$ and $D_x^{(\mathrm{l})}$ for $30 \leq x < 50$ under the assumption that both $D_x^{(\mathrm{m})}$ and $D_x^{(\mathrm{l})}$ are binomially distributed with parameters $(L^{(\tau)}(x), q_x^{(\mathrm{m})})$ and $(L^{(\tau)}(x) - D_x^{(\mathrm{m})}, q_x^{(\mathrm{l})})$, respectively.[14] Combining the resulting cash flows with the 10,000 simulated interest rate paths, we obtain the distribution of L under the consideration of random survivorship, stochastic interest rate, and interest-rate-sensitive surrender. The simulation results are shown in Fig. 4.

Fig. 4 indicates that the surrender option actually benefits life insurers. The expected value of reserves turns from positive to negative and the standard deviation as well as the 95th percentile decreases significantly. The mean reserve decreases to \$ −70,719 from the previous \$ 1,397,287. An obvious reason for the decrease is the surrender charge. If we assume that the surrender charge is zero,[15] the mean reserve bounces back to \$ 960,068 as shown in Fig. 5.

The mean in Fig. 5 is still smaller than that in Fig. 3. The drop in the mean results from surrenders occurring during low interest rate periods. The policyholders who choose to surrender their policies when the market interest rate is low relinquish the valuable fixed credit rate provision. These surrenders benefit insurers. Although surrenders that occur during high interest rate periods impair the insurer's profits, the convexity of the present value function with respect to the interest rate makes the losses smaller than the gains. Comparing Fig. 5 with Fig. 3, we observe that the shrinkage in the right tail due to surrenders occurring in low interest rate periods is greater than the shrinkage in the left tail resulting from surrenders occurring in high interest rate periods. The net effect of surrenders is thus a decrease in the mean of policy reserves.

The decrease in the mean reserve implies a negative value for a pool of surrender options, which has not been documented in previous studies such as Albizzati and Geman (1994) and Grosen and Jorgensen (1997, 2000) among others. Previous studies typically assumed that policyholders optimally exercise their surrender options in

---

[13] We assume that the lapse rates at time $t - 1$ and $t - 2$ are 8%, the average lapse rate during the sample period.

[14] The assumption that $D_x^{(\mathrm{l})}$ has a binomial distribution is equivalent to assuming that surrender decisions among policyholders are mutually independent given the market interest rate. The reason why the distribution parameter for $D_x^{(\mathrm{l})}$ is $L^{(\tau)}(x) - D_x^{(\mathrm{m})}$ instead of $L^{(\tau)}(x)$ is that policyholders who are able to surrender their policies during the age interval of x and $x + 1$ are assumed to be those who survive to age $x + 1$.

[15] We make this assumption in all the following analyses to focus on the impact of early surrender itself.

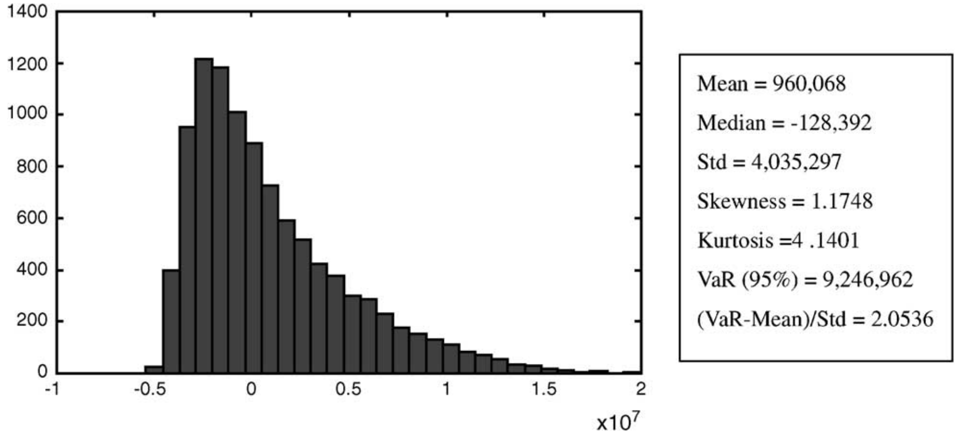

Fig. 5. Two decrements (mortality and surrender) with stochastic interest rate and zero surrender charge.

