5 Application in practical

In practical, there is always more than one advertising campaign for each merchandise, and each of them is distinct and relatively short. To appraise the benefit of each advertising strategy in certain geographic areas, we divide the time stream into several subintervals $T_j = [t_{j-1}, t_j]$ with respect to each advertising campaign, that is, the $j-th$ advertising campaign start at $t = t_{j-1}$ and end at $t = t_j$. According to proposition 2 above, for each T_j, there is a constant effect coefficient of the advertising campaign, say β_j, $j = 1, 2, 3...$, so the main equation of the model can be rewritten as

$$\frac{S_t(x,t)}{S(x,t)} = c^2 \frac{S_{xx}(x,t)}{S(x,t)} + \beta(t), \beta(t) = \beta_j,$$

as $t \in T_j$ and for each β_j, the sale intensity will be

$$S(x,t) = \exp(\beta_j t) \cdot \sum_{n=0}^{\infty} A_n \exp\left(-\mu_n^2 t\right) \sin(\lambda_n x).$$

Through this result, the sale with respect to the $j-th$ advertising campaign with x_k consumers is

$$S(x_k,t) = \exp(\beta_j t) \cdot \sum_{n=0}^{\infty} A_n \exp\left(-\mu_n^2 t\right) \sin(\lambda_n x_k).$$

Particularly, for the periods without advertising campaigns, the constant effect coefficient will be set as $\beta_j = 0$.

In the period between the store penetrations complete and the very first advertising campaign occurs, say $T_1 = [0, t_1]$, since the only effect on sale intensity is circulating of information by word of mouth, the sale intensity in this period with x_k consumers will be

$$S(x_k, t) = \sum_{n=1}^{\infty} A_n \exp\left(-\mu_n^2 t\right) \sin(\lambda_n x_k), \ t \in T_1.$$

We will be able to measure the diffusion coefficient c with the observed data of sale intensity.

After the diffusion coefficient of the merchandise is resolved in period T_1, we move on to the period forward, the sale intensity ${}_jS(x,t)$ in each individual period T_j forward can be modeled as

$$\frac{{}_jS_t(x,t)}{{}_jS(x,t)} = c^2 \frac{{}_jS_{xx}(x,t)}{{}_jS(x,t)} + \beta_j, \ 0 \le x \le L, t \in T_j,$$

subjected by

$${}_jS(x, t_{j-1}) = {}_{j-1}S(x), 0 \le x \le L,$$

$${}_jS(0,t) = 0 =_j S_x(L,t), t \in T_j,$$

with ${}_jS(x,t)$ the sale intensity at time t in T_j with consumers x;

β_j the effect coefficient of the $j-th$ advertising campaign;

c the diffusion constant of merchandise;

L the size of the market;

${}_{j-1}S(x)$ the sale intensity in the end of the period T_{j-1},

$${}_jS(x, t_{j-1}) =_{j-1} S(x) =_{j-1} S(x, t_{j-1}).$$

Since we have already figured out the diffusion coefficient, with this sub-model and the observed data, the effect coefficient β_j of the $j-th$ advertising campaign will be able to measure easily. By comparing each β_j, the efficiency of the advertising campaign will be clearly compared in practical.