Chapter 4

An Illustrative Example and Numerical Results

4.1 An Illustrative Example

Suppose, for each link e, we have a mean delay ℓ_e related to the link's speed, propagation delay, and maximal transfer unit. The maximal possible link capacity is U_e on each link $e \in E$, and the link cost is κ_e of using one unit bandwidth. A connection j in each class i should be routed through a path $p_{i,j}$ between node 1 and node 7. Under a limited available budget B, we plan to allocate the bandwidth in order to provide each class with maximal possible QoS and determine the optimal path from node 1 to node 7 under guaranteed service. Decision variables are listed as follows: q_i denotes the bandwidth allocated to each connection in class i, and $\chi_{i,j}(e)$ is a binary variable which determines whether the link e is chosen for connection j in class i. The purpose of this work is to present an mathematical model that provides the decision maker to explore a set of solutions satisfying users' preferences with fairness.

Consider Fig. 1.1, given the cost of delay, the purchasing cost of bandwidth for each link, and the maximal capacity of each link as Table 4.1.

Table 4.1: Characteristics of Each Link

Characteristics	e_1	e_2	e_3	e_4	e_5	e_6	e_7
Capacity (kbps)	230,000	350,000	100,000	250,000	210,000	220,000	200,000
Cost ($)	5	6	10	5	4	11	6
Delay (sec)	0.03	0.032	0.035	0.012	0.02	0.012	0.03
	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}
Capacity (kbps)	300,000	210,000	270,000	150,000	180,000	300,000	350,000
Cost ($)	8	6	7	12	6	5	6
Delay (sec)	0.015	0.027	0.012	0.03	0.02	0.035	0.035

Consider a sample network as shown in Figure 1.1, where V ={node 1, node 2, ..., node 7} and $E = \{e_k,\ k = 1, 2, \ldots, 14\}$ denote the set of nodes and the set of links in the network respectively. Let node 1 and node 7 be the source and destination respectively. Each connection is delivered from node 1 to node 7. Table 4.1 shows the capacity U_e, mean delay ℓ_e, and the link cost κ_e of bandwidth for each link $e \in E$. In Table 4.2, three different QoS classes are given, where class 1 has the highest priority and class 3 has the lowest priority. We assume every connection in class i, for $i = 1, 2, 3$, has the same aspiration level a_i kbps (i.e. kilobits/sec), reservation level r_i kbps, mean packet size s_i kb, maximal end-to-end delay D_i, and bandwidth requirement b_i kbps.

Suppose the number of connections in each class i is J_i for $i = 1, 2, 3$. Under the total available budget $B = \$1,000,000$, we want to allocate the bandwidths in order to provide each class with maximal possible QoS defined via the utility function [5].

Let $\chi_{i,j}(e_k)$ be the 0-1 variable to denote whether link e_k is chosen, $\forall k = 1, 2, \ldots, 14$. We let q_i be the bandwidth allocated to each connection of class i, $\forall i = 1, 2, 3$. For each class i, we consider the objective function f_i as below: $f_1(q_1) = \log_2(q_1/167)$, $f_2(q_2) = \log_2(q_2/83)$, $f_3(q_3) = \log_2(q_3/28)$. Suppose w_i, for $i = 1, 2, 3$, is the weight assigned to each objective function $f_i(q_i)$ and $\sum_{i=1}^{3} w_i = 1$.

Table 4.2: Characteristics of Each QoS Class

Class i	Bandwidth Requirement b_i (kbps)	Asp. Level a_i (kbps)	Res. Level r_i (kbps)	Max. Delay D_i (sec)
1	160	334	167	0.89
2	80	166	83	1.02
3	25	56	28	2.34

Table 4.3: Change in the Number of Connections in Class 1 (J_1) for Model I

Number of Connections (J_1, J_2, J_3)	*Optimal Value* (*satisfaction*)	*Bandwidth*(kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(190, 100, 100)	2.051	(160,80,116)	(0,0,1)
(180, 100, 100)	2.237	(160,80,132)	(0,0,1)
(170, 100, 100)	2.402	(160,80,148)	(0,0,1)
(160, 100, 100)	2.55	(160,80,164)	(0,0,1)
(150, 100, 100)	2.684	(160,80,180)	(0,0,1)
(140, 100, 100)	2.807	(160,80,196)	(0,0,1)
(130, 100, 100)	2.921	(160,80,212)	(0,0,1)
(120, 100, 100)	3.026	(160,80,228)	(0,0,1)
(110, 100, 100)	3.123	(160,80,244)	(0,0,1)
(100, 100, 100)	3.215	(160,80,260)	(0,0,1)
(90, 100, 100)	3.301	(160,80,276)	(0,0,1)
(80, 100, 100)	3.382	(160,80,292)	(0,0,1)
(70, 100, 100)	3.459	(160,80,308)	(0,0,1)
(60, 100, 100)	3.532	(160,80,324)	(0,0,1)
(50, 100, 100)	3.602	(160,80,340)	(0,0,1)
(40, 100, 100)	3.668	(160,80,356)	(0,0,1)
(30, 100, 100)	3.732	(160,80,372)	(0,0,1)
(20, 100, 100)	3.793	(160,80,388)	(0,0,1)
(10, 100, 100)	4.564	(3950,80,25)	(1,0,0)

4.2 Numerical Results of Model I

We want to know weights w_i where $w_i \in [0, 1]$, the total sum of optimal utilization and the selected path e how to change as J_i, B, a_i and r_i changing.

