

Chapter 2

The Maximum Satisfaction Method

In this chapter, we apply the **MSM** to solve a problem of QoS on All-IP networks. Especially, we focus on obtaining the maximum value of the satisfaction function and give numerical examples to illustrate an easy implementation of MSM.

2.1 The Two Phases: MBM and BRAM

The first phase

Given a total budget B , the objective of Maximum Bandwidth Model (**MBM**) is to assign bandwidth to the network as much as possible with considering the requirement of each class but under the total budget constraint. The Mathematical program of MBM is given as follows:

$$\text{Maximize} \quad M = \sum_{e \in O_1} x_e \quad (7)$$

$$\text{Subject to} \quad \sum_{e \in L} k_e x_e \leq B \quad (8)$$

$$\sum_{e \in I_j} x_e = \sum_{e \in O_j} x_e, \forall e \in L \quad (9)$$

$$j \in N$$

$$x_e \leq U_e, e \in L$$

where M is the total amount of projective bandwidth, and x_e is the bandwidth allocated to the link e . Without constraint (8), MBM is simply a maximum

flow problem. In this model, it only involves the relation between budget and bandwidth; there is no random variable. Since it is a deterministic linear programming problem, we can solve it by a popular package, such as MATLAB [5].

The second phase

Next, we introduce **BRAM** by employing mean values of number of connections in the optimization model. Let λ_i be a random variable which represents the number of connections for the i^{th} class. Denote by $E[\lambda_i]$ the expected value of $\lambda_i, i = 1, 2, \Lambda, m$. **BRAM** is constructed after M and $x_i(e)$ are obtained in (7)~(9). Given the weights $w_i \in (0, 1)$ and the bandwidth requirements for each class, i.e., r_1, r_2, Λ, r_m , **BRAM** is modeled in the following:

$$\text{Maximize} \quad F = w_1 \times f_1(\theta_1) + w_2 \times f_2(\theta_2) + \Lambda + w_m \times f_m(\theta_m) \quad (10)$$

$$\text{Subject to} \quad E[\lambda_1] \times \theta_1 + E[\lambda_2] \times \theta_2 + \Lambda + E[\lambda_m] \times \theta_m = M \quad (11)$$

$$\sum_i x_i(e) \theta_{ie} = x_e, \forall e \in L. \quad (12)$$

$$E[\lambda_i] \times \theta_i = \sum_{e \in O_i} \theta_{ie}, \forall e \in L$$

$$\theta_1 \geq r_1, \quad \theta_2 \geq r_2, \quad \Lambda, \theta_m \geq r_m,$$

where θ_i is the bandwidth allocated to each i , and θ_{ie} is the bandwidth allocated to each i on the link e . In the model, we will be able to decide how to allocate the bandwidth to each class in order to maximize the satisfaction function of the network. Substituting M in (11) by that in (7), we obtain the maximum value of the satisfaction function after solving (10).

Afterwards, there are two questions: is the output F in (10) increasing

when the input M is increasing as we fix $(E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m])$ in **BRAM**? Secondly, whether or not the value of F by applying $(E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m])$ is the same as the average value of F by taking random samples of $(\lambda_1, \lambda_2, \Lambda, \lambda_m)$ when we fix M ? We shall answer the first question by the following theorem but verify the second question by numerical simulation in next sections.

Theorem 2.1

Let $0 \leq \tau^{(1)} \leq \tau^{(2)} \leq \Lambda \leq \tau^{(n)}$ and $F(\tau^{(1)})=F^{(1)}, F(\tau^{(2)})=F^{(2)}, \Lambda, F(\tau^{(n)})=F^{(n)}$. Then the outputs $F^{(1)}, F^{(2)}, \Lambda, F^{(n)}$ are increasing when $\tau^{(1)}, \tau^{(2)}, \Lambda, \tau^{(n)}$ are increasing and $(E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m])$ are fixed in **BRAM**.

Its proof is trivial since F is assumed to be concave, e.g., a sum of utilization functions. The optimum value of (10) is increasing as M is increasing when the constraints construct a convex set. Therefore, the output F increases when M increases in **BRAM**. In section 2.4, we will check this property by numerical simulation.

MSM includes two parts, i.e., **MBM** and **BRAM**. We present it as follows:

- Step 1.** Given $\lambda_1, \lambda_2, \Lambda, \lambda_m$ compute the expected value $E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m]$.
- Step 2.** Input $(E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m])$ and solve **MBM** to decide the routing paths and to find M .
- Step 3.** Input M and solve **BRAM** to find the value of satisfaction function.

