

Chapter 3

The Minimum Cost Method

MCM includes two phases: **MCML** and **BRAM**. By solving MCML and BRAM, we introduce MCM with $Q\%$ of total loom width requirement.

3.1 A Simulation Algorithm Applied on All-IP Networks

In Chapter 2, we focus on maximizing the satisfaction function. In this chapter, we focus on how to find $Q\%$ of total bandwidth requirement? We say that the bandwidth τ satisfying $Q\%$ of total bandwidth requirement if $\frac{P^*(\mathcal{G} \in \Omega : \mathcal{G} \leq \tau)}{P^*(\Omega)} = Q\%$ holds, where Ω is the sample space of total bandwidth requirement and P^* is a probability measure on Ω . In implementation, we propose a simulation method to minimize the cost with which it satisfies $Q\%$ of total bandwidth requirement as well as maximize the satisfaction statistically. In specific, we focus on the choice of the routing paths which bring off twofold: seeking the maximum value of the satisfaction function and minimizing the cost [4]. We introduce the method as follows.

Minimum cost model with variates of connections:

Generate 30 random samples a_1, a_2, Λ, a_m of Poisson distributions with respect to each mean $E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m]$. Consider the following **MCML** model.

$$\text{Minimize} \quad \sum_{e \in L} k_e x_e \quad (13)$$

$$\text{Subject to} \quad \sum_{e \in L} k_e x_e \leq B \quad (14)$$

$$\sum_{e \in I_n} x_e = \sum_{e \in O_n} x_e, \forall e \in L. \quad (15)$$

$$M = \sum_{e \in O_1} x_e = \sum_{i=1}^m r_i a_i \quad (16)$$

$$x_e \leq U_e$$

Note (16) involves a_1, a_2, Λ, a_m . Because λ_i is a random variable, it is very difficult to find the probability of M such that $M = \sum_{e \in O_1} x_e = \sum_{i=1}^m r_i \lambda_i$ in (16) for each e . Furthermore, since x_e is a random variable for each i , $\sum_{e \in Z} k_e x_e$ is a random variable in (13). Thus, θ^i is random in (11) so is $\sum_i x_i(e) \theta_{ie} = x_e$ in (12). It is difficult to solve the model by optimality conditions of mathematical analysis. Therefore, we use simulation method to solve it.

Assume that M follows a Normal distribution. We need the following definition and theorem of Uniformly Minimum Variance Unbiased Estimator to investigate the property of sampling.

Definition 3.1 If τ is an unbiased estimator of M , $Var(\tau) < \infty$, and for any estimator τ' of M , we have $Var(\tau) < Var(\tau')$ then τ is a Uniformly Minimum Variance Unbiased Estimator (UMVUE).

Theorem 3.1 Suppose $\tau^1, \tau^2, \Lambda, \tau^n$ are random samples of M from a Normal distribution with unknown average and variance. The sample standard

deviation is S . Then $\tau_{Q\%} = \bar{\tau} + \beta_{Q\%} \times \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \times \frac{\sqrt{n-1}}{\sqrt{2}} \times S$ is an UMVUE of

M for the Q^{th} percentile, where $\bar{\tau} = \frac{\tau^1 + \tau^2 + \Lambda + \tau^n}{n}$ is the sample mean, and

$\beta_{Q\%}$ is the Q^{th} percentile of Standard Normal distribution [1].

Proof: Let $M \sim N(\mu, \sigma^2)$, where $\{\mu, \sigma^2\}$ are unknown.

$\left(\sum_{k=1}^n (\tau^{(k)})^2, \sum_{k=1}^n \tau^{(k)} \right)$ is the complete sufficient statistic for this model.

The Q^{th} percentile $\tau_{Q\%}$ of the distribution satisfies $P^* \{M \leq \tau_{Q\%}\} = Q\%$.

Namely, it is $P^* \left\{ \frac{M - \mu}{\sigma} \leq \frac{\tau_{Q\%} - \mu}{\sigma} \right\} = Q\%$.

Equivalently, we have $\tau_{Q\%} = \mu + \beta_{Q\%} \times \sigma$.

