Chapter 2

A Network Optimization Model

2.1 Utility Functions

Following the concepts of reference point methodology [22], we assume that the decision maker specifies requirements in terms of aspiration and reservation levels by introducing desired and required values for several outcomes, depending on the specified aspiration and reservation levels, a_i and r_i, respectively. Further, assumes that an utility function of q_i can be viewed as an extension of the fuzzy membership function in terms of a strictly monotonic and concave utility function as shown in Figure 2.1. (see [9], [13], [18] etc.)

$$f_i(q_i) = \log_{\delta_i} \frac{q_i}{r_i} \tag{2.1}$$

where $\delta_i = a_i/r_i$. Formally, we define $f_i(\cdot)$ over the range $[0, \infty)$, with $f_i(0) = -\infty$ and $f_i'(0) = \infty$. It is a strictly increasing function of q_i, having value 1 if $q_i = a_i$, and value 0 if $q_i = r_i$. The utility function can map the different values onto a normalized scale of the decision maker's satisfaction. Moreover, the logarithmic utility function will be intimately associated with the concept of proportional fairness (see [3], [9], and [10].)

Proposition 1 *The utility function $f_i(q_i)$ is continuous, increasing, and concave.*

The proof was given in [15]. We will formulate the mathematical model of the fair bandwidth allocation by using the utility function.

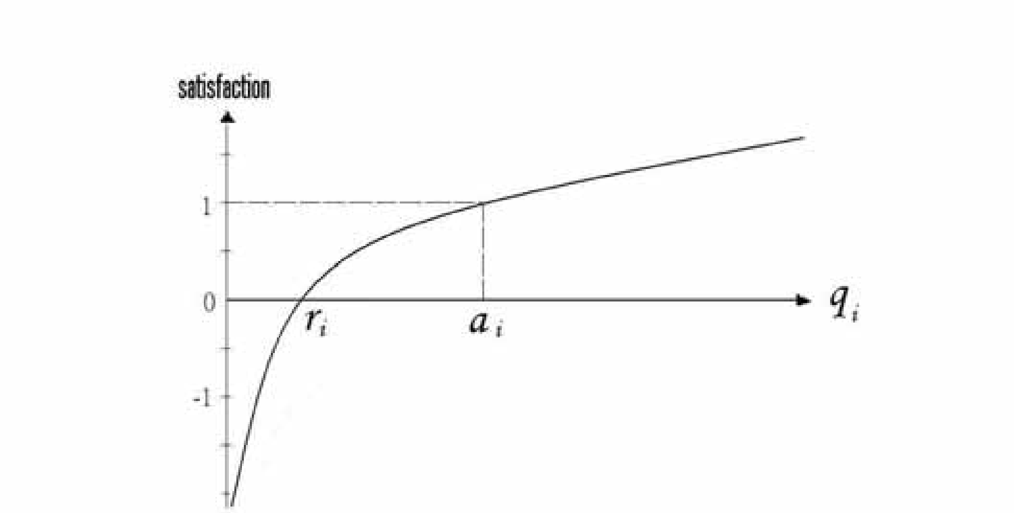

Figure 2.1: The Graph of an Utility Function $f_i(q_i)$

2.2 A Network Optimization Model

Consider a network optimization model. Let V be a set of nodes. Denote $E_\nu^{in} \subseteq E$ a subset of incoming links to the node $\nu \in V$, and $E_\nu^{out} \subseteq E$ a subset of outgoing links from the node $\nu \in V$. Namely, $E_o \subseteq E$ and $E_d \subseteq E$ be subsets of links connected with the source node o and destination node d, respectively. We may formulate the mathematical model of the fair bandwidth allocation when adopting the utility function (2.1) interpreted as a measure of QoS on networks, . In [15], the precomputation-based maximization model with its constraints can be formulated as follows:

$$
\begin{aligned}
\max \quad & \sum_{i\in I} w_i f_i(q_i) && (2.2)\\
s.t. \quad & \sum_{e\in E}\sum_{i\in I}\sum_{j\in \mathcal{K}_i} \kappa_e \cdot q_i \cdot \chi_{i,j}(e) \le B && (2.3)\\
& \sum_{i\in I}\sum_{j\in \mathcal{K}_i} q_i \chi_{i,j}(e) \le U_e,\ \forall e\in E && (2.4)\\
& \sum_{e\in E} \ell_e \chi_{i,j}(e) \le D_i,\ \forall\ j\in \mathcal{K}_i,\ i\in I && (2.5)\\
& q_i \ge b_i,\ \forall i \in I && (2.6)\\
& \sum_{e\in E_o} \chi_{i,j}(e) = 1,\ \forall\ j\in \mathcal{K}_i,\ i\in I && (2.7)\\
& \sum_{e\in E_\nu^{in}} \chi_{i,j}(e) = \sum_{e\in E_\nu^{out}} \chi_{i,j}(e),\ \forall \nu \in V,\ j\in \mathcal{K}_i,\ i\in I && (2.8)\\
& \sum_{e\in E_d} \chi_{i,j}(e) = 1,\ \forall\ j\in \mathcal{K}_i,\ i\in I && (2.9)\\
& q_i \ge 0,\ \forall i\in I && (2.10)\\
& \chi_{i,j}(e) = 0 \ or\ 1,\ \forall e\in E,\ j\in \mathcal{K}_i,\ i\in I && (2.11)
\end{aligned}
$$

κ_e : the marginal cost of bandwidth for each link e

B : the total available budget

U_e : the maximal capacity of each link e

ℓ_e : the mean delay allocated to each link e

D_i : the maximal end-to-end delay allocated to each class i

b_i : the bandwidth requirement for class i.