Suppose constraint (11) is calculated at random. That is it replaces $E[\lambda_i]$ by λ_i but keep others unchanged. In this case the objective function

value F becomes a random variable, too. According to the distribution of λ_i , we will carry out a simulation test to verify that the expected value of F is equivalent to the optimal value of (10) with the expected values of λ_i , statistically. That is $F(E[\lambda_i]) = E[F(\lambda_i)]$. The simulation procedure is provided in the following. Its flowchart is drawn in **Figure 2.1**.

Simulation Procedure

Step 0. Generate 30 random samples (a_1, a_2, Λ, a_m) of Poisson distributions with respect to each means $E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m]$.

Set $k = 0$.

Step 1. $k = k + 1$. Replace $(E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m])$ in **BRAM** by

$(a_1, a_2, \Lambda, a_m)^k$ which is the random sample at the k^{th} run in simulation.

Step 2. Input $M, (a_1, a_2, \Lambda, a_m)^k$, find F by solving BRAM and obtain the bandwidth allocated to each i .

Step 3. If $k \geq 30$, go to **Step 4**; otherwise go to **Step 1**.

Step 4. Stop and calculate the average of satisfaction function, the average of bandwidth allocated to each i .

Finally, we check whether the satisfaction function value obtained from MSM falls into the 95% confidence interval of the satisfaction obtained by simulation. We give numerical examples to illustrate our approach in the following sections.

