Abstract

The univariate c-characteristic function has been shown to be important in cases that are hard to manage using the traditional characteristic function. In this thesis, we first give its inversion formulas. We then use them to obtain (1) the probability density functions (PDFs) of a linear combination of the components of a Dirichlet random vector; (2) the PDFs of random functionals of a Ferguson-Dirichlet process with some interesting parameter measures; (3) a Lebesgue integral expression of any random functional of the Ferguson-Dirichlet process.

New properties of the multivariate c-characteristic function with a spherical distribution are given in this thesis. With them, we show that the random mean of a Ferguson-Dirichlet process over a spherical surface in n dimensions has a spherical distribution on the n-dimensional ball. Moreover, we derive its exact PDF. Furthermore, we generalize this result to any ellipsoidal surface in n-space.

We also study the issue of compatibility for specified conditional distributions. This issue is important in probability theory and Bayesian computations. Several necessary and sufficient conditions for the compatibility are provided. We also address the problem of uniqueness of the associated joint distribution when the given conditionals are compatible. In addition, we provide a method to obtain all possible joint distributions that have the given compatible conditionals. Algorithms for checking the compatibility and the uniqueness, and for constructing all associated densities are also given.

Through the related compatibility theorems, we provide a fully and cleanly unified theory of inverse Bayes formula (IBF) and construct a generalized IBF (GIBF) that is applicable in the more general measurable space. In addition, using the GIBF, we provide a marginal density fitting algorithm, which avoids the problems of convergence in iterative algorithm such as the Gibbs sampler.

Keywords: c-characteristic function, compatibility, Ferguson-Dirichlet process, generalized inverse Bayes formula, random functional

中文摘要

單變量c-特徵函數已被證明可處理一些難以使用傳統特徵函數解決的問題，在本文中，我們首先提出其反演公式，透過此反演公式，我們獲得（1）Dirichlet隨機向量之線性組合的機率密度函數；（2）以一些有趣測度為參數之Ferguson-Dirichlet過程其隨機動差的機率密度函數；（3）Ferguson-Dirichlet過程之隨機泛函的Lebesgue積分表示式。

本文給予對稱分配之多變量c-特徵函數的新性質，透過這些性質，我們證明在任何n維球面上之Ferguson-Dirichlet過程其隨機均值是一對稱分配，並且我們亦獲得其確切的機率密度函數，此外，我們將這些結果推廣至任何n維橢球面上。

我們亦探討條件分配相容性的問題，這個問題在機率理論與貝式計算上有其重要性，我們提出其充要條件。當給定相容的條件分配時，我們不但解決相關聯合分配唯一性的問題，而且也提供方法去獲得所有可能的相關聯合分配，我們亦給予檢驗相容性、唯一性及建構機率密度函數的演算法。

透過相容性的相關理論，我們提出完整且清楚地統合性貝氏反演公式理論，並建構可應用於一般測度空間的廣義貝氏反演公式。此外，我們使用廣義貝氏反演公式提供一個配適機率密度函數的演算法，此演算法沒有疊代演算法（如Gibbs取樣法）的收斂問題。

關鍵詞：c-特徵函數，相容性，Ferguson-Dirichlet過程，廣義貝氏反演公式，隨機泛函