Chapter 5

Inter-Departure Time

5.1 Idle Time Approximation

Following [12], let $N(i)$ be the inter-arrival time between $(i-1)$th and ith VBR cell arrival. Consider a MMBP/G/1 queueing system where VBR has an arrival process of MMBP and a service that may be extended by ABR cells with probability w. Let S be the service time of VBR cells, where

$$S = \begin{cases} 1, & \text{with probability } 1-w, \\ 2, & \text{with probability } w, \end{cases}$$

where

$$w = \frac{u_A}{1-u_V}.$$

$E[S] = 1 + w$ and $V[S] = 2w$, $E[S^2] = 1 + 3w$. Let $T(i)$ be the flow time of the ith VBR cell and Z be a generic steady-state waiting time. We define $I(i) = (N(i) - T(i-1))^+, i = 1, 2, \ldots$ and $\{I(i)\}, \{N(i)\}$ and $\{T(i)\}$ converges in distribution, we have

$$I \overset{d}{=} (N - T, 0)^+, \text{ where } \overset{d}{=} \text{ means equal in distribution.}$$

We also have,

$$\begin{aligned}((N-T,0)^+)^j &\stackrel{d}{=} (-1)^j((T-N)^j - ((T-N,0)^+)^j) \\ &\stackrel{d}{=} (-1)^j((T-N)^j - Z^j) \quad \text{for } j = 1,2,\ldots,n. \end{aligned} \tag{5.1}$$

We define

$$I \stackrel{\triangle}{=} N - \eta S.$$

where η denotes a scale value and S denotes service time. Hence, for any $n > 0$, and $I, S, N, \eta \geq 0$ the following equation holds

$$E[I^n] = E[(N-\eta S)^n].$$

When $n = 1$, we have

$$E[I] = E[N] - E[S]\eta = E[N] - (1+w)\eta.$$

When $n = 2$, we have

$$E[I^2] = E[N^2] - \frac{2(p(0)+p(1))}{p(1)r(0)+p(0)r(1)}(1+w) + (4w)\,\eta^2. \tag{5.2}$$

Then

$$V[I] = E[I^2] - E[I]. \tag{5.3}$$

5.2 Departure Processes of VBR

In this subsection, we consider the inter-departure time of VBR.

Suppose after an VBR is sent, the channel is occupied by a ABR, or an idle time, say a dummy idle source. Let Y be a discrete random variable for such a period of time until next VBR departs, i.e., it indicates in which there is no VBR occurrence until the $(n+1)$th sources when next VBR arrived. In addition, the VBR cell has a higher transmission non-preemptive than the ABR cell. Hence, the mean of inter-departure time between two VBR cells is equal to the mean of service time and idle time, It yields that the mean of inter-departure time is given by

$$E[Y] \quad = \quad E[S] + E[I]. \tag{5.4}$$

Next, to compute the variance of the inter-departure time, we use the formula

$$V[Y] \quad = \quad V[S] + V[I]. \tag{5.5}$$

which may be calculated from (5.2) and (5.3).

From (5.4), clearly, the expect value of inter-departure time of VBR cells is function with respect to u_V. And from (4.6), we get that the value of u_V is a function of $r(\cdot)$ and $y(\cdot)$. If probabilities of $r(\cdot)$ and $y(\cdot)$ are given which are increasing, the probability of utilization of the channel of VBR is also increasing, relatively, mean of inter-departure time of VBR is small, so is variance of inter-departure time of VBR. From math formula, VBR cells directly have effect mean of inter-departure time of ABR sources, so is variance of inter-departure time of ABR. Conversely, ABR cells do not influence anything of inter-departure time of VBR cells. Next section will also show numerical data to verify derived results.

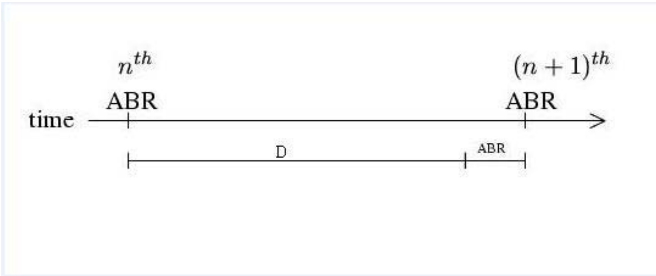
5.3 Departure Processes of ABR

To compute the inter-departure time of ABR, we consider the following condition:

Suppose after an ABR is sent, the channel is occupied by a VBR, or an idle time, say a dummy idle source. Let D be a discrete random variable for such a period of time until next ABR departs. That is, it indicates a length in which there is no ABR occurrence until the $(n+1)$th cells. It is shown in Figure 5.1. Therefore, D has a geometric distribution with parameter w, namely,

$$Pr\{D = j\} = (1-w)^j(w), \quad j = 0, 1, 2, \cdots .$$

Figure 5.1: Illustration for ABR departures



It yields that the mean of inter-departure time of ABR cells is given by

$$\begin{aligned} E[D] &= \sum_{j=0}^{\infty} j Pr\{D=j\} \\ &= \sum_{j=0}^{\infty} j(1-w)^j (w) \\ &= w(1-w)\sum_{j=1}^{\infty} j(1-w)^{j-1} \\ &= w(1-w)\sum_{j=1}^{\infty} \frac{d}{d(1-w)}(1-w)^j \\ &= w(1-w)\frac{d}{d(1-w)}\left(\sum_{j=1}^{\infty}(1-w)^j\right) \\ &= w(1-w)\frac{d}{d(1-w)}\left(\frac{1-w}{1-(1-w)}\right) \\ &= \frac{1-w}{w}. \end{aligned} \tag{5.6}$$