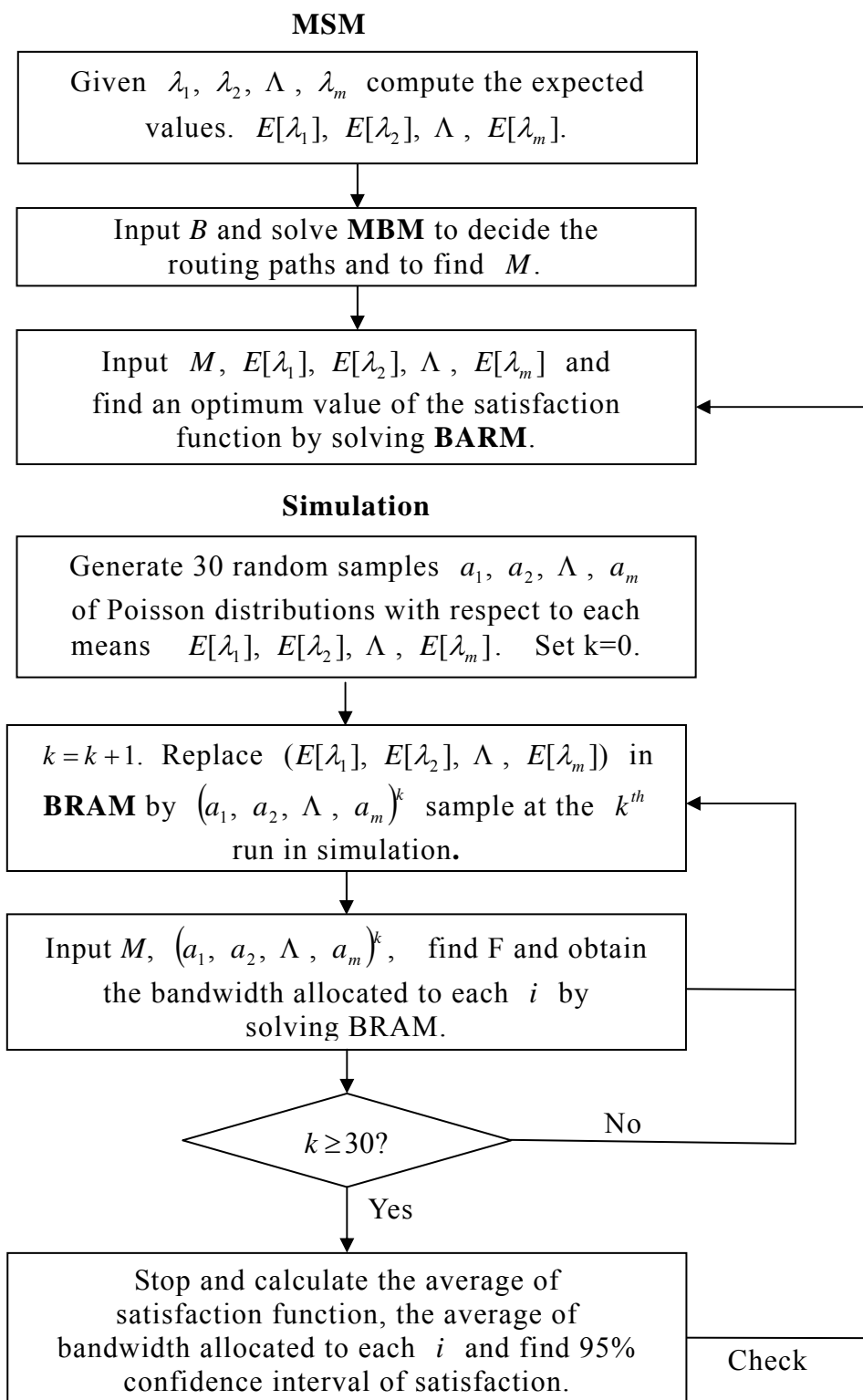


Figure 2.1 MSM

2.2 Example 1 of MBM Model

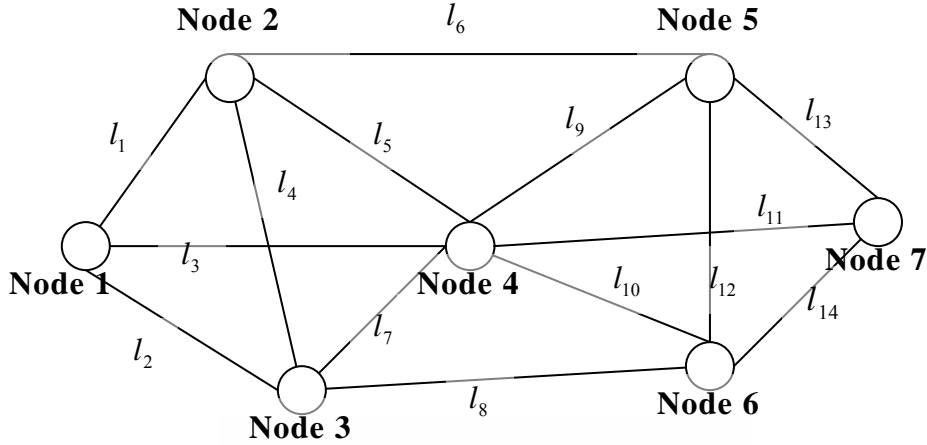


Figure 2.2 A Sample Network

A network topology $G = (N, L)$ as shown in **Figure 2.2** [13], where $N = \{\text{Node 1, Node 2, ..., Node 7}\}$ and $L = \{l_i, i = 1, 2, \dots, 14\}$. Let Node 1 and Node 7 be the source and destination respectively. Each connection is delivered from source to destination. The purchasing cost of bandwidth for each link are: $k_1 = \$5$, $k_2 = \$6$, $k_3 = \$10$, $k_4 = \$5$, $k_5 = \$4$, $k_6 = \$11$, $k_7 = \$6$, $k_8 = \$8$, $k_9 = \$6$, $k_{10} = \$7$, $k_{11} = \$12$, $k_{12} = \$6$, $k_{13} = \$5$, and $k_{14} = \$6$. The maximal capacity of each link are: $U_1 = 2,300$ kbps (i.e., kilobits/sec), $U_2 = 3,500$ kbps, $U_3 = 1,000$ kbps, $U_4 = 2,500$ kbps, $U_5 = 2,100$ kbps, $U_6 = 2,200$ kbps, $U_7 = 2,000$ kbps, $U_8 = 3,000$ kbps, $U_9 = 2,100$ kbps, $U_{10} = 2,700$ kbps, $U_{11} = 1,500$ kbps, $U_{12} = 1,800$ kbps, $U_{13} = 3,000$ kbps, and $U_{14} = 3,500$ kbps. The plan is set under the total available budget $B = \$100,000$. There are three different classes of sessions as shown in Table 2.1.

Table 2.1 Three different classes of sessions

class	Bandwidth requirement	Average number of connections
1	167 kbps	10
2	83 kbps	10
3	28 kbps	10

In **MBM**, we consider the objective function as $x_1 + x_2 + x_3$.