accordance with changes in the interest rate. When the market interest rate is lower than the policy credit rate, policyholders are not supposed to exercise their surrender options because the fixed credit rate provision is valuable. When the interest rate is higher than the credit rate, policyholders should surrender their policies to take advantage of the higher-yield alternatives in the financial markets. However, history demonstrates different surrender behavior. There are always some policyholders surrendering their policies regardless how low the market interest rate is. For instance, the 1-year *T*-bill rate was very low during the early 1960s but the lapse rate was over 5%. Even in Japan, where recent interest rates were extremely low, more than 10% of policies lapsed in 1997–1999.[16] Conversely, only a small portion of policyholders surrender their policies when the market interest rate is high. For example, the lapse rate was lower than 12% during the extraordinarily high interest rate period of the early 1980s. Hence, previous studies that assumed optimal exercises on surrender options in accordance with the interest rate would over-estimate the (aggregate) value of surrender options.[17]

To test the robustness of our findings, we experiment with alternative surrender patterns. In the first experiment, we presume that no policyholders would surrender their policies when the market interest rate is lower than the 6% policy credit rate. The mean reserve increases tremendously to $ 2,557,555, as shown in Fig. 6, and is much larger than the mean reserve of $ 1,397,287 when surrender is not considered. In other words, if we exclude the possibility of surrendering during low interest rate periods, the aggregate value of the surrender options would be about 43% of the annual premiums, which is consistent with the literature. We hence confirm that the negative value for a pool of surrender options originates from policyholders' exercising surrender options at times of low interest rate.

In the second experiment, we assume that the lapse rate is independent of the interest rate with a normal distribution with the mean and standard deviation estimated from sampled lapse rates. The results are shown in Fig. 7. The mean reserve decreases from $ 1,397,287 in Fig. 3 to $ 483,936, meaning that the aggregate value of the surrender options is $ −913,351. The surrender option has the largest negative value among the analyzed cases because the proportion of policyholders who surrender their policies when the interest rate is low/high is the greatest/smallest in this case. The results from the second experiment are consistent with our reasoning for the negative surrender option value.

The third experiment simulates the policy reserve distribution under the assumption that the lapse rate has a normal distribution, as in the second experiment, but the interest rate is constant. This case corresponds to the multiple decrement models in Bowers et al. (1986, 1997). The results shown in Fig. 8 illustrate that early surrender does

[16] The number is estimated based on the data from the Life Insurance Business in Japan (http://www.seiho.or.jp/english/index.html).
[17] Although Albizzati and Geman (1994) set boundaries for the lapse rate to account for non-economic surrenders and non-surrenders, the assumed range for the lapse rate (3–60%) is much wider than that actually observed (5.0–11.6%).

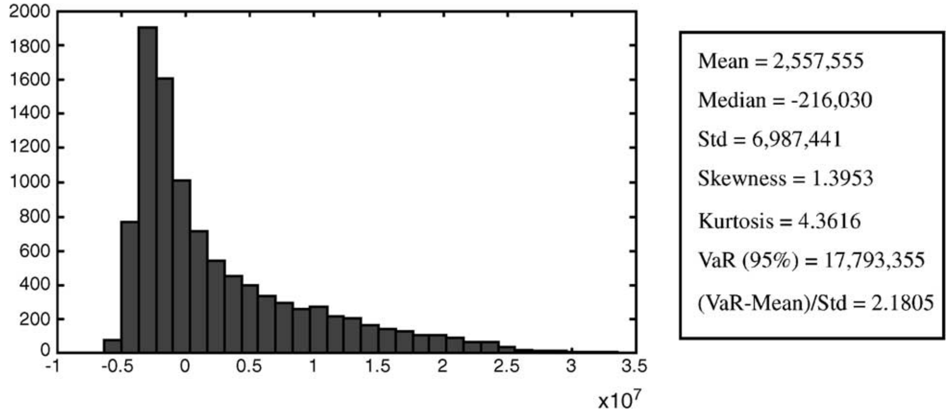

Fig. 6. Two decrements (mortality and surrender) with stochastic interest rate, zero surrender charge, and "truncated" surrenders.