Given $B = 1,000,000$, $a_1 = 334(kbps)$, $a_2 = 166(kbps)$, $a_3 = 56(kbps)$, $r_1 = 167(kbps)$, $r_2 = 83(kbps)$, and $r_3 = 28(kbps)$ where $\delta_i = a_i/r_i = 2$, $\forall i = 1, 2, 3$. Just like in Table 4.2, three different QoS classes are given, where class 1 has the highest priority and class 3 has the lowest priority. We change the numbers of connections in each class, and the numerical results are shown in Tables 4.3, 4.4, and 4.5. We draw graphs of Tables 4.3, 4.4, and 4.5, named as Figures 4.1, 4.2, and 4.3.

Table 4.4: Change in the Number of Connections in Class 2 (J_2)for Model I

Number of Connections (J_1, J_2, J_3)	*Optimal Value* (*satisfaction*)	*Bandwidth*(kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(100, 190, 100)	2.747	(160,80,188)	(0,0,1)
(100, 180, 100)	2.807	(160,80,196)	(0,0,1)
(100, 170, 100)	2.865	(160,80,204)	(0,0,1)
(100, 160, 100)	2.921	(160,80,212)	(0,0,1)
(100, 150, 100)	2.974	(160,80,220)	(0,0,1)
(100, 140, 100)	3.026	(160,80,228)	(0,0,1)
(100, 130, 100)	3.075	(160,80,236)	(0,0,1)
(100, 120, 100)	3.123	(160,80,244)	(0,0,1)
(100, 110, 100)	3.17	(160,80,252)	(0,0,1)
(100, 100, 100)	3.215	(160,80,260)	(0,0,1)
(100, 90, 100)	3.259	(160,80,268)	(0,0,1)
(100, 80, 100)	3.301	(160,80,276)	(0,0,1)
(100, 70, 100)	3.342	(160,80,284)	(0,0,1)
(100, 60, 100)	3.382	(160,80,292)	(0,0,1)
(100, 50, 100)	3.421	(160,80,300)	(0,0,1)
(100, 40, 100)	3.459	(160,80,308)	(0,0,1)
(100, 30, 100)	3.661	(160,1050,25)	(0,1,0)
(100, 20, 100)	4.246	(160,1575,25)	(0,1,0)
(100, 10, 100)	5.246	(160,3150,25)	(0,1,0)

Table 4.5: Change in the Number of Connections in Class 3 (J_3) for Model I

Number of Connections (J_1, J_2, J_3)	*Optimal Value* (*satisfaction*)	*Bandwidth*(kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(100, 100, 190)	2.289	(160,80,136)	(0,0,1)
(100, 100, 180)	2.367	(160,80,144.444)	(0,0,1)
(100, 100, 170)	2.449	(160,80,152.941)	(0,0,1)
(100, 100, 160)	2.537	(160,80,162)	(0,0,1)
(100, 100, 150)	2.63	(160,80,173.333)	(0,0,1)
(100, 100, 140)	2.73	(160,80,185.714)	(0,0,1)
(100, 100, 130)	2.837	(160,80,200)	(0,0,1)
(100, 100, 120)	2.925	(160,80,216.667)	(0,0,1)
(100, 100, 110)	3.078	(160,80,236.364)	(0,0,1)
(100, 100, 100)	3.215	(160,80,260)	(0,0,1)
(100, 100, 90)	3.367	(160,80,288.889)	(0,0,1)
(100, 100, 80)	3.537	(160,80,325)	(0,0,1)
(100, 100, 70)	3.73	(160,80,371.429)	(0,0,1)
(100, 100, 60)	3.952	(160,80,433.333)	(0,0,1)
(100, 100, 50)	4.215	(160,80,520)	(0,0,1)
(100, 100, 40)	4.537	(160,80,650)	(0,0,1)
(100, 100, 30)	4.952	(160,80,866.667)	(0,0,1)
(100, 100, 20)	5.537	(160,80,1300)	(0,0,1)
(100, 100, 10)	6.537	(160,80,2600)	(0,0,1)

Table 4.6: Change in the Total Budget (B) for Model I

Total Budget B	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(2000, 000)	4.763	(160,80,760)	(0,0,1)
(1900, 000)	4.664	(160,80,710)	(0,0,1)
(1800, 000)	4.559	(160,80,660)	(0,0,1)
(1700, 000)	4.445	(160,80,610)	(0,0,1)
(1600, 000)	4.322	(160,80,560)	(0,0,1)
(1500, 000)	4.187	(160,80,510)	(0,0,1)
(1400, 000)	4.038	(160,80,460)	(0,0,1)
(1300, 000)	3.872	(160,80,410)	(0,0,1)
(1200, 000)	3.684	(160,80,360)	(0,0,1)
(1100, 000)	3.469	(160,80,310)	(0,0,1)
(1000, 000)	3.215	(160,80,260)	(0,0,1)
(900, 000)	2.907	(160,80,210)	(0,0,1)
(800, 000)	2.515	(160,80,160)	(0,0,1)
(700, 000)	1.974	(160,80,110)	(0,0,1)
(600, 000)	1.100	(160,80,60)	(0,0,1)

