1 Introduction

1.1 Introduction

We know three classical questions described in Section 1.2 had been solved by some methods such as Recurrence Relation[1], Observation and Induction[2], Difference Equation[3], Euler's Formula[4], etc.. In this article, we will use another way to resolve the questions by giving a combinatorial argument and trying to generalize its formula to higher dimensional space.

In Section 2, we show how recurrence relation solves these questions by adjusting a small part of its original solution with a useful equation[5]. Then, we will get an idea of the general question of higher dimension.

In Section 3, we generalize the three classical question into n-dimensional space. Then we generalize the properties of lower dimensional spaces by a system of equations called Standard Partition System of n-dimensional Space for this general question. This system tells us the form of recurrence relation in Section 2 is suitable for the general question and shows a more directly way to infer the structure of regions in Section 4.4. At the end of this section, we use the same form of recurrence relation as Section 2 to solve the general question.

In Section 4, we provide a Combinatorial Argument by Algorithm which labels each region once by levels. And the same form of these Algorithms provides a way to label each region in the higher dimensional spaces. Hence, collecting all the labels we will get the same formula as in Section 3. At last, we offer a list of all the numbers which is easily obtained from the recurrence relation.

In Section 5 and Section 6, we divide the formula into two parts -bounded regions and unbounded regions- which are also in the sense of maximizing the number of region.

1.2 Description of Three Original Questions

We describe three original questions from lower dimension to higher dimension as follows:

Question 1: Partition Line with k Points

If there are k different points in an 1-dimensional line, then what is the largest number of the different regions which are formed by these k different points?

Question 2: Partition Plane with k Lines

If there are k different 1-dimensional lines in an 2-dimensional plane, then what is the largest number of the different regions which are formed by these k different lines?

Question 3: Partition Space with k Planes

If there are k different 2-dimensional planes in an 3-dimensional space, then what is the largest number of the different regions which are formed by these k different planes?