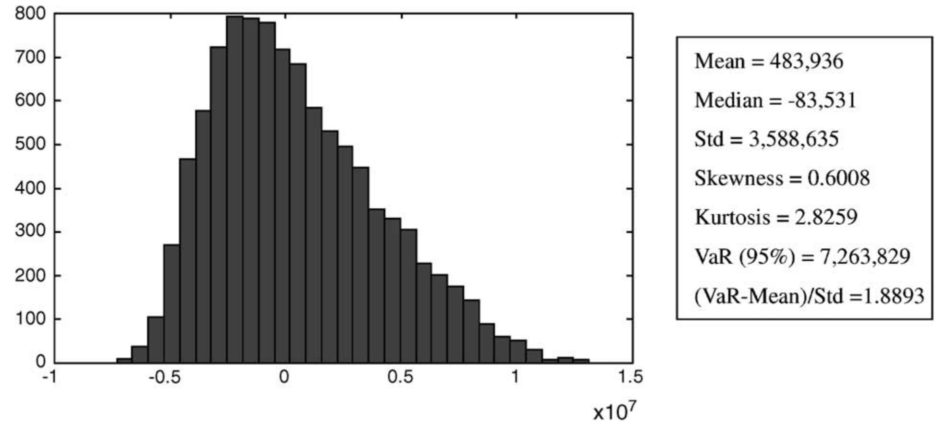

Fig. 7. Two decrements (mortality and surrender) with stochastic interest rate, zero surrender charge, and independent surrenders.

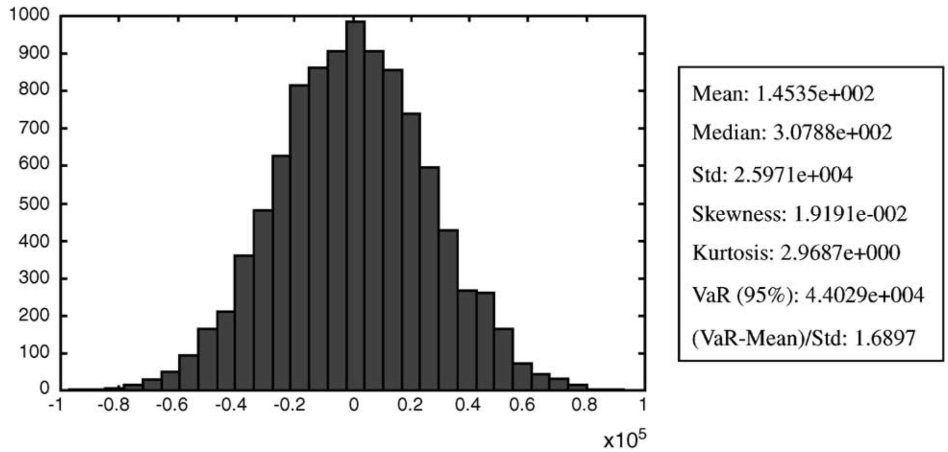

Fig. 8. Two decrements (mortality and surrender) with constant interest rate, zero surrender charge, and independent surrenders.

Table 4
Summarized results

	No surrenders (Fig. 3)	VAR surrenders (Fig. 5)	Truncated surrenders (Fig. 6)	Independent surrenders (Fig. 7)
Mean	1,397,287	960,068	2,557,555	483,936
Standard deviation	9,072,683	4,035,297	6,987,441	3,588,635
VaR (95%)	18,819,865	9,246,962	17,793,355	7,263,829

not matter if the interest rate remains constant. All statistics in Fig. 8 are close to those in Fig. 2. Early surrender in a constant interest rate environment acts like an additional cause for termination to death and introduces an inconsequential risk to insurers. The effect of early surrender therefore hinges on the randomness of the interest rate. The surrender option is worthless if the interest rate is fixed. The consistent results from these three experiments ensure the robustness of our simulations.