Because of $E[\bar{\tau}] = \mu$, by **Scheffe theorem** (see **Appendix A**), we obtain that $\bar{\tau}$ is an UMVUE for μ .

Let $T = \frac{(n-1)S^2}{\sigma^2} \rightarrow \chi^2(n-1)$

$$\therefore E[\sqrt{T}] = \int_0^\infty t^{\frac{1}{2}} \frac{t^{\frac{n-1}{2}-1} e^{-\frac{1}{2}t}}{\Gamma\left(\frac{n-1}{2}\right) 2^{\frac{n-1}{2}}} dt = \frac{\Gamma\left(\frac{n}{2}\right) 2^{\frac{1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)}$$

$$\text{which implies } E \left[\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) 2^{\frac{1}{2}}} \sqrt{n-1} S \right] = \sigma$$

Hence, $\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) 2^{\frac{1}{2}}} \sqrt{n-1} S$ is an unbiased estimator of σ . and is a

function of complete sufficient statistic.

Again, by Rao-Blackwell-Lehmann-Scheffe theorem, we have

$$\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)2^{\frac{1}{2}}}\sqrt{n-1}S \text{ is an UMVUE for } \sigma.$$

Hence, $\tau_{Q\%} = \bar{\tau} + \beta_{Q\%} \times \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)2^{\frac{1}{2}}} \times \sqrt{n-1} \times S$ is an UMVUE of M for the

Q^{th} percentile.

Consider in BRAM the satisfaction value F in (10) is a function of a_1, a_2, Λ, a_m . The following theorem states a simulation result in **BRAM** model may be formulated as an UMVUE of F for the Q^{th} percentile.

Theorem 3.2

Given $\{(a_1, a_2, \Lambda, a_m)^1, (a_1, a_2, \Lambda, a_m)^2, \dots, (a_1, a_2, \Lambda, a_m)^n\}$ is a set of random samples of $(\lambda_1, \lambda_2, \Lambda, \lambda_m)$. Let $(r_1, r_2, \Lambda, r_m) \bullet (a_1, a_2, \Lambda, a_m)^1 = \tau^1, (r_1, r_2, \Lambda, r_m) \bullet (a_1, a_2, \Lambda, a_m)^2 = \tau^2, \dots, (r_1, r_2, \Lambda, r_m) \bullet (a_1, a_2, \Lambda, a_m)^n = \tau^n$. Then $\tau_{Q\%} = \bar{\tau} + \beta_{Q\%}$

$\times \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)2^{\frac{1}{2}}} \times \sqrt{n-1} \times S$ is an UMVUE of M for the Q^{th} percentile. For inserting

$\tau_{Q\%}$ and $(a_1, a_2, \Lambda, a_m)^k$ to **BRAM**, we obtain the value of the satisfaction function

F^k for $k = 1, \dots, n$. Then the average of $\{F^k\}_{k=1}^n$ is an UMVUE of F satisfying

$Q\%$ of total bandwidth requirement.

3.1.1 A Statistical inference process of our study

We describe the procedure and the Statistical inference process of our study as follows.

