1 Introduction

The theory of factorization is a study of ways in which a given function can be written as a composition of other functions. Usually, the given functions are meromorphic functions or entire functions, including the polynomials, rational functions, elliptic functions, $\cdots$, etc. In this respect, Nevanlinna's theory plays an important role, especially in the growth and the value distribution of meromorphic functions and entire functions. In this thesis, we are mainly interested in the factorization of rational functions and the Weierstrass $\wp$-function, which is a kind of elliptic functions.

We will study some factorizations of several interesting functions. In fact, we are concerned in prime functions (i.e., for any factorization $f \circ g(z)$, one of $f(z)$ and $g(z)$ must be bilinear) or pseudo-prime functions (i.e., for any factorization $f \circ g(z)$, either $f(z)$ is rational or $g(z)$ is a polynomial). Particular results of this type are the following: (i) All polynomials are pseudo-prime, in particular, all polynomials of prime degree are prime. (ii) All periodic entire functions of exponential type are pseudo prime. (iii) $e^z + p(z)$, where $p(z)$ is a nonconstant polynomial, is prime. Of course, we only mention some of them in the literature, for more details, one can find in [1, 2].

There are three sections in this thesis. In section 1, we give a brief introduction. In section 2, we study, in detail, the factorization of rational functions. In particular, we prove a new class of rational functions which are prime composite, this is the main work of this thesis. In section 3, we discuss the factorization of the Weierstrass $\wp$-function, including an example, which says that there exist a rational function which has two factorizations into prime functions, each having a different number of factors.