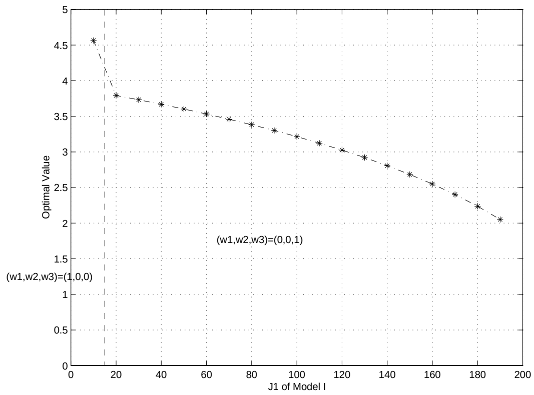

Figure 4.1: Change in the Number of Connections in Class 1 (J_1) for Model I

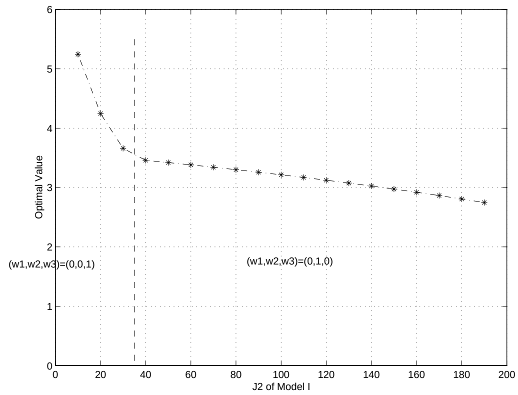

Figure 4.2: Change in the Number of Connections in Class 2 (J_2)for Model I

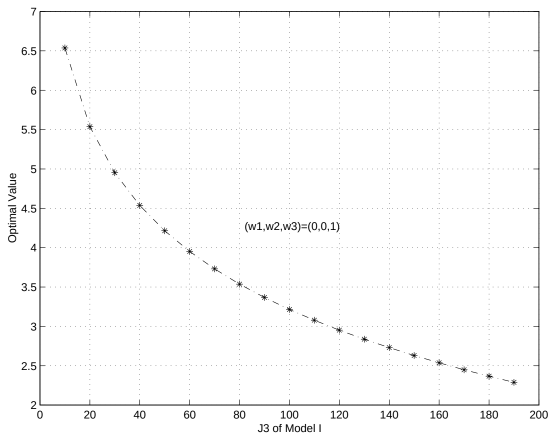

Figure 4.3: Change in the Number of Connections in Class 3 (J_3) for Model I

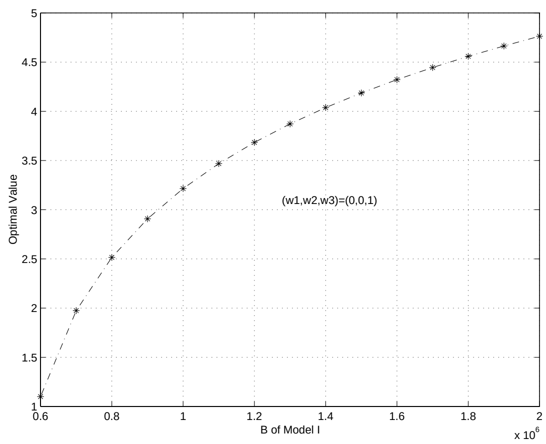

Figure 4.4: Change in the Total Budget (B) for Model I

Table 4.7: Change in the Aspiration Level in Class 1 for Model I

Aspiration Level (a_1, a_2, a_3)	*optimal value* (*satisfaction*)	*bandwidth* (kbps) (q_1, q_2, q_3)	*weight* (w_1, w_2, w_3)
(668, 166, 56)	3.215	(160,80,260)	(0,0,1)
(620, 166, 56)	3.215	(160,80,260)	(0,0,1)
(584.5, 166, 56)	3.215	(160,80,260)	(0,0,1)
(540, 166, 56)	3.215	(160,80,260)	(0,0,1)
(501, 166, 56)	3.215	(160,80,260)	(0,0,1)
(460, 166, 56)	3.215	(160,80,260)	(0,0,1)
(417.5, 166, 56)	3.215	(160,80,260)	(0,0,1)
(370, 166, 56)	3.215	(160,80,260)	(0,0,1)
(334, 166, 56)	3.215	(160,80,260)	(0,0,1)
(290, 166, 56)	3.215	(160,80,260)	(0,0,1)
(250.5, 166, 56)	3.215	(160,80,260)	(0,0,1)
(220, 166, 56)	3.215	(160,80,260)	(0,0,1)
(200, 166, 56)	4.774	(395,80,25)	(1,0,0)
(190, 166, 56)	6.672	(395,80,25)	(1,0,0)
(180, 166, 56)	11.484	(395,80,25)	(1,0,0)

