Chapter 6

Conclusions and Future Researches

In the preceding five chapters we have presented theoretical and computational results of differential approaches for valuating American options or AEOs. We now summarize the contributions of this work and present a number of directions for future research.

6.1 Conclusions

6.1.1 Single Asset American Option Pricing Problems

We consider a parabolic FBP. Under some given assumptions, we have shown that the solutions of this problem are strictly increasing. Moreover, we have obtained that the free boundary $s(t)$ is a concave function. The American call pricing model derived by Merton [59] is a special case of this parabolic FBP. By using our results, a rigorous verification of the concavity of the optimal exercise boundary is obtained. The concavity of the optimal exercise boundary provides a useful information to derive its asymptotic formula.

6.1.2 American Exchange Option Pricing Problems

An AEO pricing model together with the optimal exercise ratio are modelled as an FBP. This FBP can be converted into an IE by using the Green's function. Meanwhile, the formula of this optimal exercise ratio is implicit in the solution of the IE. For the perpetual AEO, we obtained the value of optimal exercise ratio and the explicit pricing formula. For the finite time horizon AEO, we provide a rigorous verification of some properties for the free boundary and propose an asymptotic solutions of the IE for the cases of $q_1 > q_2$ and $q_1 = q_2$, respectively. We also extend the numerical method of one variable integral recursive method proposed by Kim [36] to the case of two variables. Compared with this numerical solution, our asymptotic solution of the IE is very close to the numerical solution as time near to maturity.

6.1.3 Optimization Approaches for Pricing an Option

We have proposed two MINLP models for valuating a European option and an American option. We have shown that both models can be solved by their NLP relaxations. The use of mathematical programming framework can easily extend to model certain exotic options. For a modest size of model, we can easily get a solution from NLP software package. The numerical results reveal that the NLP models reduces the computational time rapidly. Moreover, a self-finance model is developed for the writer's problems. When the market price is less than the fair price, the solution of this model will provide an optimal portfolio, that minimizes the expected loss for writers.

6.2 Future Researches

We now discuss a number of possible directions for future research.

6.2.1 Incomplete Markets

In Chapter 3, we have obtained the properties of the optimal exercise boundary and derived the accurate approximate or numerical solutions for pricing of the American options under the perfect market assumptions. However, the transaction costs or the liquid constraints are not considered in our models. Under the assumption of the discrete time hedge policy, Hoggard et al. [30] derived a nonlinear BS equation, which extends the BS equation to incorporate the transaction costs. Therefore, we would like to investigate the FBP of this nonlinear parabolic equation.

In Chapter 5, we have developed MINLP model for valuating the American option in the perfect market. To investigate the effect of liquidity or the bid-ask spread for American option valuation problems, we would like to limit the volume of holding the American option or replace $\mathbf{S} = \{S_t^i\}$ to $\mathbf{S}_{\text{bid}}$ and $\mathbf{S}_{\text{ask}}$ in model A and analyze the properties of this modified model.

6.2.2 American Spread Option Valuation Problems

The final payoff of a spread option is given as

$$\max(S_1 - S_2 - K, 0).$$

The American spread option (ASO) valuation problems are model as a two variable parabolic FBP. In Chapter 4, we have provided the properties and an asymptotic formula of the FBP for valuating the AEO, which is a special case of the ASO with $K = 0$. Therefore, we have an interest in investigation asymptotic formulas for valuating an ASO.

On the other hand, we have shown that the free boundary of a single variable parabolic FBP is a convex set in Chapter 3. Therefore, it is interesting to investigate the convexity of the optimal exercise region for the ASO.

6.2.3 Martingale Probability Measure for the American Option

In Chapter 5, we have proposed an no-arbitrage American option pricing model and shown that this model can be solved by its NLP relaxation. Rogers [64] and king [38] showed the equivalence between the no-arbitrage condition and the martingale representation. Therefore, there may exist a martingale probability that makes the asset price is a martingale by analyzing the duality of its NLP relaxation. Furthermore, we will show that the value of the American option at each state is equal to the conditional expectation of all the possible future payoff under the synthesis probability measures in my future studies.