Appendix C

Proof of Theorem 5.3.6 :

From (5.13) and (5.14), we can easily derive

$$\mathbf{u}_0^{(\alpha)}\ \mathbf{a}(\omega_\alpha) = \mathbf{u}_1^{(\alpha)} + x_\alpha \omega_\alpha \mathbf{u}_0^{(\alpha)}, \tag{C.1}$$

$$\mathbf{v}_0^{(\alpha)}\ \mathbf{b}(\omega_\alpha) = \mathbf{v}_1^{(\alpha)} - x_\alpha \omega_\alpha \mathbf{v}_0^{(\alpha)}, \tag{C.2}$$

$$\mathbf{u}_0^{(\alpha)}\ \mathbf{a}'(\omega_\alpha) = \frac{1}{\omega_\alpha}\mathbf{u}_1^{(\alpha)} + x_\alpha \mathbf{u}_0^{(\alpha)},$$

$$\mathbf{v}_0^{(\alpha)}\ \mathbf{b}'(\omega_\alpha) = \frac{1}{\omega_\alpha}\mathbf{v}_1^{(\alpha)} - x_\alpha \mathbf{v}_0^{(\alpha)}.$$

From (5.56), (5.57) and $\boldsymbol{\beta}_1(\mathbf{S}_1 - x_\alpha \mathbf{I}_1)^{-3}\boldsymbol{\gamma}_1 = 0$, we have

$$\mathbf{u}_{-1}^{(\alpha)}(\mathbf{a}(\omega_\alpha) - x_\alpha \omega_\alpha \mathbf{I}_1) = \mathbf{u}_0^{(\alpha)}, \tag{C.3}$$

and

$$\mathbf{v}_{-1}^{(\alpha)}(\mathbf{b}(\omega_\alpha) + x_\alpha \omega_\alpha \mathbf{I}_2) = \mathbf{v}_0^{(\alpha)}. \tag{C.4}$$

Then (5.55) follows by (C.1), (C.2), (C.3), and (C.4).

Use a similar approach in the proof of Theorem 5.3.1, we can prove that $\mathbf{u}_1^{(\alpha)} \otimes \mathbf{v}_1^{(\alpha)}$, $\mathbf{u}_0^{(\alpha)} \otimes \mathbf{v}_1^{(\alpha)} - \mathbf{u}_1^{(\alpha)} \otimes \mathbf{v}_0^{(\alpha)}$, and $\mathbf{u}_{-1}^{(\alpha)} \otimes \mathbf{v}_1^{(\alpha)} - \mathbf{u}_0^{(\alpha)} \otimes \mathbf{v}_0^{(\alpha)} + \mathbf{u}_1^{(\alpha)} \otimes \mathbf{v}_{-1}^{(\alpha)}$ are linearly independent. $\square$