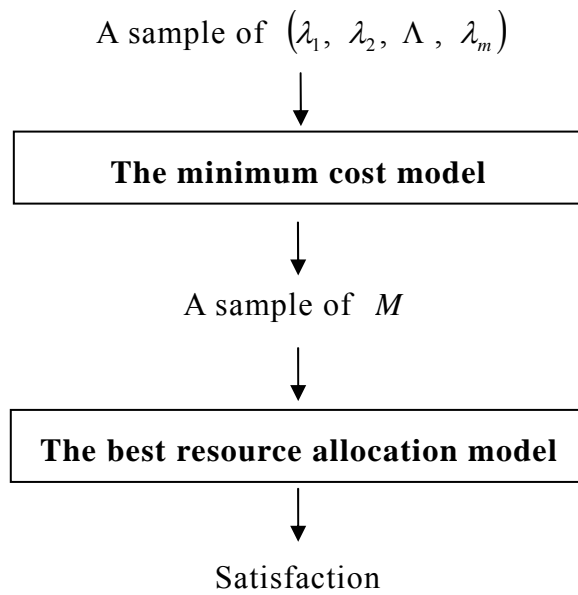


Figure 3.1 The Statistical inference process of our study

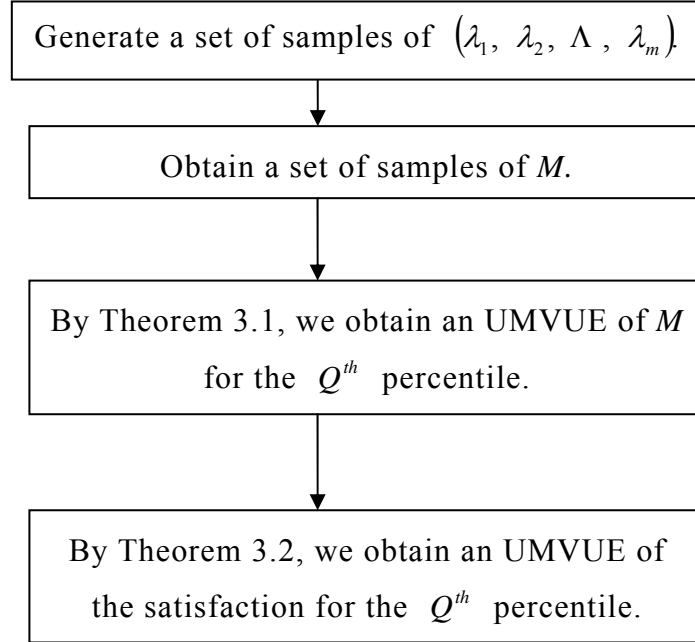


Figure 3.2 The structure of our study

3.1.2 The simulation process

We propose a method which combines both simulation and optimization to reach the goal of $Q\%$ satisfaction.

Generate random 30 samples a_1, a_2, Λ, a_m of Poisson distribution with respect to each mean $E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m]$. Denote by $(a_1, a_2, \Lambda, a_m)^k$ the value of $(\lambda_1, \lambda_2, \Lambda, \lambda_m)$ at the k th run of the simulation. For each k , we solve the Minimum cost model to obtain the total bandwidth τ^k and the bandwidth of each links we purchase.

We use a simulation method for solving All-IP Networks Problem, and state our algorithm as follows:

- Step 1.** Given $\lambda_1, \lambda_2, \Lambda, \lambda_m$ compute the expected value $E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m]$.
- Step 2.** Generate 30 random samples a_1, a_2, Λ, a_m of Poisson distributions with respect to each mean $E[\lambda_1], E[\lambda_2], \Lambda, E[\lambda_m]$. Set $k=0$.
- Step 3.** $k=k+1$.
- Step 4.** Calculate $(r_1, r_2, \Lambda, r_m) \bullet (a_1, a_2, \Lambda, a_m)^k = \tau^k$.
- Step 5.** $k \leq 30$? If not, go to **Step 6**, otherwise $k=k+1$. Go to **Step 4**.
- Step 6.** Stop and calculate the $Q\%$ of total bandwidth requirement by Theorem 3.1 and the bandwidth $\tau_{Q\%}$.
- Step 7.** Input $\tau_{Q\%}$ and solve **BRAM** to find the value of satisfaction.
- Next, we introduce the algorithm of calculating the $Q\%$ of total bandwidth requirement.

Calculating the $Q\%$ of total bandwidth requirement

- Step 1.** Calculate $\bar{\tau} = \frac{\tau^1 + \tau^2 + \Lambda + \tau^{30}}{n}$.
- Step 2.** Calculate the sample variance S^2 of $\tau^1, \tau^2, \Lambda, \tau^{30}$.
- Step 3.** Find $\beta_{Q\%}$.
- Step 4.** Calculate $\tau_{Q\%}$.

By **Theorem 3.1** $\tau_{Q\%}$ is an UMVUE of M for the Q^{th} percentile.