Given $J_1 = J_2 = J_3 = 100$, $a_1 = 334(kbps)$, $a_2 = 166(kbps)$, $a_3 = 56(kbps)$, $r_1 = 167(kbps)$, $r_2 = 83(kbps)$, and $r_3 = 28(kbps)$ where $\delta_i = a_i/r_i = 2$, $\forall i = 1, 2, 3$. Just like in Table 4.2, three different QoS classes are given, where class 1 has the highest priority and class 3 has the lowest priority. We change the total budget B, and the numerical results are shown in Table 4.6. When $B \leq 500,000$, we get infeasible solutions. We draw the graph of the Table 4.6, named as Figure 4.4.

Given $J_1 = J_2 = J_3 = 100$, $B = 1,000,000$, $r_1 = 167(kbps)$, $r_2 = 83(kbps)$, and $r_3 = 28(kbps)$ where $\delta_i = a_i/r_i$, $\forall i = 1, 2, 3$. We change the aspiration level, just like in Table 4.2, three different QoS classes are given, where class 1 still has the highest priority and class 3 has the lowest priority, and the numerical results are shown in Tables 4.7, 4.8, and 4.9.

Given $J_1 = J_2 = J_3 = 100$, $B = 1,000,000$, $a_1 = 334(kbps)$, $a_2 = 166(kbps)$, and $a_3 = 56(kbps)$ where $\delta_i = a_i/r_i$, $\forall i = 1, 2, 3$. We change the reservation level, just like in Table 4.2, three different QoS classes are given, where class 1 still has the highest priority and class 3 has the lowest priority, and the numerical results are shown in Tables 4.10, 4.11, and 4.12.

By numerical results in Tables 4.3-4.12, we have a proposition of structures of optimal weights as follow:

Table 4.8: Change in the Aspiration Level in Class 2 for Model I

Aspiration Level (a_1, a_2, a_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(334, 332, 56)	3.215	(160,80,260)	(0,0,1)
(334, 310, 56)	3.215	(160,80,260)	(0,0,1)
(334, 290.5, 56)	3.215	(160,80,260)	(0,0,1)
(334, 270, 56)	3.215	(160,80,260)	(0,0,1)
(334, 249, 56)	3.215	(160,80,260)	(0,0,1)
(334, 220, 56)	3.215	(160,80,260)	(0,0,1)
(334, 207.5, 56)	3.215	(160,80,260)	(0,0,1)
(334, 180, 56)	3.215	(160,80,260)	(0,0,1)
(334, 166, 56)	3.215	(160,80,260)	(0,0,1)
(334, 140, 56)	2.551	(160,315,25)	(0,1,0)
(334, 124.5, 56)	3.289	(160,315,25)	(0,1,0)
(334, 100, 56)	7.158	(160,315,25)	(0,1,0)
(334, 90, 56)	16.472	(160,315,25)	(0,1,0)

Table 4.9: Change in the Aspiration Level in Class 3 for Model I

Aspiration Level (a_1, a_2, a_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(334, 166, 112)	1.924	(160,315,25)	(0,1,0)
(334, 166, 108)	1.924	(160,315,25)	(0,1,0)
(334, 166, 98)	1.924	(160,315,25)	(0,1,0)
(334, 166, 94)	1.924	(160,315,25)	(0,1,0)
(334, 166, 84)	1.924	(160,315,25)	(0,1,0)
(334, 166, 80)	1.924	(160,315,25)	(0,1,0)
(334, 166, 70)	1.924	(160,315,25)	(0,1,0)
(334, 166, 66)	1.924	(160,315,25)	(0,1,0)
(334, 166, 56)	3.215	(160,80,260)	(0,0,1)
(334, 166, 52)	3.6	(160,80,260)	(0,0,1)
(334, 166, 42)	5.496	(160,80,260)	(0,0,1)

Table 4.10: Change in the Reservation Level in Class 1 for Model I

Reservation Level (r_1, r_2, r_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(20.875, 83, 28)	3.215	(160,80,260)	(0,0,1)
(30, 83, 28)	3.215	(160,80,260)	(0,0,1)
(41.75, 83, 28)	3.215	(160,80,260)	(0,0,1)
(50, 83, 28)	3.215	(160,80,260)	(0,0,1)
(60, 83, 28)	3.215	(160,80,260)	(0,0,1)
(70, 83, 28)	1.924	(160,315,25)	(0,1,0)
(85, 83, 28)	1.391	(160,315,25)	(0,0,1)
(87, 83, 28)	1.924	(160,315,25)	(0,1,0)
(95, 83, 28)	3.215	(160,80,260)	(0,0,1)
(110, 83, 28)	3.215	(160,80,260)	(0,0,1)
(135, 83, 28)	3.215	(160,80,260)	(0,0,1)
(167, 83, 28)	3.215	(160,80,260)	(0,0,1)