In addition to reducing the expected value of policy reserves, early surrender also acts to mitigate the policy reserve risk. Specifically, the standard deviation drops more than 50% from $ 9,072,683 in Fig. 3 to $ 4,035,297 in Fig. 5. So does the 95th percentile of the distribution. Early surrender reduces the reserve risk because it makes the losses and gains to insurers resulting from variations in the interest rate smaller. The surrender options exercised under high interest rate conditions reduce the insurer's profits and those exercised during low interest rate periods mitigate the insurer's losses. When surrenders in low interest rate periods are assumed away, the risk-reducing effect of early surrender becomes weaker, as shown in Fig. 6. If policyholders surrender their policies independently, the risk-reducing effect of early surrender turns out to be more significant. As shown in Fig. 7, the standard deviation of policy reserve distribution is reduced to be $ 3,588,635. According to these results, we conclude that early surrender moderates the interest rate risk of policy reserves, which is in line with the findings on the decreased effective duration of policy reserves in Babbel (1995), Briys and de Varenne (1997), and Santomero and Babbel (1997). The results in this section are summarized in Table 4.

3.5. *Alternative assumptions on the policy credit rate and initial values*

We impose various assumptions on the policy credit rate and the initial values for the interest rate and lapse rate in this section to conduct further robustness checks on our results.[18] Figs. 9–11 display the mean reserves simulated with different initial interest rates and lapse rates for policies with credit rates of 6, 10, and 3%.[19] For instance, the first left point in Fig. 9 is the mean reserve for a policy issued with a credit rate of 6% when the market interest rate and lapse rate are set to zero. These figures show that higher initial values for the market interest rate result in lower mean reserves, which is reasonable according to the present value nature of reserves.

Figs. 9–11 suggest that the effect of the initial lapse rate, or more generally the effect of the lapse rate, depends on the difference between the policy credit rate and market interest rate.[20] Early surrender decreases/increases the mean reserve when the credit rate is higher/lower than the market interest rate. When the credit rate is lower than the market interest rate, the mean reserves tend to be negative. Negative mean reserves imply that the insurer expects to profit from these policies. Early surrenders would make the insurer earn less because policyholders leave the pool sooner and thus increase mean reserves. On the other hand, mean reserves are usually positive when the credit

[18] The initial values could be important in terms of their enduring impacts because of the partial adjustment to the long-term level in our cointegration system.

[19] Here we adopt the deterministic survivorship group concept rather than the probabilistic interpretation of decrement rates to reduce computation burden. Specifically, we assume that the number of people leaving the pool during the age interval of x and $x+1$ is equal to $L^{(\tau)}(x)(q_x^{(\mathrm{m})}+q_x^{(\mathrm{l})})$. The deterministic interpretation of the decrement rates leads to minor differences in the previous analyses.

[20] Since the average lapse rate and the initial lapse rate are positively correlated due to the gradual adjustment in the lapse rate in our cointegration system, we interpret the effect of the initial lapse rate broadly as the lapse rate effect.

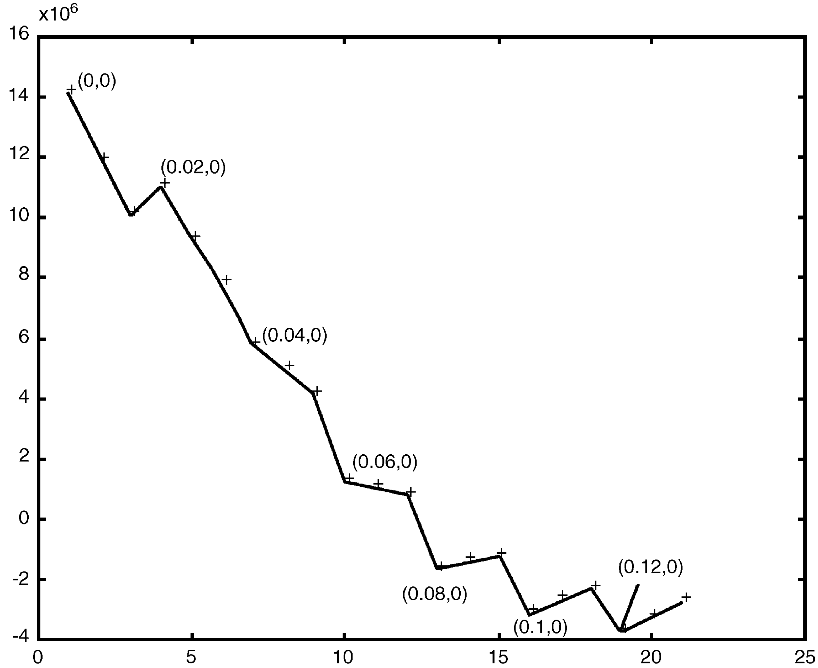

Fig. 9. Mean reserves when the credit rate is 6%, market interest rate ranges from 0 to 12%, and initial lapse rate is 0, 8 or 16% (the first coordinate in the parenthesis represents the initial market interest rate and the second the initial lapse rate).