Maximize $x_1 + x_2 + x_3$

Subject to

$$5x_1 + 6x_2 + 10x_3 + 5x_4 + 4x_5 + 11x_6 + 6x_7 + 8x_8 + 6x_9 + 7x_{10} + 12x_{11}$$

$$+ 6x_{12} + 5x_{13} + 6x_{14} \leq 100000$$

$$x_1 - x_4 - x_5 - x_6 = 0$$

$$x_3 + x_5 + x_7 - x_9 - x_{10} - x_{11} = 0$$

$$x_2 + x_4 - x_7 - x_8 = 0$$

$$x_6 + x_9 + x_{12} - x_{13} = 0$$

$$x_8 + x_{10} - x_{12} - x_{14} = 0$$

$$x_e \leq U_e$$

In order to maximize the objective function in **MBM**, we assume the budget B shall be used to buy the total amount of bandwidth as much as possible.

We solve **MBM** by MATLAB 6.5 [5], and set:

$$A = \begin{bmatrix} 5 & 6 & 10 & 5 & 4 & 11 & 6 & 8 & 6 & 7 & 12 & 6 & 5 & 6 \\ 1 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; b = \begin{pmatrix} 100000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2300 \\ 3500 \\ 1000 \\ 2500 \\ 2100 \\ 2200 \\ 2000 \\ 3000 \\ 2100 \\ 2700 \\ 1500 \\ 1800 \\ 3000 \\ 3500 \end{pmatrix}$$

After applying `FMINCON(inline('-x1-x2-x3'), [1 ; 1], A, b, [], [], [0; 0], [])` we obtain the optimal solution in the following table:

Table 2.2 The optimal solution

x_1	x_2	x_3	x_4	x_5
2000 kbps	3000 kbps	0 kbps	0 kbps	2000 kbps
x_6	x_7	x_8	x_9	x_{10}
0 kbps	0 kbps	3000 kbps	2000 kbps	0 kbps
x_{11}	x_{12}	x_{13}	x_{14}	M
0 kbps	0 kbps	2000 kbps	3000 kbps	5000 kbps

The cost of the resource allocation is \$100000. There are two optimal paths:

1. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 2000 kbps.
2. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 3000 kbps.

2.3 Example 1 of BRAM Model with $w_1=w_2=w_3$

2.3.1 Calculate the satisfaction by the expected number of connections

Given Topology in **Figure 2.2**, suppose the expected value of λ_i is 10 for the i^{th} class and each class is regarded as important as each other. Thus, each objective function has equal weight $w_1 = w_2 = w_3 = \frac{1}{3}$. Set $M = 5000$ kbps as given in Section 2.2.

In the **BRAM**, given the requirements for each class i , we consider the objective function $w_1 f_1(\theta_1) + w_2 f_2(\theta_2) + w_3 f_3(\theta_3)$, where $f_1(\theta_1) = \ln(\theta_1/167)$, $f_2(\theta_2) = \ln(\theta_2/83)$, $f_3(\theta_3) = \ln(\theta_3/28)$, and we set the constraint: $10 \times \theta_1 + 10$

$\times \theta_2 + 10 \times \theta_3 = M$. By computing FMINCON (inline(' -1/3 * ln(θ_1 /167) -1/3 * ln(θ_2 /83) -1/3 * ln(θ_3 /28)'), [1 ; 1 ; 1], [10 10 10], [5000], [], [], [167; 83; 28], []), we obtain the optimal solution as follows:

$$\theta_1 = 185.0181 \text{ kbps}$$

$$\theta_2 = 130.0312 \text{ kbps}$$

$$\theta_3 = 184.9507 \text{ kbps.}$$

Hence we select a path $l_1, l_2, l_5, l_8, l_9, l_{13}, l_{14}$, and obtain the optimal value that is $\frac{1}{3} \ln(\theta_1 /167) + \frac{1}{3} \ln(\theta_2 /83) + \frac{1}{3} \ln(\theta_3 /28) = 0.8113$. The routing path and bandwidth allocation to each class are:

Class 1. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 1850.181 kbps.

Class 2. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 149.819 kbps.

$l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 1150.493 kbps.

Class 3. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds Bandwidth of 1849.507 kbps.

Since the path $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$ is projected with 2000 kbps in total from MBM, it may be shared by Class 1 and part of Class 2. Likewise, $l_2 \rightarrow l_8 \rightarrow l_{14}$ is assigned for Class 2 and Class 3.