Table 4.11: Change in the Reservation Level in Class 2 for Model I

Reservation Level (r_1, r_2, r_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(167, 10.375, 28)	1.231	(160,315,25)	(0,1,0)
(167, 15, 28)	1.266	(160,315,25)	(0,1,0)
(167, 20.75, 28)	1.308	(160,315,25)	(0,1,0)
(167, 25, 28)	1.338	(160,315,25)	(0,1,0)
(167, 30, 28)	1.374	(160,315,25)	(0,1,0)
(167, 35, 28)	3.215	(160,80,260)	(0,0,1)
(167, 41.5, 28)	1.462	(160,315,25)	(0,1,0)
(167, 50, 28)	1.534	(160,315,25)	(0,1,0)
(167, 60, 28)	1.639	(160,315,25)	(0,1,0)
(167, 70, 28)	3.215	(160,80,260)	(0,0,1)
(167, 83, 28)	3.215	(160,80,260)	(0,0,1)

Table 4.12: Change in the Reservation Level in Class 3 for Model I

Reservation Level (r_1, r_2, r_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	*Weight* (w_1, w_2, w_3)
(167, 83, 3.5)	1.924	(160,315,25)	(0,1,0)
(167, 83, 5)	1.924	(160,315,25)	(0,1,0)
(167, 83, 7)	1.924	(160,315,25)	(0,1,0)
(167, 83, 10)	1.924	(160,315,25)	(0,1,0)
(167, 83, 12)	1.997	(160,80,260)	(0,0,1)
(167, 83, 14)	2.108	(160,80,260)	(0,0,1)
(167, 83, 18)	2.353	(160,80,260)	(0,0,1)
(167, 83, 22)	2.643	(160,80,260)	(0,0,1)
(167, 83, 26)	3.001	(160,80,260)	(0,0,1)
(167, 83, 28)	3.215	(160,80,260)	(0,0,1)

Table 4.13: Change in the Number of Connections in Class 1 (J_1) for Model II

Number of Connections (J_1, J_2, J_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(190, 100, 100)	0.662	(160,98,98)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(180, 100, 100)	0.737	(160,106,106)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(170, 100, 100)	0.807	(160,114,114)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(160, 100, 100)	0.872	(160,122,122)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(150, 100, 100)	0.933	(160,130,130)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(140, 100, 100)	0.991	(160,138,138)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(130, 100, 100)	1.045	(160,146,146)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(120, 100, 100)	1.096	(160,154,154)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 100)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(90, 100, 100)	1.243	(185.185,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(80, 100, 100)	1.299	(208.333,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(70, 100, 100)	1.364	(238.095,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(60, 100, 100)	1.438	(277.778,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(50, 100, 100)	1.525	(333.333,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(40, 100, 100)	1.633	(416.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(30, 100, 100)	1.771	(555.556,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(20, 100, 100)	1.966	(833.333,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(10, 100, 100)	2.299	(1666.666,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

Proposition 2 *For Optimization Model I, there exists a unit vector, such that the total weighted utilization function value in Model I is maximized.*

Proof:

For each $\mathbf{x} \in Q^*$,

there exists $k \in \{1, 2, \ldots, m\}$, such that $f_k(\mathbf{x}) \geq f_i(\mathbf{x})$, for each $i = 1, 2, \ldots, m$.

Because of $0 \leq w_i \leq 1$ and $f_i(\mathbf{x}) \geq 0$, for each $i = 1, 2, \ldots, m$,

we have $\sum_{i=1}^{m} w_i f_i(\mathbf{x}) \leq \sum_{i=1}^{m} w_i f_k(\mathbf{x}) \leq f_k(\mathbf{x}) \cdot \sum_{i=1}^{m} w_i \leq f_k(\mathbf{x})$.

It implies that $w_k = 1$ and $w_j = 0$, for all $j \in \{1, 2, \ldots, m\} - \{k\}$.

4.3 Numerical Results of Model II

We want to know weights w_i where $w_i \in [0, 1]$, the total sum of optimal utilization and the selected path e how to change as J_i, B, a_i and r_i changing.

Given $B = 1,000,000$, $a_1 = 334(kbps)$, $a_2 = 166(kbps)$, $a_3 = 56(kbps)$, $r_1 =$

Table 4.14: Change in the Number of Connections in Class 2 (J_2) for Model II

Number of Connections (J_1, J_2, J_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(100, 190, 100)	0.883	(166.667,87.719,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 180, 100)	0.909	(166.667,92.593,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 170, 100)	0.937	(166.667,98.039,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 160, 100)	0.966	(166.667,104.167,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 150, 100)	0.997	(166.667,111.111,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 140, 100)	1.030	(166.667,119.048,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 130, 100)	1.066	(166.667,128.205,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 120, 100)	1.104	(166.667,138.889,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 110, 100)	1.146	(166.667,151.515,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 100)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 90, 100)	1.243	(166.667,185.185,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 80, 100)	1.300	(166.667,208.333,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 70, 100)	1.364	(166.667,238.095,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 60, 100)	1.438	(166.667,277.778,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 50, 100)	1.525	(166.667,333.333,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 40, 100)	1.633	(166.667,416.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 30, 100)	1.771	(166.667,555.556,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 20, 100)	1.966	(166.667,833.333,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 10, 100)	2.300	(166.667,1666.663,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