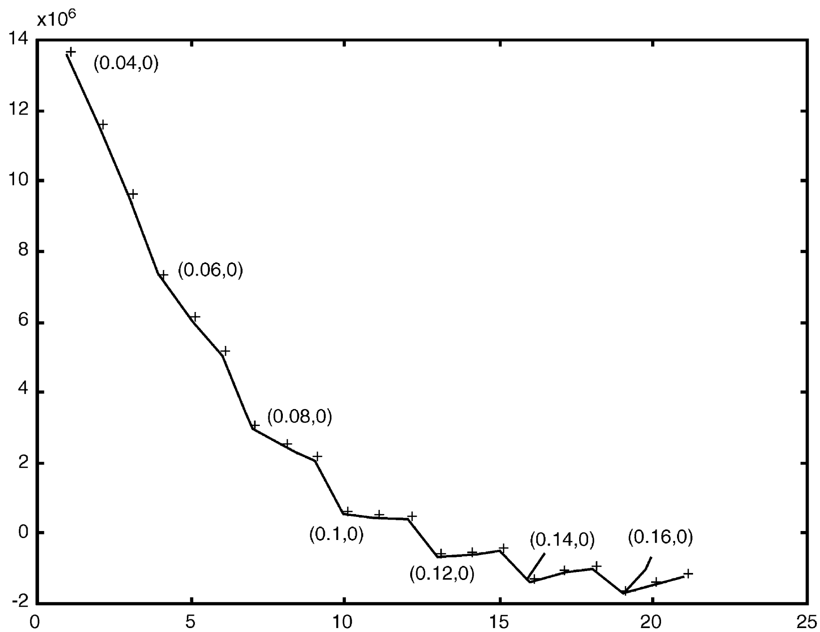

Fig. 10. Mean reserves when the credit rate is 10%, market interest rate ranges from 4 to 16%, and initial lapse rate is 0, 8 or 16% (the first coordinate in the parenthesis represents the initial market interest rate and the second the initial lapse rate).

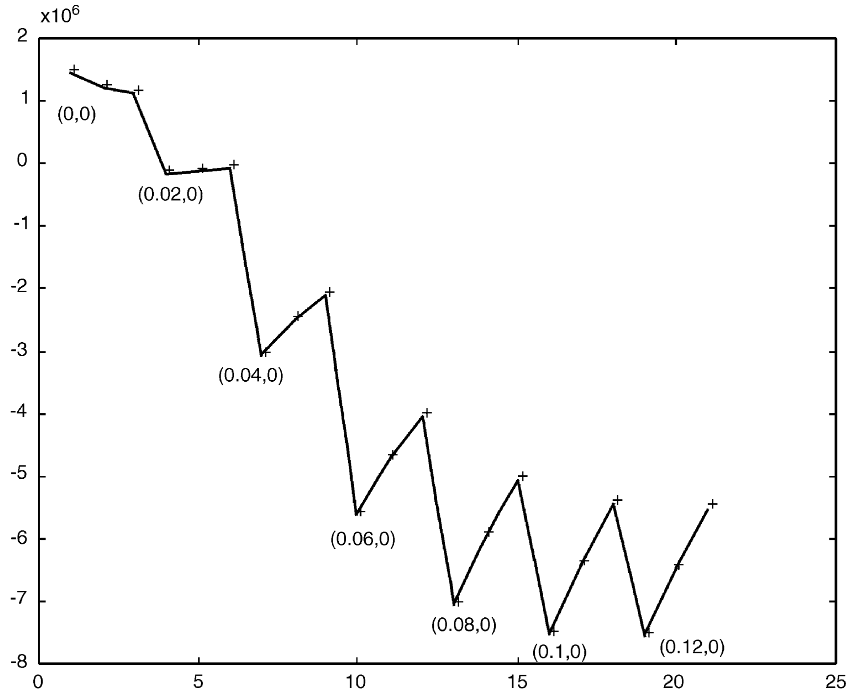

Fig. 11. Mean reserves when the credit rate is 3%, market interest rate ranges from 0 to 12%, and initial lapse rate is 0, 8 or 16% (the first coordinate in the parenthesis represents the initial market interest rate and the second the initial lapse rate).

rate is higher than the market interest rate. Positive mean reserves represent negative expected net present values. Early surrenders partially release the insurer from his high credit rate promise and consequently decrease the mean reserves. This new finding is consistent with our previous results that the surrender option benefits the insurer by decreasing the positive mean reserve.