Next, the test problem is set for different values of M . Thus, we input M as 2500, 3000, 3500, 4000, 4500 and 5000 kbps, respectively. We obtain the results in the following table:

Table 2.3 The relation between M and the value of the satisfaction function

$M=$	2500 kbps	3000kbps	3500kbps
The value of the satisfaction function	-0.0653	0.2918	0.5018
$M=$	4000 kbps	4500kbps	5000kbps
The value of the satisfaction function	0.6451	0.7714	0.8113

From Table 2.3, it verifies the monotonic property that the output of **BRAM** is increasing when the input of M is increasing.

2.3.2 Simulation of example 1 with the same weights

In **BRAM**, the objective function is

$$\frac{1}{3}\ln(\theta^1/167) + \frac{1}{3}\ln(\theta^2/83) + \frac{1}{3}\ln(\theta^3/28).$$

Generate 30 random samples

(a_1, a_2, a_3) of Poisson distribution. The expected value of a_i is 10, for all i .

After simulation, we have that the average of the bandwidth allocated to class 1 is 1917.842342 kbps, the average of the bandwidth allocated to class 2 is 1354.681251 kbps and the average of the bandwidth allocated to class 3 is 1727.476333 kbps. The average of the objective function is 0.78, and a sample variance is 0.042399. Assume satisfaction follows a normal distribution. Therefore, the 95% confidence interval of the satisfaction is $[0.7063, 0.853684]$. In Section 2.3.1, the satisfaction is 0.8113 which falls in the interval $[0.7063, 0.853684]$.

2.4 Example 1 of BRAM model with different weights

2.4.1 Calculate the satisfaction by the expected number of connections

Given Topology in **Figure 2.2**, suppose the expected value of λ_i is 10 for all i . Each objective function has different weights. Assume $w_1 = 0.6$, $w_2 = 0.2$, $w_3 = 0.2$, and $M = 5000$ kbps.

We solve **BRAM** models by MATLAB 6.5. After applying `FMINCON`(`inline('-1/3*ln(θ_1 /167)-1/3*ln(θ_2 /83)-1/3*ln(θ_3 /28)'),[1 ; 1 ; 1],[10 10 10],[5000],[],[],[167; 83; 28],[,]) we obtain the optimal solution as follows:`

$$\theta_1 = 221.8607 \text{ kbps}$$

$$\theta_2 = 121.2385 \text{ kbps}$$

$$\theta_3 = 156.9008 \text{ kbps.}$$

Hence we select a path $l_1, l_2, l_5, l_8, l_9, l_{13}, l_{14}$, and the obtain optimal value is $0.6 * f_1(\theta_1) + 0.2 * f_2(\theta_2) + 0.2 * f_3(\theta_3) = 0.639$.

The routing path and bandwidth allocation to each class are:

Class 1. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 2218.607 kbps.

Class 2. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 430.992 kbps.

$$l_2 \rightarrow l_8 \rightarrow l_{14}, \text{ which holds bandwidth of } 781.393 \text{ kbps.}$$

Class 3. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 1569.008 kbps.

2.4.2 Simulation of example 1 with different weights

After simulation, we have that the average of the bandwidth allocated to class 1 is 2747.9 kbps, the average of the bandwidth allocated to class 2 is 1064.5 kbps, and the average of the bandwidth allocated to class 3 is 1187.6 kbps. The average of the objective function is 0.65, and its variance is 0.039. Thus, the 95% confidence interval of the satisfaction is $[0.5793, 0.7207]$, assuming the distribution is normal. In Section 2.4.1, the satisfaction is 0.639 which falls in the interval $[0.5793, 0.7207]$.

The routing path and bandwidth allocation to each class are:

Class 1. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 2747.9 kbps.

Class 2. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 812.4 kbps.

$l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 252.1 kbps.

Class 3. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 1187.6 kbps.

2.5 Example 2 with MBM Model

Let a network topology $G = \langle N, L \rangle$ as given in **Figure 2.2**, each connection flows from source to destination. Suppose the cost implicitly includes delay. The purchasing cost of bandwidth for each link and the maximal capacity of each link are the same as that at Section 2.2. Bandwidth requirements and the average number of connections are given in Table 2.4.