Table 4.15: Change in the Number of Connections in Class 3 (J_3) for Model II

Number of Connections (J_1, J_2, J_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(100, 100, 190)	0.883	(166.667,166.667,87.719)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 180)	0.909	(166.667,166.667,92.593)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 170)	0.937	(166.667,166.667,98.039)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 160)	0.966	(166.667,166.667,104.167)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 150)	0.997	(166.667,166.667,111.111)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 140)	1.031	(166.667,166.667,119.048)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 130)	1.066	(166.667,166.667,128.025)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 120)	1.104	(166.667,166.667,138.889)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 110)	1.146	(166.667,166.667,151.515)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 100)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 90)	1.243	(166.667,166.667,185.185)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 80)	1.299	(166.667,166.667,208.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 70)	1.364	(166.667,166.667,238.095)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 60)	1.438	(166.667,166.667,277.778)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 50)	1.526	(166.667,166.667,333.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 40)	1.633	(166.667,166.667,416.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 30)	1.771	(166.667,166.667,555.556)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 20)	1.966	(166.667,166.667,833.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(100, 100, 10)	2.300	(166.667,166.667,1666.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

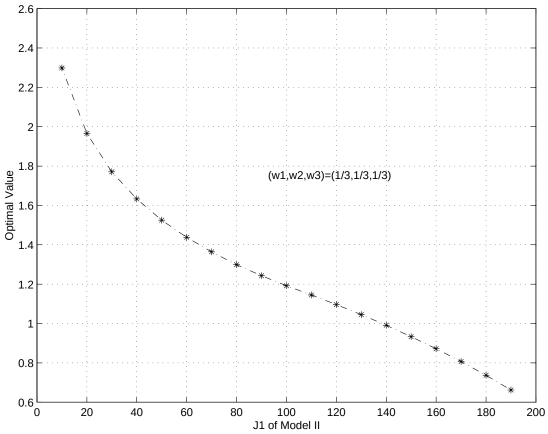

Figure 4.5: Change in the Number of Connections in Class 1 (J_1) for Model II

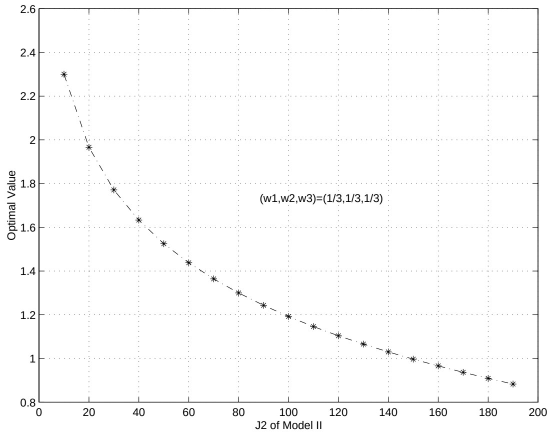

Figure 4.6: Change in the Number of Connections in Class 2 (J_2) for Model II

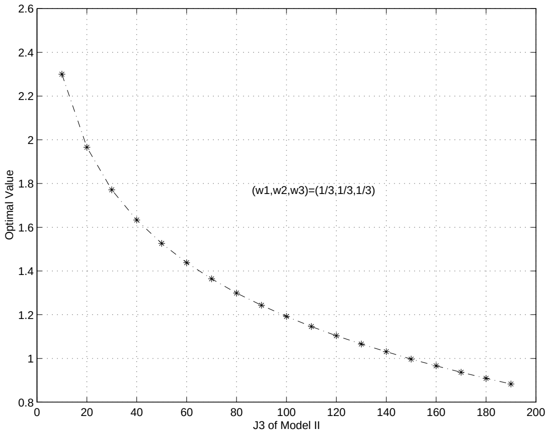

Figure 4.7: Change in the Number of Connections in Class 3 (J_3) for Model II

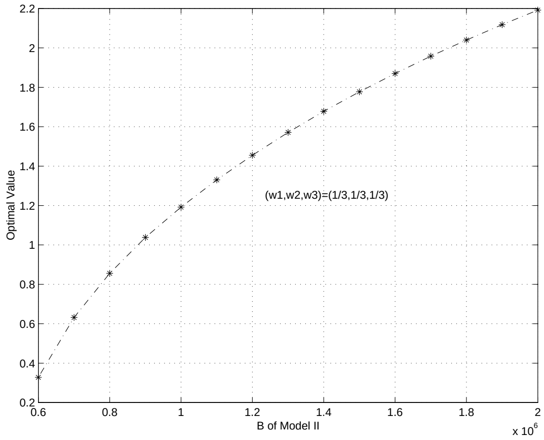

Figure 4.8: Change in the Total Budget (B) for Model II

Table 4.16: Change in the Total Budget (B) for Model II

Total Budget B	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
$(2000,000)$	2.192	(333.333,333.333,333.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1900,000)$	2.118	(316.667,316.667,316.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1800,000)$	2.040	(300.000,300.000,300.000)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1700,000)$	1.958	(283.333,283.333,283.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1600,000)$	1.870	(266.667,266.667,266.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1500,000)$	1.777	(250,250,250)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1400,000)$	1.678	(233.333,233.333,233.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1300,000)$	1.571	(216.667,216.667,216.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1200,000)$	1.455	(200,200,200.)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1100,000)$	1.330	(183.333,183.333,183.333)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(1000,000)$	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(900,000)$	1.038	(160,145,145)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(800,000)$	0.856	(160,120,120)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(700,000)$	0.632	(160,95,95)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
$(600,000)$	0.328	(160,80,60)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