Figs. 9–11 also demonstrate that the impact of early surrenders on mean reserves increases with the size of the difference between the policy credit rate and market interest rate. For instance, the two left-top points in Figs. 9 and 10 suggest that early surrender dramatically decreases the mean reserves when the credit rate is 6% higher than the market interest rate. The right-bottom part of Fig. 11 demonstrates the significant boost effect of early surrender on the mean reserves when the credit rate is much lower than the market interest rate. Conversely, the middle part of Fig. 10 illustrates that the mean reserves change little in accordance with early surrender when the credit rate is close to the market interest rate. Therefore, the impact of the surrender option on mean reserves depends on both the sign and magnitude of the difference between the policy credit rate and the market interest rate.

4. Conclusions

Risk management of policy reserves is essential to the solvency of life insurers. To portray the risk profile of policy reserves, one needs to model the relevant cash flow and discount rate stochastically. Scholars have developed reserving techniques in a stochastic mortality and interest rate environment. We contribute to the literature by incorporating early surrender into the estimation for the policy reserve distribution. This is important in the sense that the surrender option could be valuable and significantly change the policy sensitivity to the interest rate.

We first establish an empirical lapse rate model. The cointegration technique is used to identify a long-term relation between the interest rate and lapse rate. Based on the estimated error-correction model, we perform Monte Carlo simulations considering mortality, the interest rate risk, and early surrender on a pool of level-premium endowment policies with a fixed policy credit rate. We find that the surrender option would decrease/increase the mean reserve when the mean reserve is positive/negative due to surrenders during low/high interest rate periods. The effects increase with the difference between the policy credit rate and market interest rate. We also find that the surrender option actually mitigates the interest rate risk of the life insurance policy, which is consistent with the decreased effective duration documented in the literature.

Our findings suggest that early surrender has a significant impact on the values and reserves of life insurance policies. Life insurers should incorporate surrender behaviors into policy pricing and reserving. Our findings also imply that minimizing the lapse rate might not be optimal because early surrender could benefit insurers in reducing the value and risk of reserves, which is different from the conventional view. The legitimacy of pursuing high persistent rates may need to be reconsidered. Finally, actuaries should adopt stochastic interest rates in pricing and reserving. Traditional pricing or reserving methods that assume a deterministic interest rate would result in serous under-estimation errors when applied in a random interest rate environment.

For researchers, a direct extension of our study involves incorporating the term structures of the lapse rate and expense. Younger policies tend to have a higher propensity to lapse and the cost of procuring, underwriting, and issuing new business all incur at the beginning. The combination of these two features might significantly reduce the beneficial effect of early surrender found in this study. Furthermore, models including more macro or even micro factors to better explain surrender behaviors need to be established. Why do some policyholders surrender their policies without regard to the interest rate level? Is there a “natural” lapse rate due to certain frictions in insurance transactions? How would concern of policyholders for the solvency of the insurer affect surrender behavior? Will we observe a “run on the insurer” in the future? How are policyholders deterred from surrendering their policies at high interest rate levels? Relevant research issues are abundant.

Acknowledgements

The authors are grateful to Tse-Chien Hsu and Chien-Hsiu Lin for their competent programming assistance, to Khari Cook and Michele Tibbs at the American Council of Life Insurance for the lapse rate data before 1965, and to the National Science Council of Taiwan for its financial support (project number NSC 89-2416-H-004-069).

References

ACLI (American Council of Life Insurance), 1997. Life Insurance Fact Book. ACLI, Washington, DC.

Albizzati, M., Geman, H., 1994. Interest rate risk management and valuation of the surrender option in life insurance policies. Journal of Risk and Insurance 61, 616–637.