MCM algorithm

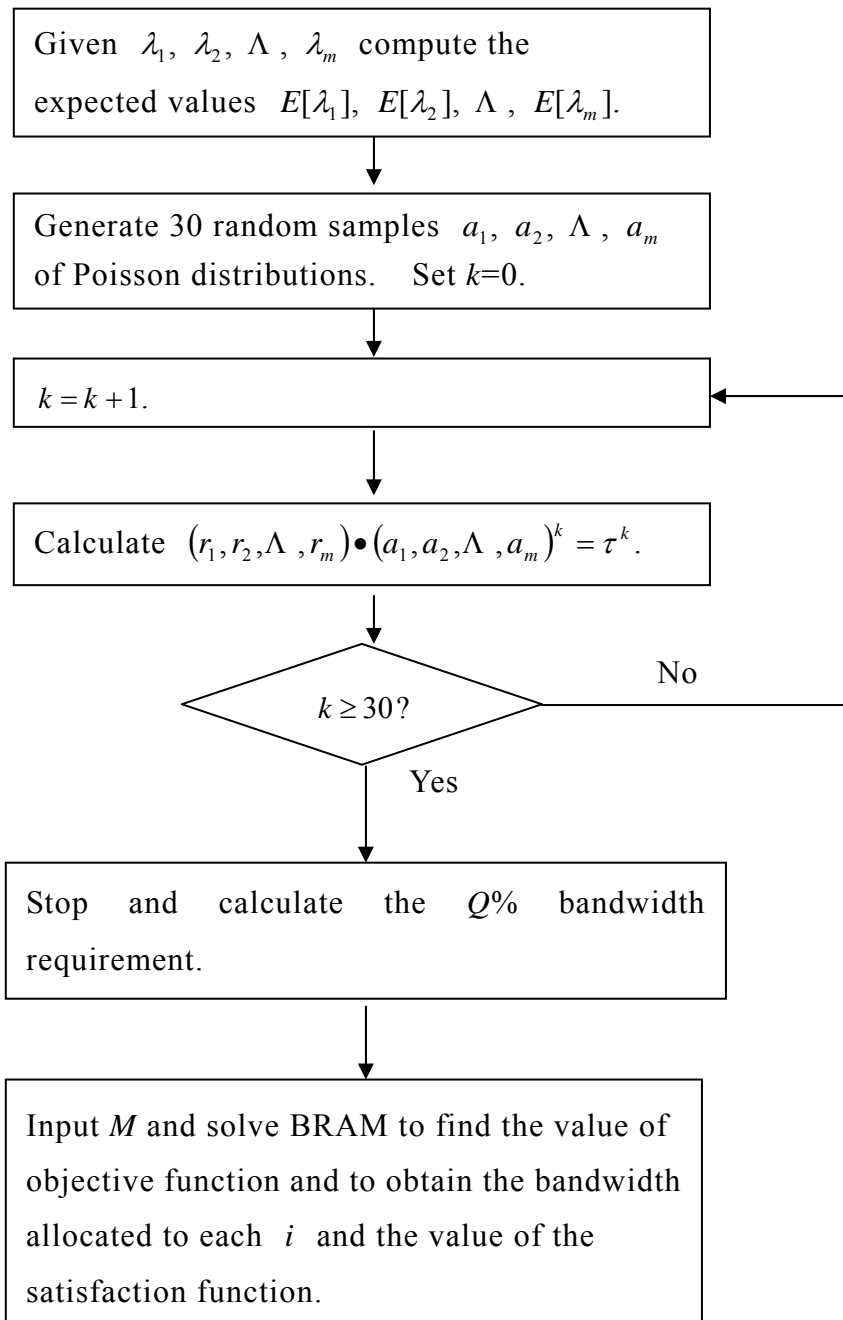


Figure 3.3 MCM algorithm

Next, consider BRAM with simulation. Different from MBM in Section 2.1 we replace (11) by $a_1 \times \theta_1 + a_2 \times \theta_2 + \Lambda + a_m \times \theta_m = M$. Input M and solve BRAM to find the value of satisfaction F and to obtain the bandwidth allocated to each class.

3.2 The Minimum Cost Method

3.2.1 Example 1 of MCM model

Consider a network topology $G = \langle N, L \rangle$ as shown in **Figure 2.2**. All data remain the same as given in Section 2.2.