Decision variables in this model are $\mathbf{x} = (q_i, \chi_{i,j}(e))$, for all $i \in I,\ j\in \mathcal{K}_i,\ e\in E$.

In [15], it proposed the analogue model, where the constraints of its model are complex. It was not able to be solved by a general software but has been reformulated in a piecewise linear type problem and being solved in ILOG [2].

In the following, we adopt the network constraints of the above model. By understanding the network constraints, it would be clear about the model and attributes of each weight for each class. We apply the above model for computation by solver BARON in software GAMS (see [11] and [12]) to obtain the numerical results because GAMS is more flexible in nonlinear optimization.

2.3 Network Constraints

For each connection j of class i, we denote the routing path connecting the source node o and destination node d by $p_{i,j}$. To determine whether link e is chosen we define the binary decision variable

$$\chi_{i,j}(e) = \begin{cases} 1 & \text{if link } e \in p_{i,j} \\ 0 & \text{if link } e \notin p_{i,j}. \end{cases} \tag{2.12}$$

In our thesis, the network problems which we discuss is that each connection of the same class chooses the same and the only one routing path.

Given the total available budget B and the marginal cost κ_e of bandwidths for each link $e \in E$, we want to allocate the bandwidths in order to provide each class with maximal possible QoS. Denoted by $\mathcal{K}_i$ a set of connections in class i. Suppose there is the number of connections in $\mathcal{K}_i$ is J_i, i.e., $|\mathcal{K}_i| = J_i$. Then, these decision variables must be nonnegative:

$$q_i \geq 0, \ \forall j \in \mathcal{K}_i, \text{ for } i = 1, \dots, m. \tag{2.13}$$

First, let $q_{i,j}$ be the bandwidth allocated to the connection j of class i, respectively. Suppose that every connection in the same class uses the same bandwidth and has the same bandwidth requirement, so we have $q_{i,1} = q_{i,2} = \cdots = q_{i,J_i}$.

We denote $q_i (= q_{i,1} = q_{i,2} = \cdots = q_{i,J_i})$ be the bandwidth allocated to each connection of class i. Thus, the constraint follows

$$q_i \geq b_i \tag{2.14}$$

where b_i is the bandwidth requirement for class i. It shows that every connection in the same class uses the same bandwidth and has the same bandwidth requirement.

Due to the limited budget on network planning, We have the budget constraint on the network:

$$\sum_{e\in E}\sum_{i\in I}\sum_{j\in\mathcal{K}_i} \kappa_e \cdot q_i \cdot \chi_{i,j}(e) \leq B \tag{2.15}$$

and

$$\sum_{i\in I}\sum_{j\in\mathcal{K}_i} q_i \chi_{i,j}(e) \leq U_e, \quad \forall e \in E \tag{2.16}$$

where U_e is the maximal capacity of each link e. The above constraint says that the aggregate bandwidth of all connections at any link does not exceed the capacity.

Moreover, for each class i, since every connection has the maximal end-to-end delay constraint, we have the end-to-end delay constraint:

$$\sum_{e\in E} \ell_e \chi_{i,j}(e) \leq D_i, \ \forall i, j \tag{2.17}$$

where ℓ_e is a mean delay allocated to each link e and D_i is maximal end-to-end delay allocated to each class i.

Let $E_o \subseteq E$ be the subset of links connected with the source node o, then we have

$$\sum_{e\in E_o} \chi_{i,j}(e) = 1, \ \forall i, j. \tag{2.18}$$

Let $E_d \subseteq E$ be the subset of links connected with the destination node d, then we have

$$\sum_{e\in E_d} \chi_{i,j}(e) = 1, \ \forall i, j. \tag{2.19}$$

Let $E_\nu^{in} \subseteq E$ be the subset of links flowed into the node ν and $E_\nu^{out} \subseteq E$ be the set of links flowed out of the node ν, then we have

$$\sum_{e\in E_\nu^{in}} \chi_{i,j}(e) = \sum_{e\in E_\nu^{out}} \chi_{i,j}(e), \ \forall i, j. \tag{2.20}$$

Constraints (2.18), (2.19) and (2.20) express the node conservation relations indicating that flow in equals flow out for every connection j in class i.

Let $\mathbf{x} = \{(q_i, \chi_{i,j}(e)) | \, \forall j \in \mathcal{K}_i, \text{ for } i = 1, \ldots, m, \forall e \in E\} \in \mathbb{R}^n$ denote the vector of decision variables and $Q^* = \{\mathbf{x} | \mathbf{x} \text{ satisfies constraints } (2.3) - (2.11)\}$ denote the feasible set. We consider a resource allocation problem defined as an optimization problem with m objective functions $f_i(\mathbf{x})$:

$$\max\{\, \mathbf{f}(\mathbf{x}) : \mathbf{x} \in Q^* \,\} \tag{2.21}$$

where $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))$ is a vector-function that maps the decision space $\mathbb{R}^n$ into the criterion space $\mathbb{R}^m$.