1 Introduction

The Calligraphy Equation

Neglecting the friction force of the paper on which a calligrapher produces his work through a handwriting brush with mass $m(t)$ at time t, we suppose that $u(t)$ be the displacement of the motion of the brush at time t, then from the Newton's second law of motion we know that the force $F(t)$ at time t can be measured by $(m(t)u'(t))'$, thus we obtain the equation

$$(m(t)u'(t))' = F(t). \tag{1.1}$$

Normally, the force $F(t)$ depends on $u(t)$ and $u'(t)$, that is $F(t) = F(u(t), u'(t))$. Experimentally, for some people the change rate of the force is proportional to the change rate of velocity in a motion, that is, there is a positive real q so that

$$\frac{dF(u(t))}{dt} \diagup F(u(t)) = q\frac{du'(t)}{dt} \diagup u'(t). \tag{1.2}$$

By some computations we find that $F(u(t)) = cu'(t)^q$ for some constant c, is one of forms of the force and thus the equation (1.1) becomes

$$(m(t)u'(t))' = cu'(t)^q. \tag{1.3}$$

Where q is called the temper-index of the equation (1.3). In this paper we consider the mass of that brushes as fixed, in another word, $m(t) = m$ for some fixed finite number m. Consider a calligrapher with temper-index q creates his work who is disturbed by a person with the same temper-index as well as characteristic p; that is, the disturbed force from that person is $c_2(u')^q u^p$, then the equation (1.1) can be translated to the form as follows:

$$\begin{cases} u'' = (u')^q(c_1 + c_2u^p), \\ u(0) = u_0,\ u'(0) = u_1. \end{cases} \tag{1.4}$$

We are interested in properties of solutions of the problem, particularly in phenomena on blow-up, blow-up rates, blow-up constants and life-spans. In next section,

[1] [2] is a paper which researched in blow-up character of solution of the equation $(\mid u' \mid^{m-2} u')' = u^p$.

we separate q into three parts, $1 \leq q < 2$, $q = 2$ and $q > 2$. And we find the blow-up time, blow-up rate and blow-up constant of u. For further informations on calligraphy equation we refer the reader to [1].