Appendix D

Proof of Theorem 5.3.7 :

We only need to show

$$\boldsymbol{\varphi}_{21}^{(\alpha)}\mathbf{Q}'(\omega_\alpha) + \boldsymbol{\varphi}_{22}^{(\alpha)}\mathbf{Q}(\omega_\alpha) = \mathbf{0}.$$

From (5.56) and (5.57), we have

$$\mathbf{u}_{-1}^{(\alpha)}(\mathbf{a}(\omega_\alpha) - x_\alpha\omega_\alpha\mathbf{I}_1) = \mathbf{u}_0^{(\alpha)} + \frac{a_1\boldsymbol{\beta}_1(\mathbf{S}_1 - x_\alpha\mathbf{I}_1)^{-3}\boldsymbol{\gamma}_1}{\omega_\alpha^2}\boldsymbol{\beta}_1,$$

and

$$\mathbf{v}_{-1}^{(\alpha)}(\mathbf{b}(\omega_\alpha) + x_\alpha\omega_\alpha\mathbf{I}_2) = \mathbf{v}_0^{(\alpha)} + a_2\boldsymbol{\beta}_2(\mathbf{S}_2 + x_\alpha\mathbf{I}_2)^{-3}\boldsymbol{\gamma}_2\boldsymbol{\beta}_2.$$

Since $\boldsymbol{\beta}_1(\mathbf{S}_1 - x_\alpha\mathbf{I}_1)^{-2}\boldsymbol{\gamma}_1 = 0$, we obtain

$$\frac{\boldsymbol{\beta}_1(\mathbf{S}_1 - x_\alpha\mathbf{I}_1)^{-3}\boldsymbol{\gamma}_1}{-\omega_\alpha^2} = \boldsymbol{\beta}_2(\mathbf{S}_2 + x_\alpha\mathbf{I}_2)^{-3}\boldsymbol{\gamma}_2$$

from (5.42). Then we can derive

$$\boldsymbol{\varphi}_{22}^{(\alpha)}\mathbf{Q}(\omega_\alpha) = \frac{a_1\boldsymbol{\beta}_1(\mathbf{S}_1 - x_\alpha\mathbf{I}_1)^{-3}\boldsymbol{\gamma}_1}{\omega_\alpha^2}(\boldsymbol{\beta}_1 \otimes \mathbf{v}_1^{(\alpha)}) + a_2\boldsymbol{\beta}_2(\mathbf{S}_2 + x_\alpha\mathbf{I}_2)^{-3}\boldsymbol{\gamma}_2(\mathbf{u}_1^{(\alpha)} \otimes \boldsymbol{\beta}_2).$$

and

$$\boldsymbol{\varphi}_{21}^{(\alpha)}\mathbf{Q}'(\omega_\alpha) = -\frac{a_1\boldsymbol{\beta}_1(\mathbf{S}_1 - x_\alpha\mathbf{I}_1)^{-3}\boldsymbol{\gamma}_1}{\omega_\alpha^2}(\boldsymbol{\beta}_1 \otimes \mathbf{v}_1^{(\alpha)}) - a_2\boldsymbol{\beta}_2(\mathbf{S}_2 + x_\alpha\mathbf{I}_2)^{-3}\boldsymbol{\gamma}_2(\mathbf{u}_1^{(\alpha)} \otimes \boldsymbol{\beta}_2).$$

Hence, the theorem follows. $\square$