The process of our study is described in the following:

Generate 30 random samples a_1, a_2, a_3 of Poisson distributions with respect to each mean 10, 10, 10 and calculate $r_1 a_1 + r_2 a_2 + r_3 a_3$ simultaneously.

Then by Theorem 3.1 we obtain the solutions in Table 3.1:

Table 3.1 The relation between Bandwidth and $Q\%$ of total bandwidth requirement

Bandwidth	2345	2765	2952
$Q\%$ of total bandwidth requirement	17%	34%	50%
Bandwidth	3131	3470	3988
$Q\%$ of total bandwidth requirement	66%	83%	95%
Bandwidth	4002	4570	
$Q\%$ of total bandwidth requirement	97%	99.5%	

If it requires 95% of total bandwidth requirement, then we can choose 3988 units for total budget. Next consider MCML. In order to minimize the cost, we insert the data to FMINCON as follows:

$$A = \begin{bmatrix} 5 & 6 & 10 & 5 & 4 & 11 & 6 & 8 & 6 & 7 & 12 & 6 & 5 & 6 \\ 1 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; b = \begin{pmatrix} 100000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2300 \\ 3500 \\ 1000 \\ 2500 \\ 2100 \\ 2200 \\ 2000 \\ 3000 \\ 2100 \\ 2700 \\ 1500 \\ 1800 \\ 3000 \\ 3500 \\ -3988 \end{pmatrix}$$

FMINCON (inline('5* x₁+6* x₂+10* x₃+5* x₄+4* x₅+11* x₆+6* x₇+8* x₈+6* x₉+7* x₁₀+12* x₁₁+6* x₁₂+5* x₁₃+6* x₁₄'), [1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1 ; 1], A, b, [], [], [0; 0], []).

We obtain the bandwidth of each link as follows:

Table 3.2 The optimal solution

x_1	x_2	x_3	x_4	x_5
1720.5 kbps	2267.5 kbps	0 kbps	0 kbps	1720.5 kbps
x_6	x_7	x_8	x_9	x_{10}
0 kbps	0 kbps	2267.5 kbps	1720.5 kbps	0 kbps
x_{11}	x_{12}	x_{13}	x_{14}	M
0 kbps	0 kbps	1720.5 kbps	2267.5 kbps	3988 kbps

With 95% of total bandwidth requirement, the cost of the resource allocation is 79767.4 dollars, compared with the cost of the resource allocation in Section 3.2 which is 100000 dollars, we save 20232.6 dollars. Moreover, it saves bandwidth projected about 20% in total, i.e., $(5000-3988)/5000$.

3.2.2 Example 1 of MCM with $w_1 = w_2 = w_3$

Suppose the number of connection is λ_i and the expected value of λ_i is 10 for the i th class, each objective is regarded as important as each other. Suppose each class has equal weight $w_1 = w_2 = w_3 = \frac{1}{3}$, and $M = 3988$ kbps after solving MCML.

For each τ and $(a_1, a_2, \Lambda, a_m)^k$ to **BRAM**, we are to obtain the value of the satisfaction function F^k for $k=1, \Lambda, 30$. We set the data to FMINCON as follows:

FMINCON(inline(' - $\frac{1}{3}*\log(\theta_1/167)$ - $\frac{1}{3}*\log(\theta_2/83)$ - $\frac{1}{3}*\log(\theta_3/28)$ '), [1 ; 1 ; 1], [(a_1, a_2, Λ, a_m)^k], [3988], [], [], [167; 83; 28], []).

We obtain the average of $\{F^k\}_{k=1}^{30}$ is 0.6073. It is compared with the value of the satisfaction by BRAM in Section 2.3 that is 0.8113. Obviously, it reduces about 25% which is computed by $(0.8113-0.6073)/0.8113$

The resource allocation to each class and each path are:

Class 1. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 1720.5 kbps.

$l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 19.195 kbps.

Class 2. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 1083.393 kbps.

Class 3. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 1164.912 kbps.

