

1 Introduction

Sale intensity is always one of the major subjects that business is concerned about. Undoubtedly, there are various kinds of factors that affect the changing of sale intensity. By the progressing of technology in mass media, the force of mass communication becomes elemental; therefore the leverage of every type of advertising is highly valued.

Since advertising plays an important role in marketing in the last two decades, there are more and more investigations into marketing focusing on the effect of advertising campaigns on sale intensity. Vidale-Wolfe's mathematical model is a starting point of some practical and theoretical studies in marketing. The model provides a rudimental opinion of the relation between advertising expenditure and sale.

The classical paper of Vidale and Wolfe (1957) [1] introduced a model of result for major industrial concerns on the sale response to advertising. In their model, Vidale and Wolfe observe two main facts concerning the relation between sale and advertising. They believe that the sale intensity decreases in time if no advertising is done. Hence, at first, the sale intensity at time t of a non-promoted product is suggested as

$$S(t) = S(0) \exp(-\kappa t),$$

where $S(0)$ is the initial sale intensity and κ is the sale decay constant. So the

changing rate of sale intensity is

$$S'(t) = -\kappa S(0) \exp(-\kappa t) = -\kappa S(t).$$

In addition, if an adequate advertising effort is done over a time period, then sale intensity increases, but a saturation effort may emerge. Therefore, it is suggested that the sale intensity $S(t)$ might satisfy the equation

$$S'(t) = -\kappa S(t) + r \left(1 - \frac{S(t)}{M}\right) u(t),$$

where $u(t)$ is the expenditure of advertising investment, r , κ and M are parameters. This is a differential equation for the state variable $S(t)$.

At the right hand side, the term $-\kappa S(t)$ represents the spontaneous decay of the state. For the second term, the parameter r is the response constant of the advertising expenditure and M is the saturation threshold, an upper bound to the sale intensity. This claims that the expenditure of advertising investment increase the sale intensity in the rate $r \left(1 - \frac{S(t)}{M}\right)$. Further, when $1 - \frac{S(t)}{M}$ is small and positive, the advertising investment $u(t)$ must be large to sustain the sale intensity.

Here we propose a new mathematical model based on the conception of Vidale-Wolfe's model. Denote S the sale intensity, S_t the derivative of S in time and F_i the i -th factors that have effect on the sale intensity, then we get the equation

$$S_t = \sum F_i.$$

First of all, we shall consider the "spontaneous sale intensity". While the

empirical marketing research pays increasingly growing attention to the multiple-market aspects of the advertising strategy (Bronnenberg and Mahajan 2001, [7] Dube and Machanda 2004 [8]), the model supports for the studies of sale intensity over geographic region has been lacking (see Feichtinger, Hartl and Sethi(1994) [9] for the most comprehensive review of the related literature). The studies of sale intensity over geographic region are especially of significance in spontaneous sale intensity. As we know, the spontaneous sale is caused by the circulating of the information; therefore, we view the “spontaneous sale” as the sale “by word-of-mouth” in practical. By using the sense of diffusion in heat equation, we characterize the behavior of the sale by word-of-mouth. In this way, it will be able to consider both empirical aspect and geographic regions.

On the other hand, because of the elemental leverage of advertising, the effect of advertising is set as our second factor. However, there are different kinds of merchandise and various types of advertising forms, the part of effect of advertising in the model can hardly be general, here we will discuss the effect in some particular situations.