2 Preliminaries and notation

We give a brief account of the heat equation under the considered boundary condition and initial condition.

The problem of heat transfer in a bar of length L is described as fallow. Let $U(x,t)$, $0 \le x \le L$, $t \ge 0$ represent the temperature of the point x of the bar at time t. By the law of heat conduction, also known as Fourier's law, the function $U(x,t)$ satisfies the equation

$$U_t(x,t) = c^2 U_{xx}(x,t), \quad 0 \le x \le L, t \ge 0,$$

where c denotes the diffusion constant of the heat transfer.

If the distribution of temperature of the bar at time $t_0 = 0$ is given as U_0, then we have the initial condition

$$U(x,0) = U_0(x), \text{ for } 0 \le x \le L.$$

If the temperature at one end $(x = 0)$ is kept at 0 and no heat lose at the other end $(x = L)$, we have the boundary conditions

$$U(0,t) = 0 = U_x(L,t), \ t \ge 0.$$

The solution to this equation under the condition above can be given by

$$U(x,t) = \sum_{n=0}^{\infty} a_n \exp\left(-c^2 \lambda_n^2 t\right) \sin(\lambda_n x), \quad \lambda_n = \frac{(2n+1)\pi}{2L} \quad \forall n \in \mathbb{N}.$$