3.2.3 Three classes have different weights

Suppose for each class their weights are $w_1 = 0.6$, $w_2 = 0.2$, $w_3 = 0.2$. However, let $M = 3988$ kbps after solving MCML.

Next we solve BRAM by FMINCON:

FMINCON(inline(' -0.6*log($\theta_1/167$)-0.2*log($\theta_2/83$)-0.2*log($\theta_3/28$)'), [1; 1; 1], [(a_1, a_2, Λ, a_m)^k], [3988], [], [], [167; 83; 28], []).

We obtain the average of $\{F^k\}_{k=1}^{30}$ 0.3981. It is compared with the value of satisfaction in Section 2.4 which is 0.639. Its satisfaction is cut down by about 30%, i.e., $(0.639-0.3981)/0.639$.

The resource allocation for each class with a possible path is:

Class 1. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth 1951.364 kbps.

Class 2. $l_2 \rightarrow l_8 \rightarrow l_{14}$, which holds bandwidth of 316.136 kbps.

$l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 716.753 kbps.

Class 3. $l_1 \rightarrow l_5 \rightarrow l_9 \rightarrow l_{13}$, which holds bandwidth of 1003.747 kbps.

3.3 Example 2 of MCM Model

Consider a network topology $G = (N, L)$ as shown in **Figure 2.2**. All data remain the same as that given in Section 2.5.

Generate 30 random samples a_1, a_2, a_3 of Poisson distributions with respect to each mean 10, 30, 110 and calculate $\gamma_1 a_1 + \gamma_2 a_2 + \gamma_3 a_3$ for (16).

The total bandwidth and $Q\%$ of total bandwidth requirement is given in

Table 3.3:

Table 3.3 The relation between Bandwidth and $Q\%$ of total bandwidth requirement

Bandwidth	6310 kbps	6610 kbps	6791.7 kbps
$Q\%$ of total bandwidth requirement	20%	30%	55%

The maximum flow from source to destination is 6800 kbps under the current capacity of network. However, it indicates that 6791 kbps is only able to reach 55% of total bandwidth requirement in simulation. Thus, we suggest the system manager to add some links on the network or increase the capacity of some links to reach a specified $Q\%$ of total bandwidth requirement.

For solving MCML in this case, we set $M = 6791.7$ and provide data to FMINCON as follows:

$$A = \begin{bmatrix} 5 & 6 & 10 & 5 & 4 & 11 & 6 & 8 & 6 & 7 & 12 & 6 & 5 & 6 \\ 1 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; b = \begin{pmatrix} 140000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2300 \\ 3500 \\ 1000 \\ 2500 \\ 2100 \\ 2200 \\ 2000 \\ 3000 \\ 2100 \\ 2700 \\ 1500 \\ 1800 \\ 3000 \\ 3500 \\ 6791 \end{pmatrix}$$

FMINCON(inline('5* x₁+6* x₂+10* x₃+5* x₄+4* x₅+11* x₆+6* x₇+8* x₈+6* x₉+7* x₁₀+12* x₁₁+6* x₁₂+5* x₁₃+6* x₁₄'), [1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1], A, b, [], [], [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0], [])

Then, the bandwidth of each link is obtained and given in Table 3.4.

Table 3.4 The optimal solution

x_1	x_2	x_3	x_4	x_5
2300 kbps	3491.7 kbps	1000 kbps	0 kbps	1400 kbps
x_6	x_7	x_8	x_9	x_{10}
900 kbps	491.7 kbps	3000 kbps	2100 kbps	0 kbps
x_{11}	x_{12}	x_{13}	x_{14}	M
791.7 kbps	0 kbps	3000 kbps	3000 kbps	6791.7 kbps

The cost of the resource allocation is \$140000 dollars and we buy bandwidth of 6791.7 kbps, which is the same as that in Section 2.5.