Table 2.4 The bandwidth requirement and Average number of connections

Class	Bandwidth requirement	Average number of connections
1	167 kbps	10
2	83 kbps	30
3	28 kbps	110

Under the total available budget $B = \$140000$, we solve **MBM** by MATLAB 6.5, and set:

$$A = \begin{bmatrix} 5 & 6 & 10 & 5 & 4 & 11 & 6 & 8 & 6 & 7 & 12 & 6 & 5 & 6 \\ 1 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; b = \begin{pmatrix} 140000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2300 \\ 3500 \\ 1000 \\ 2500 \\ 2100 \\ 2200 \\ 2000 \\ 3000 \\ 2100 \\ 2700 \\ 1500 \\ 1800 \\ 3000 \\ 3500 \end{pmatrix}$$

After applying FMINCON(inline('-x₁-x₂-x₃'),[1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1],A,b,[],[],[0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0],[]), we obtain the optimal solution as follows:

Table 2.5 The optimal solution

x_1	x_2	x_3	x_4	x_5
2300 kbps	3491.7 kbps	1000 kbps	0 kbps	1400 kbps
x_6	x_7	x_8	x_9	x_{10}
900 kbps	491.7 kbps	3000 kbps	2100 kbps	0 kbps
x_{11}	x_{12}	x_{13}	x_{14}	M
791.7 kbps	0 kbps	3000 kbps	3000 kbps	6791.7 kbps

2.5.1 Three classes have different weights

Suppose each class has different weights which are $w_1 = 0.6$, $w_2 = 0.2$, and $w_3 = 0.2$. After solving **BRAM**, we obtain the optimal solution as follows:

$$\theta_1 = 187.82 \text{ kbps}$$

$$\theta_2 = 40.847 \text{ kbps}$$

$$\theta_3 = 33.53 \text{ kbps.}$$

Hence we select a path $l_1, l_2, l_5, l_8, l_9, l_{13}, l_{14}$, and obtain the

optimal satisfaction value is 0.0285. In total, the bandwidth allocated to class 1 is 1878.2 kbps, the bandwidth allocated to class 2 is 408.47 kbps, and the bandwidth allocated to class 3 is 335.3 kbps.

2.5.2 Simulation of example 2

Generate 30 random samples a_1, a_2, a_3 with Poisson distribution. The expected value of λ_1 is 10, λ_2 is 30, and λ_3 is 110.

After simulation, we have that the average of the bandwidth allocated to class 1 is 1878.3 kbps, the average of the bandwidth allocated to class 2 is 1225.2 kbps, and the average of the bandwidth allocated to class 3 is 3688.2 kbps. The average value of the objective function is 0.0277, and the sample variance is 0.0019, so that the 95% confidence interval of the satisfaction is $[0.01974, 0.03566]$

In Section 2.6.1, the satisfaction is 0.0285 which falls in the interval $[0.01974, 0.0296]$

