Chapter 2

Literature Review

In the past three decades, a considerable number of research has been proposed to valuate the American options. In this chapter, we explore historical and intellectual developments in areas related to the study in this dissertation.

2.1 American Option Pricing Problems

Free Boundary Problems

Merton [59] first proposed an FBP for pricing the American option and derived an exact value of the perpetual American option. Gerber and Shiu [24] applied the method of Esscher transforms and the martingale approach to pricing one or two assets perpetual American options and obtained the same results as Merton. Ekström [17] showed that optimal exercise boundary for the American put is convex in the standard Black-Scholes model.

Asymptotic Solutions for the Free Boundary

Several researchers such as [9], [11], [18], [32], [41] converted the FBP into an IE. They use this IE to find an analytic approximation to the early exercise boundary when the expiration date is finite. Kuske and Keller [41] as well as Evans et al. [18] solved this integral equation asymptotically when the remaining time is near to zero. Chen [11] derived rigorously high order asymptotic expansions for the early exercise boundary near to expiration. Goodman and Ostrov [26] studied the short time behavior of the optimal exercise boundary for American put and obtained an asymptotic expansion through iteration using a boundary integral equation. Knessl [37] analyzed the price of an American option in various asymptotic limits, that includes situations where the interest and dividend rate are large or small, compared to the volatility of the asset, by using the perturbation methods.

Numerical Solutions

Recently, the numerical method and the simulation-based approaches are proposed to valuate the American-style options. Koloder [39] pointed out that the IE suitable for iteration-like nonlinear Volterra equations, so that they are amenable to numerical techniques. MacMillan [56], and Barone-Adesi and Whaley [3] used numerical methods such as the fixed-point method or Newton's method to obtain the early exercise boundary and provided other approximate solutions of this FBP. Kim [36] solved this integral equation numerically and recursively. Huang [31] followed this spirit to compute option hedge parameters and extended this method to price other kinds of options. These approaches provide a formula to valuate the long-term American option. However, one disadvantage of these approaches is time-consumption.

2.2 American Exchange Option Pricing Problems

No Dividends AEO

An American exchange option is an option which gives the holder the right but not obligation to exchange one asset to another at any time prior to the expiration date. Margrabe [55] provided a pricing formulate of European exchange option (EEO) and also showed that the value of the AEO on the non-dividend underlying assets price is equal to the European counterpart. Naturally, exercising AEO may not only lose the right to prevent the risk that comes from the variation of the price of asset 2 but also take another risk from asset 1. Hence there is no reason to exercise AEO prematurely. However, if the underlying assets pay the dividends, then the holder may exercise early the expiry at a proper time for receiving the remuneration from asset 1. This means that the holder of AEO have a nonnegative probability to obtain more return than holding the corresponding EEO. So the price of this AEO is generally higher than the European counterpart.

Asymptotic and Numerical Solutions

Finding the explicit representation of the early exercise boundary becomes a challenge to solve the FBP. Liu and Liu [46] extended the method of Evans et al. [18] and provided an asymptotic pricing formula as time nears to the maturity date. For the perpetual AEO, Liu and Liu [47] propose an exact solution of this FBP.

For the numerical methods, Carr [9] generalized the Gesker-Johnson approach [23] to AEO on dividend-paying assets. The value of a general pseudo-AEO is represented as a formula of the standard multi-normal distribution. The valuation formula becomes cumbersome for a large number of exercise points. Longstaff and Schwartz [53] proposed the least squares Monte Carlo that can obtain a good price estimates very fast in practice. Rogers [65] proposed a dual representation of American option prices and computed the upper bounds for several types of American option using Monte Carlo simulation.

2.3 Optimization Approaches for Option Pricing Problems

There are a great number of studies have been made in dealing with the theoretical price of an option by using the technique of mathematical programming.

Linear Programming on Finite Difference Methods

The American option pricing problems can usually be formulated as an FBP. Dempster and Hutton [15] proposed a linear programming formulation for solving an FBP by using the finite difference approximations. It is easy to implement using a commercial solver such as CPLEX or XPRESS. However, a commercial solver is a general purpose algorithm which may not carry out a solution in an acceptable time. Dempster, Hutton, and Richards [16] proposed a special simplex solver for tridiagonal constraint matrices, exploiting the rapid LU decomposition algorithms for such matrices, which produces dramatic speed-ups.

Stochastic Programming Perspective on Contingent Claims

King [38] modelled the asset price process as a scenario tree and proposed a stochastic programming model for the hedging of contingent claim in the discrete time and discrete state setting. By analyzing the duality of the stochastic model, he obtain the equivalence of absence of arbitrage and the existence of a probability measure that makes the price process into a martingale. An extension of the model to incorporate pre-existing liabilities and endowments reveals the reasons why buyers and sellers trading in options are also proposed in [38].

Following King's framework, Liu and Liu [51] proposed a stochastic programming for concerning a European contingent claim (ECC). They showed that the value of ECC satisfies the conditional expectation of the discounted final payoff. Moreover, the solution of their model can be used as the perfect hedging strategies of ECC.