$167(kbps)$, $r_2 = 83(kbps)$, and $r_3 = 28(kbps)$ where $\delta_i = a_i/r_i = 2,\ \forall i = 1, 2, 3$. Just like in Table 4.2, three different QoS classes are given, where class 1 has the highest priority and class 3 has the lowest priority. We change the numbers of connections in each class, and the numerical results are shown in Tables 4.13, 4.14, and 4.15. We draw graphs of Tables 4.13, 4.14, and 4.15, named as Figures 4.5, 4.6, and 4.7.

Given $J_1 = J_2 = J_3 = 100$, $a_1 = 334(kbps)$, $a_2 = 166(kbps)$, $a_3 = 56(kbps)$, $r_1 = 167(kbps)$, $r_2 = 83(kbps)$, and $r_3 = 28(kbps)$ where $\delta_i = a_i/r_i = 2,\ \forall i = 1, 2, 3$. Just like in Table 4.2, three different QoS classes are given, where class 1 has the highest priority and class 3 has the lowest priority. We change the total budget B, and the numerical results are shown in Table 4.16. When $B \leq 500,000$, we get infeasible solutions. We draw the graph of Tables 4.16, named as Figure 4.8.

Given $J_1 = J_2 = J_3 = 100$, $B = 1,000,000$, $r_1 = 167(kbps)$, $r_2 = 83(kbps)$, and $r_3 = 28(kbps)$ where $\delta_i = a_i/r_i,\ \forall i = 1, 2, 3$. We change the aspiration level, just like in Table 4.2, three different QoS classes are given, where class 1 still has the highest priority and class 3 has the lowest priority, and the numerical results are shown in Tables 4.17, 4.18, and 4.19.

Given $J_1 = J_2 = J_3 = 100$, $B = 1,000,000$, $a_1 = 334(kbps)$, $a_2 = 166(kbps)$, and $a_3 = 56(kbps)$ where $\delta_i = a_i/r_i,\ \forall i = 1, 2, 3$. We change the reservation level,

Table 4.17: Change in the Aspiration Level in Class 1 for Model II

Aspiration Level (a_1, a_2, a_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(668, 166, 56)	1.202	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(620, 166, 56)	1.201	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(584.5, 166, 56)	1.201	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(540, 166, 56)	1.200	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(501, 166, 56)	1.199	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(460, 166, 56)	1.199	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(417.5, 166, 56)	1.197	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(370, 166, 56)	1.194	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 56)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(290, 166, 56)	1.201	(192.871,153.565,153.565)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(250.5, 166, 56)	1.254	(230.423,134.789,134.789)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(220, 166, 56)	1.418	(278.503,110.748,110.748)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(200, 166, 56)	1.805	(328.883,85.564,85.564)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(190, 166, 56)	2.336	(354.286,80,65.914)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(180, 166, 56)	3.810	(379.01,80,40.99)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

Table 4.18: Change in the Aspiration Level in Class 2 for Model II

Aspiration Level (a_1, a_2, a_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(334, 332, 56)	1.077	(200,100,200)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 310, 56)	1.080	(197.941,104.119,197.940)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 290.5, 56)	1.083	(195.825,108.369,195.825)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 270, 56)	1.088	(193.228,113.545,193.228)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 249, 56)	1.095	(190.017,119.962,190.021)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 220, 56)	1.111	(184.428,131.142,184.430)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 207.5, 56)	1.121	(181.390,137.217,181.393)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 180, 56)	1.159	(172.687,154.625,172.687)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 56)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 140, 56)	1.315	(160,193.816,146.184)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 124.5, 56)	1.481	(160,214.516,125.484)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 100, 56)	2.531	(160,267.966,72.034)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 90, 56)	5.445	(160,304.438,35.562)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

just like in Table 4.2, three different QoS classes are given, where class 1 still has the highest priority and class 3 has the lowest priority, and the numerical results are shown in Tables 4.20, 4.21, and 4.22.

From (3.2), it is easy to determine the permutation τ when $\hat{w}_k$ is given, $\forall\ k$. Then the optimal weights w_i, $\forall\ i$, are obtained. First of all, we use q_i to obtain the value $f_i(q_i)$. Because there exists a permutation τ of set $S = \{1, 2, 3\}$ such that $\hat{\Psi}_k(\mathbf{f}(\mathbf{x})) = f_{\tau(k)}(\mathbf{x})$ for $k = 1, 2, 3$, we have $\hat{\Psi}_k(\mathbf{f}(\mathbf{x}))$. For example, $\hat{\Psi}_1(\mathbf{f}(\mathbf{x})) = f_3(q_3)$, $\hat{\Psi}_2(\mathbf{f}(\mathbf{x})) = f_1(q_1)$, and $\hat{\Psi}_3(\mathbf{f}(\mathbf{x})) = f_2(q_2)$, and we know $\hat{w}_1 = w_3$, $\hat{w}_2 = w_1$, $\hat{w}_3 = w_2$. Thus, the relation between $\hat{w}_i$ and w_i is obtained.

