Abstract

In the past three decades, a great deal of effort has been made on solving the free boundary problem (FBP) arising from American option valuation problems. In this dissertation, we show that the solutions, the price and the free boundary, of this FBP are increasing functions. Furthermore, we provide a rigorous verification that the free boundary of this problem is concave. Our results imply that the optimal exercise boundary of an American call is a strictly decreasing concave function of time. These results will provide a useful information to obtain an asymptotic formula for the optimal exercise boundary.

For pricing of American exchange options (AEO), we convert the associated FBP into a single variable integral equation (IE) and provide a formula for valuating the perpetual AEO. For the finite horizon AEO, we propose an asymptotic solution as time is near to expiration and develop a numerical method for its optimal exercise boundary. Compared with the computational results, the values of our asymptotic solution are close to the computational results as time is near to expiration.

For valuating American options, we develop a mixed integer nonlinear programming (MINLP) model. The solution of the MINLP model provides a hedging portfolio for writers, the optimal trading strategy for buyers, and the fair price for American options at the same time. We show that it can be solved by its nonlinear programming (NLP) relaxation. The numerical results reveal that the use of NLP relaxation reduces the computation time rapidly. Moreover, when the market price is less than the fair price, we propose a minimum expected loss model. The solution of this model provides a hedging strategy that minimizes the expected loss for the writer.