To analyze the American contingent claim in the incomplete markets, Pennanen and King [62] proposed a convex programming model, that is an extension of King's model, and obtained the martingale-expressions for seller's and buyer's prices. Liu and liu [50] proposed a stochastic arbitrage model, which involves trading the American option and asset in the frictionless market, and analyzed the arbitrage model by using the duality theory of mathematical programming. They showed that the initial value of the American option is equal to the expectation of all the future possible payoff.

By using the stochastic programming approach, Wu and Sen [69] developed currency option hedging models, which incorporate constraints on sensitivity measures such as Delta and Gamma, in an imperfect market. They illustrated that their model can provide better performance than other myopic hedging model and is a practical computational tool for realistic hedging problems.

Recovering Probability Measure form Observed Market Prices

To recover the risk-neutral probability distribution of an underlying asset price from the contemporaneous price of its associated options, Rubistein [66] proposed a quadratic program, that is minimizing the sum of the squared differences between the prior and posterior probabilities. Taking the example of stock market crash of October 1987, Jackwerth and Rubistein [33] pointed out that the risk-neutral probability of three standard deviation decline in the index is about 10 time more likely than under the assumption of lognormality. Based on the quadratic program, they proposed a fast optimization model, that maximizes the smoothness of the resulting distribution, for estimating probability distributions. The advantage of these methods are that they do not make the assumptions of the stochastic processes for the asset's price and the investors' preferences.

In recent years, a considerable number of research is concentrated on finding the new approaches to recover the asset price's probability from the observed market option price. Liu [45] proposed a single-period, finite states arbitrage model and derived a feasible problem from the arbitrage model by using the Lagrangian multiplier method. Based on the feasible problem, he constructed a linear programming model to recover the risk-neutral probability from the observed asset prices. Chang [10] proposed a linear programming model to recover the risk-neutral probability distribution of asset price from its associated market option price and evaluated the fair price of options by the resulting risk-neutral probability distribution. Liao [44] proposed a biobjective nonlinear programming model to derive risk-neutral probability distribution of the underlying asset. Given a non-arbitrage observed option price, this model recovers a risk-neutral probability distribution consistent with the observed price. By using the option's price, Chen [13] developed a two-person zero sum game to formula the participant behavior in the market and recover an implied risk neutral probability distribution for the asset. By applying data from Taiwan's option market, their empirical results shows that the recovered risk neutral probability does not satisfy the lognormal distribution.

From American option prices, Melick and Thomas [58] and Flamours and Giamouridis [19] developed a method for estimating the probability density function for futures prices. Melick and Thomas applied the resulting distribution for crude oil during the Persian Culf crisis and found that the distribution differs significantly from recovered using standard techniques. Flamours and Giamouridis relaxed the restriction of the lognormality for the underlying asset and found that the recovered distributions are more robust than those recovered with a model, which assumes a mixture of two lognormal distributions.

Optimal Trading Strategy of Option Portfolios

Liu et al. [49] proposed an integer programming model which will provide an optimal trading strategy for constructing a portfolio with a series of same expiration options. Liu and Liu [48] investigated the option portfolio selection problem. They derived an arbitrage theorem and proposed two arbitrage models. The theorem provides a simple way to construct an arbitrage portfolio. The arbitrage models in their work eliminate the uncertainty successfully due to dynamic of asset prices.

There are many research investigate the dynamic asset allocation by maximizing the return and/or minimizing the risk under a given utility function. The choose of utility function of an individual is depending on his or her risk tolerance and financial condition. Luenberger [54] provides a systematical way for assigning an appropriate utility function to an investor, some of which are quite elaborate. Guo and Yen [28] derive a closed-form solution of optimal dynamic asset allocation for institutional investors. They determine the extra risk by empirical results and define the utility function of the institutional investor as a negative exponential utility function. Pedersen [61] proposes a utility function for capturing trade-offs between return and a large body of risk and shows that this function forms a basis for an extension to the capital asset pricing model.

2.4 Applications to Real Options

There are a considerable number of studies [4, 27] that model the real option as an American option or an AEO when early exercise is positive possibility. Rhys et al. [63] investigated the time of real option exercise and provided the expected waiting time before an option is exercised by using the first passage time approach. McDonald and Siegel [57] first modelled an investment timing problem as an AEO and described the installing price and the project value as two Geometric Brownian motion. Lee and Paxson [52] also valuated the R&D program as an American exchange option. Their models assume that both the R&D values and the R&D cost follow geometric diffusion processes.

Mathematical programming is the most important tool to deal with the path-dependency problem of real options valuation. Wnag and Neufville [68] developed a stochastic mixed-integer programming model, that introduces a framework for exploring real option in physical systems. Their model forcus on identifying the best way to build flexibility into the design but not on valuating individual options. Messian and Bosetti [60] consider land allocation problems. They develop a discrete model that includes both environmental and economic uncertainty treated and used Arrow-Fisher [1] quasi option value to derived decision rules that account for different levels of flexibility of land allocation probabilities.