Suppose classes have different weights: $w_1 = 0.6$, $w_2 = 0.2$, $w_3 = 0.2$ and let $M = 6791.7$ kbps. Inserting τ and $(a_1, a_2, \Lambda, a_m)^k$ to **BRAM**, we obtain the value of the satisfaction function F^k for $k = 1, \Lambda, 30$. Setup the data for FMINCON as follows:

FMINCON(inline(' -0.6*ln($\theta_1/167$)-0.2*ln($\theta_2/83$)-0.2*ln($\theta_3/28$)'), [1 ; 1 ; 1], [(a_1, a_2, Λ, a_m)^(k)], [6791.7], [], [], [167; 83; 28], []).

In average, θ_i , $i=1,2,3$, are given as follows:

$$\theta_1 = 187.82 \text{ kbps}$$

$$\theta_2 = 40.847 \text{ kbps}$$

$$\theta_3 = 33.53 \text{ kbps}$$

Hence we select a path $l_1, l_2, l_5, l_8, l_9, l_{13}, l_{14}$, and obtain the optimal value of satisfaction is $0.6 * f_1(\theta_1) + 0.2 * f_2(\theta_2) + 0.2 * f_3(\theta_3) = 0.0205$.

3.4 The Application on another All-IP Networks Problem

Consider the same network topology $G = \langle N, L \rangle$ as given in Section 2.6. All data are kept the same as that in Section 2.6.

Generate 30 random samples a_1, a_2, a_3 of Poisson distributions with respect to each mean 5, 5, and 10. The maximum flow of the network from the source to the destination is 3500 kbps. Since the total bandwidth requirement is less than 3500 kbps, the total bandwidth and $Q\%$ of total bandwidth requirement for different Q 's are given in Table 3.5.

Table 3.5 The relation between Bandwidth and $Q\%$ of total bandwidth requirement

Bandwidth	1133 kbps	1446 kbps	1663 kbps
$Q\%$ of total bandwidth requirement	17%	34%	50%
Bandwidth	1791 kbps	1961 kbps	2484 kbps
$Q\%$ of total bandwidth requirement	66%	83%	95%
Bandwidth	2492 kbps	2720 kbps	
$Q\%$ of total bandwidth requirement	97%	99.5%	

If it requires a 95% of total bandwidth requirement, then we can setup a total bandwidth of 2484 kbps. In order to minimize the cost, we set the data to FMINCON as follows:

The bandwidth of each link is given as follows:

Table 3.6 The optimal solution

x_1	x_2	x_3	x_4	x_5
384 kbps	384 kbps	384 kbps	2100 kbps	2100 kbps
x_6	x_7	x_8	x_9	x_{10}
0 kbps	0 kbps	0 kbps	0 kbps	0 kbps
x_{11}	x_{12}	x_{13}	x_{14}	M
0 kbps	0 kbps	0 kbps	2484 kbps	2484kbps

The cost for the resource allocation is 42030 dollars with which we buy bandwidth 2484 kbps. Suppose classes have different weights: $w_1 = 0.6$, $w_2 = 0.2$, $w_3 = 0.2$, and let $M = 2484$ kbps. Input τ^* and $(a_1, a_2, \Lambda, a_m)^k$ to **BRAM**, we obtain the value of the satisfaction function F^k for $k = 1, \Lambda, 30$. After applying `FMINCON`(`inline('-0.6*ln($\theta_1/167$)-0.2*ln($\theta_2/83$)-0.2*ln($\theta_3/28$)')`, [1 ; 1 ; 1], [$(a_1, a_2, \Lambda, a_m)^k$], [2490], [], [], [167; 83; 28], []).

In average, we obtain θ_i , $i=1,2,3$, as follows:

$$\theta_1 = 274.6538 \text{ kbps}$$

$$\theta_2 = 125.9787 \text{ kbps}$$

$$\theta_3 = 48.6837 \text{ kbps.}$$

Hence we select a path $l_1, l_2, l_5, l_8, l_9, l_{13}, l_{14}$, and obtain the

optimal value of the satisfaction is $0.6 \times f_1(\theta_1) + 0.2 \times f_2(\theta_2) + 0.2 \times f_3(\theta_3) = 0.4926$. That is, the bandwidth allocated to class 1 is 1371.269 kbps, the bandwidth allocated to class 2 is 627.8935 kbps, and the bandwidth allocated to class 3 is 484.837 kbps.