Lemma 5.1 *The following equation is valid and the value of its range is* $[0, \infty)$*,*

$$E[D] = \frac{1-w}{w}, \quad 0 < w \leq 1.$$

Proof:
It is obviously that $\frac{1-w}{w}$ is greater than zero for w between 0 and 1 and $\eta > 0$, then $E[D]$ exists. And claim that the value of its range be between 0 and ∞.

Because $\frac{1-w}{w}$ is continuous on $(0, 1]$, it is a monotone function on $(0, 1]$ and is greater than zero. The range of its value is $[0, \infty)$. □

Next, to compute the variance of the inter-departure time, we use the formula

$$\begin{aligned} V[D] &= E[D^2] - E^2[D] \\ &= \sum_{j=0}^{\infty} j^2(1-w)^j(w) - \left(\frac{1-w}{w}\right)^2 \\ &= \frac{2-3w+w^2}{w^2} - \left(\frac{1-w}{w}\right)^2 \\ &= \frac{1-w}{w^2}. \end{aligned} \tag{5.7}$$

By (5.6) and (5.7), we could summarize that the values of $E[D]$ and $V[D]$ both depends on w. However, w can be determined by parameters g_1, g_2, $r(\cdot)$ and $y(\cdot)$. In other words, the matrix has constructed and $\pi_{(s,q,z)}$ are calculated while parameters g_1, g_2, $r(\cdot)$ and $y(\cdot)$ are given. Then, the w also be obtained immediately from (4.7). From (5.6), we see that if the value of the w be small, then the mean of inter-departure time $E[D]$ is large and if the value of the w is large, then the mean of inter-departure time $E[D]$ is small. From (5.7), we get that if the value of the w be small, then the variance of inter-departure time $V[D]$ be large and if the value of the w is large, then the variance of inter-departure time $V[D]$ is small. This w is small or large means which indicates the utilization of the channel of ABR. In this subsection, we obtain some results that the w is concerned with g_1 and g_2, namely, g_1 and g_2 would control the value of w. Intuitively, we observe under the rules of all systems to discovery if the probability of the utilization of the channel of ABR is small, the mean of inter-departure time $E[D]$ is large, and the variance of inter-departure time $V[D]$ is large. Next section will show numerical results to verify derived results.

From (5.6) again, we obtain that $E[D]$ is a function of w, and w is a function of g_1 and g_2. It is a composite function of g_1 and g_2, i,e,

$$E_D(g_1, g_2) = E[D]. \tag{5.8}$$

Having g_2 fixed, we find the partial derivative of (5.8) with respect to g_1,

$$\begin{aligned}\frac{\partial E_D(g_1, g_2)}{\partial g_1} &= \frac{dE_D(w)}{dw} \cdot \frac{\partial w(g_1, g_2)}{\partial g_1} \\ &= \frac{-1}{w^2(g_1, g_2)} \cdot \frac{\partial w(g_1, g_2)}{\partial g_1}.\end{aligned}$$

Lemma 5.2 *If* $\frac{\partial w(g_1,g_2)}{\partial g_1} > 0$*, then* $E_D(g_1, g_2)$ *is non-increasing as* g_1 *increases.*

We observe that the probability of the system without ABR cells at buffer decreases as g_1 increases, implying the utilization of ABR cells on channel is increasing. Then, the partial derivative of (5.8) with respect to g_1 is negative and the mean of inter-departure time of ABR sources decreases.

If one has g_1 fixed, to find the partial derivative of (5.8) with respect to g_2, it gives

$$\begin{aligned}\frac{\partial E_D(g_1, g_2)}{\partial g_2} &= \frac{dE_D(w)}{dw} \cdot \frac{\partial w(g_1, g_2)}{\partial g_2} \\ &= \frac{-1}{w^2(g_1, g_2)} \cdot \frac{\partial w(g_1, g_2)}{\partial g_2}.\end{aligned}$$

Lemma 5.3 *If* $\frac{\partial w(g_1,g_2)}{\partial g_2} < 0$*, then* $E_D(g_1, g_2)$ *is increasing as* g_2 *decreases.*

We also observe that the probability of the system without ABR cells at buffer increases as g_2 decreases, implying the utilization of ABR cells at channel is decreasing and the mean of inter-departure time of ABR cells increases.

Suppose g_1 and g_2 are fixed. We observe u_A decreases as u_V increases which is a function of $r(\cdot)$. Thus, u_A decreases as $r(\cdot)$ increases. However, $E[D]$ increases when $r(\cdot)$ increases and w decreases.