附錄

證明式子(2.3.1)：考慮$i<k$的情況：

$$\because \quad \frac{P(t,t_{i+1})}{P(t,t_i)}=\frac{1}{1+\tau_{i+1}L_{i+1}(t)}$$

$$\therefore \frac{P(t,t_k)}{P(t,t_i)}=\frac{P(t,t_{i+1})}{P(t,t_i)}\times\frac{P(t,t_{i+2})}{P(t,t_{i+1})}\times\ldots\ldots\ldots\ldots\times\frac{P(t,t_{k-1})}{P(t,t_{k-2})}\times\frac{P(t,t_k)}{P(t,t_{k-1})}$$

$$=\frac{1}{1+\tau_{i+1}L_{i+1}(t)}\times\frac{1}{1+\tau_{i+2}L_{i+2}(t)}\times\ldots\ldots\ldots\times\frac{1}{1+\tau_kL_k(t)}$$

$$\Rightarrow \ln\frac{P(t,t_k)}{P(t,t_i)}=\ln\left(\frac{1}{\prod_{j=i+1}^{k}\left(1+\tau_jL_j(t)\right)}\right)$$

$$= -\sum_{j=i+1}^{k}\ln(1+\tau_jL_j(t)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(*)$$

又當計價單位改變時，$dL_k(t)$的擴散項並不受影響，但飄浮項將有所改變如下式(可參照 Brigo，2001)：

$$\mu_i(t)=\mu_k(t)-\frac{1}{dt}d\ln L_k(t)d\ln\left(\frac{P(t,T_k)}{P(t,T_i)}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(**)$$

其中$\mu_i(t)$代表新測度（Q^i Measure)下之飄浮項，$\mu_k(t)=0$代表原始測度（Q^k Measure)下之飄浮項，$P(t,t_i)$代表新測度的計價單位，$P(t,t_k)$代表原始測度的計價單位。

將$\mu_k(t)=0$及$(*)$式子代入$(**)$式子可得：

$$\mu_i(t)=0dt-d\ln L_k(t)d\left(-\sum_{j=i+1}^{k}\ln(1+\tau_jL_j(t))\right)$$

$$=\sum_{j=i+1}^{k}d\ln L_k(t)d\ln(1+\tau_jL_j(t))$$

$$=\sum_{j=i+1}^{k}\Big((\cdot)dt+\sigma_k(t)L_k(t)dZ_k(t)\Big)\left((\cdot)dt+\frac{\tau_j}{1+\tau_jL_j(t)}\sigma_j(t)L_j(t)dZ_j(t)\right)$$

(by Ito's formula)

$$= \sum_{j=i+1}^{k} \frac{\rho_{j,k}\sigma_k(t)\tau_j\sigma_j(t)L_j(t)}{1+\tau_j L_j(t)}$$

$$(\because dt \cdot dt = 0, dt \cdot dZ_k(t) = dt \cdot dZ_j(t) = 0)$$

故得證。

相同的方法亦可得證$i > k$的情況。