4 Solution for the relatively short advertising campaign

Theoretically, with sufficient data, it is possible to figure out the proper function to represent the effect of advertising and predict the further sale intensity by solving the model. But in the reality, most manufacture cannot and have no willing to afford the failure in marketing, so the advertising strategy is changed rapidly, and each period that an advertising strategy lasts, here we call it an advertising campaign, is relatively short to the period of a merchandise lasts. Hence it is hard to get enough data to figure out the proper function to represent the effect of advertising. However, except the sale predicting, there is another use of this model, to appraise the benefit of each advertising strategy in some certain geographic areas.

Since the advertising campaigns are relatively short to the period of the merchandise lasts, the effect function of advertising is observed as a constant, say β, and this constant represents the effect coefficient of each advertising campaign. The equation will then become

$$\frac{S_t\left(x,t\right)}{S\left(x,t\right)} = c^2 \frac{S_{xx}\left(x,t\right)}{S\left(x,t\right)} + \beta.$$

Similar to solving the heat equation, we start with looking for the product solution of the form

$$S\left(x,t\right) = X\left(x\right) T\left(t\right).$$

Where $X(x)$ is a function of x and $T(t)$ is a function of t. Plugging into the equation above and separating variables, we obtain

$$\frac{T'(t)}{T(t)} = c^2 \frac{X''(x)}{X(x)} + \beta.$$

For the equation to hold, we must let

$$\frac{T'(t)}{T(t)} - \beta = c^2 \frac{X''(x)}{X(x)} = k,$$

where k is the separation constant.

As $k = 0$, $X(x) = Ax + B$, A and B are constants; and $X(x)$ will be zero under the boundary condition and lead to the trivial solution $S(x,t) = 0$ for all x and t . This is a worthless solution in practical since it has no sale intensity.

As $k = \mu^2 > 0$ by the boundary condition, we also have the trivial solution $S(x,t) = 0$ for all x and t .

As $k = -\mu^2 < 0$, we set

$$T_n(t) = B_n \exp\left(\left(-\mu_n^2 + \beta\right) t\right),$$

$$X_n(x) = C_n \sin(\lambda_n x),$$

where $\mu_n = c\lambda_n = c\frac{(2n+1)\pi}{2L}$, and

$$S_n(x,t) = T_n(t) X_n(x) = A_n \exp\left(\left(-\mu_n^2 + \beta\right) t\right) \cdot \sin(\lambda_n x), \; A_n = B_n C_n,$$

According to the superposition principle, we obtain the general solution

$$S(x,t) = \sum_{n=0}^{\infty} S_n(x,t).$$

By applying the initial condition and the sine series expansion, the coefficient can be determined

$$A_n = \frac{1}{L}\int_0^{2L} S_0(x)\sin(\lambda_n x)\,dx,\ \ n = 0, 1, 2, 3, \cdots .$$

So the sale intensity as the number of consumers reaches x in the target market at time t is

$$S(x,t) = \exp(\beta t)\cdot\sum_{n=0}^{\infty} A_n \exp\left(-\mu_n^2 t\right)\cdot\sin(\lambda_n x),$$

where

$$A_n = \frac{1}{L}\int_0^{2L} S_0(x)\sin(\lambda_n x)\,dx, n = 0, 1, 2, 3, \cdots,$$

$$\mu_n = \frac{c(2n+1)\pi}{2L}, n = 0, 1, 2, 3, \cdots,$$

$$\lambda_n = \frac{(2n+1)\pi}{2L}, n = 0, 1, 2, 3, \cdots .$$

Thus we have:

Proposition 2: *When the advertising campaign is relatively short to the period of the merchandise lasts, the function $A(x,t)$ represents the effect of advertising campaign is deduced to a constant β , and the function of sale intensity in time and quantity of consumers will be null; or*

$$S(x,t) = \exp(\beta t)\sum_{n=0}^{\infty} A_n \exp\left(-\mu_n^2 t\right)\sin(\lambda_n x),$$

where

$$A_n = \frac{1}{L}\int_0^{2L} S_0(x)\sin(\lambda_n x)\,dx, n = 0, 1, 2, 3, \cdots,$$

$$\mu_n = \frac{c\,(2n+1)\,\pi}{2L}, \lambda_n = \frac{(2n+1)\,\pi}{2L}, n = 0, 1, 2, 3, \cdots .$$