By numerical results in Tables 4.13-4.22, we have a proposition of structures of

Table 4.19: Change in the Aspiration Level in Class 3 for Model II

Aspiration Level (a_1, a_2, a_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(334, 166, 112)	0.816	(200,200,100)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 108)	0.824	(198.928,198.928,102.143)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 98)	0.849	(195.825,195.825,108.349)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 94)	0.862	(194.386,194.352,111.262)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 84)	0.902	(190.047,190.047,119.906)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 80)	0.923	(187.952,187.952,124.096)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 70)	0.994	(181.391,181.391,137.217)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 66)	1.034	(178.038,178.038,143.923)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 56)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 52)	1.297	(160.271,160.271,179.458)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(334, 166, 42)	1.852	(160,125.484,214.516)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

Table 4.20: Change in the Reservation Level in Class 1 for Model II

Reservation Level (r_1, r_2, r_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(20.875, 83, 28)	1.457	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(30, 83, 28)	1.444	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(41.75, 83, 28)	1.427	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(50, 83, 28)	1.416	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(60, 83, 28)	1.403	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(70, 83, 28)	1.388	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(83.5, 83, 28)	1.368	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(95, 83, 28)	1.350	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(110, 83, 28)	1.325	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(135, 83, 28)	1.275	(160,170,170)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 28)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

Table 4.21: Change in the Reservation Level in Class 2 for Model II

Reservation Level (r_1, r_2, r_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(167, 10.375, 28)	1.325	(210,80,210)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 15, 28)	1.311	(210,80,210)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 20.75, 28)	1.295	(210,80,210)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 25, 28)	1.284	(210,80,210)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 30, 28)	1.271	(207.886,84.228,207.886)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 35, 28)	1.258	(204.478,91.05,204.472)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 41.5, 28)	1.244	(200,100,200)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 50, 28)	1.227	(193.976,112.048,193.976)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 60, 28)	1.211	(186.485,127.026,186.489)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 70, 28)	1.199	(178.398,143.204,178.398)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 28)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

Table 4.22: Change in the Reservation Level in Class 3 for Model II

Reservation Level (r_1, r_2, r_3)	*Optimal Value* (*satisfaction*)	*Bandwidth* (kbps) (q_1, q_2, q_3)	v_i (v_1, v_2, v_3)	*Ordered Weight* $(\hat{w}_1, \hat{w}_2, \hat{w}_3)$
(167, 83, 3.5)	0.943	(222.222,222.222,55.556)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 5)	0.944	(218.672,218.561,62.767)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 7)	0.948	(214.286,214.286,71.429)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 10)	0.959	(208.05,208.167,83.482)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 12)	0.969	(204.085,204.085,91.831)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 14)	0.982	(200,200,100)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 18)	1.018	(191.518,191.519,116.963)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 22)	1.069	(182.357,182.356,135.287)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 26)	1.143	(172.211,172.211,155.578)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$
(167, 83, 28)	1.192	(166.667,166.667,166.667)	$(0,0,\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

optimal weights as follow:

Proposition 3 *For Optimization Model II, the form of optimal weights is a vector* $(w_1, w_2, \ldots, w_m)$ *with* $w_i = \frac{1}{m}$, *for each* $i = 1, 2, \ldots, m$.

Proof:

Because of $v_m = \hat{w}_m$ and $v_k = \hat{w}_k - \hat{w}_{k+1},$ for each $k = 1, 2, \ldots, m-1,$

and $v_k \geq 0,$ for all $k = 1, 2, \ldots, m,$

it yields $\hat{w}_1 \geq \hat{w}_2 \geq \ldots \geq \hat{w}_m \geq 0.$

Since it holds $\hat{\Psi}_1(\mathbf{f}(\mathbf{x})) \leq \hat{\Psi}_2(\mathbf{f}(\mathbf{x})) \leq \ldots \leq \hat{\Psi}_m(\mathbf{f}(\mathbf{x})),$

and $\displaystyle\sum_{k=1}^{m} v_k \mathbf{y}_k(\mathbf{f}(\mathbf{x})) = \sum_{k=1}^{m} \hat{w}_k \hat{\Psi}_k(\mathbf{f}(\mathbf{x})),$ for all $\mathbf{x} \in Q^*,$

we have $\hat{w}_1 \leq \hat{w}_2 \leq \ldots \leq \hat{w}_m.$

Because of $\displaystyle\sum_{k=1}^{m} \hat{w}_k = 1,$

it gives $\hat{w}_1 = \hat{w}_2 = \ldots = \hat{w}_m = \dfrac{1}{m},$

and $w_1 = w_2 = \ldots = w_m = \dfrac{1}{m}.$