Babbel, D.F., 1995. Asset-liability matching in the life insurance industry. In: Altman, E.I., Vanderhoof, I.T. (Eds.), The Financial Dynamics of the Insurance Industry. IRWIN Professional Publishing, New York.

Beekman, J.A., Fuelling, C.P., 1990. Interest and mortality randomness in some annuities. Insurance: Mathematics and Economics 9, 185–196.

Beekman, J.A., Fuelling, C.P., 1991. Extra randomness in certain annuity models. Insurance: Mathematics and Economics 10, 275–287.

Beekman, J.A., Fuelling, C.P., 1993. One approach to dual randomness in life insurance. Scandinavian Actuarial Journal 2, 173–182.

Bellhouse, D.R., Panjer, H.H., 1981. Stochastic modeling of interest rates with applications to life contingencies. Part II. Journal of Risk and Insurance 48, 628–637.

Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A., Nesbitt, C.J., 1986. Actuarial Mathematics. Society of Actuaries, Itasca, IL.

Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A., Nesbitt, C.J., 1997. Actuarial Mathematics, 12th ed. Society of Actuaries, Schaumburg, IL.

Briys, E., de Varenne, F., 1997. On the risk of insurance liabilities: debunking some common pitfalls. Journal of Risk and Insurance 64, 673–694.

Cox, S.H., Laporte, P.D., Linney, S.R., Lombardi, L., 1992. Single-premium deferred annuity persistency study. Transactions of Society of Actuaries Reports, pp. 281–332.

De Schepper, A., Goovaerts, M., 1992. Some further results on annuities certain with random interest. Insurance: Mathematics and Economics 11, 283–290.

Engle, R.F., Granger, C.W.J., 1987. Co-integration and error correction representation, estimation and testing. Econometrica 55, 251–276.

Frees, E.W., 1990. Stochastic life contingencies with solvency considerations. Transaction of the Society of Actuaries 42, 91–129.

Giaccotto, C., 1986. Stochastic modeling of interest rates: actuarial vs. equilibrium approach. Journal of Risk and Insurance 53, 435–453.

Grosen, A., Jorgensen, P.L., 1997. Valuation of early exercisable interest rate guarantees. Journal of Risk and Insurance 64, 481–503.

Grosen, A., Jorgensen, P.L., 2000. Fair valuation of life insurance liabilities: the impact of interest guarantees, surrender options, and bonus policies. Insurance: Mathematics and Economics 26, 37–57.

Hamilton, J.D., 1994. Time Series Analysis. Princeton University Press, Princeton, NJ.

Johansen, S., 1991. Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. Econometrica 59, 1551–1580.

Jordan, C.W., 1967. Life Contingencies. Society of Actuaries, Chicago.

MacKinnon, J.G., 1991. Critical values for cointegration tests. In: Engle, R.F., Granger, C.W.J. (Eds.), Long-run Economic Relationships: Readings in Cointegration. Oxford University Press, New York.

Marceau, E., Gaillardetz, P., 1999. On life insurance reserves in a stochastic mortality and interest rates environment. Insurance: Mathematics and Economics 25, 261–280.

Outreville, J.F., 1990. Whole-life insurance lapse rates and the emergency fund hypothesis. Insurance: Mathematics and Economics 9, 249–255.

Panjer, H.H., Bellhouse, D.R., 1980. Stochastic modeling of interest rates with applications to life contingencies. Journal of Risk and Insurance 47, 91–110.

Parker, G., 1994a. Moments of the present value of a portfolio of policies. Scandinavian Actuarial Journal 1, 53–67.

Parker, G., 1994b. Stochastic analysis of a portfolio of endowment insurance policies. Scandinavian Actuarial Journal 2, 119–130.

Parker, G., 1994c. Limiting distribution of the present value of a portfolio. ASTIN Bulletin 24, 47–60.

Parker, G., 1996. A portfolio of endowment policies and its limiting distribution. ASTIN Bulletin 26, 25–33.

Parker, G., 1997. Stochastic analysis of the interaction between investment and insurance risks. North American Actuarial Journal 1, 55–84.

Santomero, A.M., Babbel, D.F., 1997. Financial risk management by insurers: an analysis of the process. Journal of Risk and Insurance 64, 231–270.