2.6 The Application on another All-IP Network

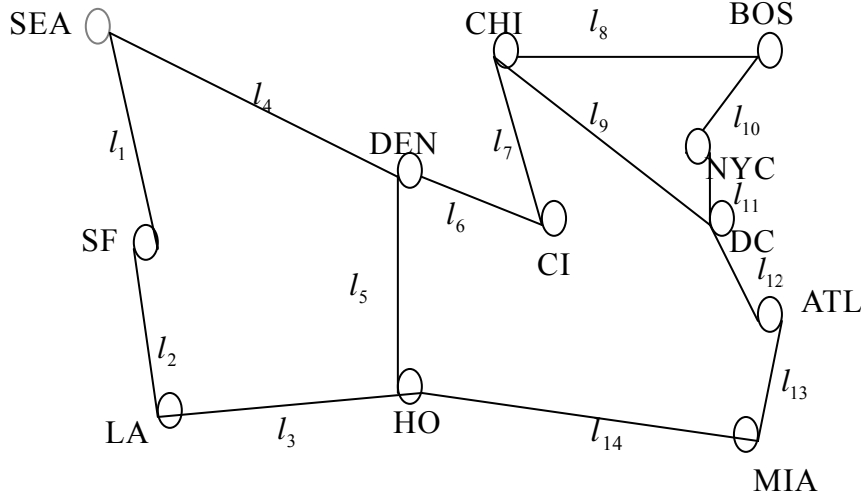


Figure 2.3 A Sample Network

A network topology $G = \langle N, L \rangle$ is depicted in [6], where $N = \{\text{SEA}, \text{SF}, \dots, \text{MIA}\}$ and $L = \{l_i, i = 1, 2, \dots, 14\}$. Let SEA and MIA be the source and the destination respectively. Each connection is delivered from source to destination. Given the purchasing cost of bandwidth for each link are: $k_1 = \$5$, $k_2 = \$6$, $k_3 = \$10$, $k_4 = \$5$, $k_5 = \$4$, $k_6 = \$11$, $k_7 = \$6$, $k_8 = \$8$, $k_9 = \$6$, $k_{10} = \$7$, $k_{11} = \$12$, $k_{12} = \$6$, $k_{13} = \$5$, and $k_{14} = \$6$. Given the maximal capacity of each link are: $U_1 = 2,300$ kbps (i.e. kilobits/sec), $U_2 = 3,500$ kbps, $U_3 = 1,000$ kbps, $U_4 = 2,500$ kbps, $U_5 = 2,100$ kbps, $U_6 = 2200$ kbps, $U_7 = 2000$ kbps, $U_8 = 3000$ kbps, $U_9 = 2100$ kbps, $U_{10} = 2700$ kbps, $U_{11} = 1500$ kbps, $U_{12} = 1800$ kbps, $U_{13} = 3000$ kbps, and $U_{14} = 3500$ kbps. The total budget available is set to $B = \$100000$. There are given three classes as shown in Table 2.6.

Table 2.6 The bandwidth requirement and Average number of connections

class	Bandwidth requirement	Average number of connections
1	167 kbps	5
2	83 kbps	5
3	28 kbps	10

Then in the network topology, the **MBM** is written in the following:

Maximize $x_1 + x_4$

Subject to

$$5x_1 + 6x_2 + 10x_3 + 5x_4 + 4x_5 + 11x_6 + 6x_7 + 8x_8 + 6x_9 + 7x_{10} + 12x_{11} + 6x_{12} + 5x_{13} + 6x_{14} = 100000$$

$$x_1 - x_2 = 0$$

$$x_1 - x_3 = 0$$

$$x_4 - x_5 - x_6 = 0$$

$$x_6 - x_7 = 0$$

$$x_7 - x_8 - x_9 = 0$$

$$x_8 - x_{10} = 0$$

$$x_8 - x_{11} = 0$$

$$x_9 + x_{11} - x_{12} = 0$$

$$x_{12} - x_{13} = 0$$

$$x_3 - x_5 - x_{14} = 0$$

$$x_e \leq U_e$$

Table 2.7 The optimal solution

x_1	x_2	x_3	x_4	x_5
1000 kbps	1000 kbps	1000 kbps	2500 kbps	2100 kbps
x_6	x_7	x_8	x_9	x_{10}
400 kbps	400 kbps	0 kbps	400 kbps	0 kbps
x_{11}	x_{12}	x_{13}	x_{14}	M
0 kbps	400 kbps	400 kbps	3100 kbps	3500 kbps

The cost for total bandwidth allocation is 74100 dollars.

2.6.1 Calculate the satisfaction with the expected number of connections

First, we solve **BRAM**. After applying FMINCON (inline ('-0.6*ln(θ_1 /167)-0.2*ln(θ_2 /83)-0.2*ln(θ_3 /28)'), [1 ; 1 ; 1], [5 5 10], [3500], [], [], [167; 83; 28], []) we obtain the optimal solution as follows:

$$\theta_1 = 384.8410 \text{ kbps}$$

$$\theta_2 = 177.5656 \text{ kbps}$$

$$\theta_3 = 68.7967 \text{ kbps.}$$

Hence we select a path $l_1, l_2, l_5, l_8, l_9, l_{13}, l_{14}$, and obtain the optimal satisfaction value is 0.83283. In total, the bandwidth allocated to class 1 is 1924.205 kbps, the bandwidth allocated to class 2 is 887.828 kbps, and the bandwidth allocated to class 3 is 687.967 kbps.

2.6.2 Simulation

Generate 30 random samples a_1, a_2, a_3 with Poisson distribution. The expected value of λ_1 is 5, λ_2 is 5, and λ_3 is 10.

After simulation, we have that the average of the bandwidth allocated to class 1 is 1933.3 kbps, the average of the bandwidth allocated to class 2 is 882.8 kbps, and the average of the bandwidth allocated to class 3 is 683.9 kbps. The average of the satisfaction is 0.8333, and the sample variance is 0.0361, so that the 95% confidence interval of the satisfaction is $[0.7653, 0.9013]$

In Section 2.7.1, the satisfaction is 0.83283 which falls in the interval $[0.7653, 0.9013]$