附錄 A：

另外一種方法求 $b_{k,i}$

還沒有求 $b_{k,i}$ 之前我們先來求 (x - 1)(x - 2)……(x - k) 的各項係數

(x - 1)(x - 2)……(x - k) 的
x^{k-2} 次的係數爲

$$\frac{1}{2}[(1+2+\cdots\cdots+k)^2-(1^2+2^2+\cdots\cdots+k^2)]$$
$$=\frac{1}{2}\{[\frac{k(k+1)}{2}]^2-\frac{1}{6}k(k+1)(2k+1)\}$$
$$=\frac{1}{2}\{\frac{1}{12}k(k+1)[3k^2+3k-4k-2]\}$$
$$=\frac{1}{24}k(k+1)(k-1)(3k+2)$$

x^{k-3} 次的係數爲

$$-\sum_{r=1}^{k-1}(r+1)[\frac{1}{24}r(r+1)(r-1)(3r+2)]$$
$$=-\frac{1}{24}\sum_{r=1}^{k-1}[3r^5+5r^4-r^3-5r^2-2r]$$
$$=-\frac{1}{24}(3\cdot\sum_{r=1}^{k-1}r^5+5\cdot\sum_{r=1}^{k-1}r^4-\sum_{r=1}^{k-1}r^3-5\cdot\sum_{r=1}^{k-1}r^2-2\sum_{r=1}^{k-1}r)$$
$$=-\frac{1}{24}\{[\frac{(k)(k-1)}{2}]^2[2k^2-2k-1]+\frac{1}{6}(k-1)(k)(2k-1)(3k^2-3k-1)-[\frac{k(k-1)}{2}]^2-5\frac{(k-1)k(2k-1)}{6}$$
$$-2\cdot\frac{k(k-1)}{2}\}$$

$$=-\frac{1}{24}\{\frac{k(k-1)}{4}[k(k-1)(2k^2-2k-1)-k(k-1)]+\frac{1}{6}(k-1)k(2k-1)(3k^2-3k-6)-k(k-1)\}$$
$$=-\frac{1}{24}\{\frac{k(k-1)}{2}[k(k-1)(k^2-k-1)]+\frac{k(k-1)}{2}(2k-1)(k+1)(k-2)-k(k-1)\}$$
$$=-\frac{1}{24}[\frac{k(k-1)}{2}(k^4-2k^3+k+2k^3-3k^2-3k)]$$
$$=-\frac{1}{24}[\frac{k(k-1)}{2}\cdot k(k+1)^2(k-2)]$$
$$=-C_2^{k+1}C_4^{k+1}$$

x^{k-4}次的係數爲

$$\sum_{r=1}^{k-1}(r+1)\cdot\frac{1}{48}[(r-1)(r-2)r^2(r+1)^2]$$
$$=\frac{1}{2!}\cdot\frac{1}{4!}\cdot\frac{1}{5!}\cdot(k-3)(k-2)(k-1)(k)(k+1)[-8-10k+15k^2+15k^3]$$
$$=\frac{1}{2!}\cdot\frac{1}{4!}\cdot C_5^{k+1}[-8-10k+15k^2+15k^3]$$

$$\because n^k=b_{k,0}C_0^n+b_{k,1}C_1^n+b_{k,2}C_2^n+\cdots\cdots+b_{k,k}C_k^n$$
$$=b_{k,0}\cdot 1+b_{k,1}\cdot n+b_{k,2}\cdot(n)(\frac{n-1}{2})+\cdots\cdots+b_{k,k-1}(n)(\frac{n-1}{2})\cdots\cdots[\frac{n-(k-2)}{k-1}]$$
$$+b_{k,k}(n)(n-1)\cdots\cdots[\frac{n-(k-1)}{k}]$$

比較等式兩邊n^k的係數

$$\therefore 1=\frac{b_{k,k}}{k!}\Rightarrow b_{k,k}=k!$$

比較等式兩邊n^{k-1}的係數

$$\frac{b_{k,k-1}}{(k-1)!}=\frac{k(k-1)}{2}$$
$$\Rightarrow b_{k,k-1}=(k-1)!\cdot\frac{k(k-1)}{2}=k!\cdot\frac{k-1}{2}$$

比較等式兩邊n^{k-2}的係數

$$\frac{b_{k,k-2}}{(k-2)!}=\frac{k(k-1)}{2}\cdot\frac{(k-2)(k-1)}{2}-\frac{1}{24}(k-1)(k)(k-2)(3k-1)$$
$$\Rightarrow b_{k,k-2}=k![\frac{(k-2)(k-1)}{4}-\frac{1}{24}(k-2)(3k-1)]$$
$$=k!\cdot\frac{1}{24}[6k^2-18k+12-3k^2+7k-2]$$
$$=\frac{k!}{4!}(k-2)(3k-5)$$

比較等式兩邊n^{k-3}的係數

$$\frac{b_{k,k-3}}{(k-3)!}=\frac{(k-3)(k-2)}{2}\cdot\frac{k(k-1)(k-2)(3k-5)}{4!}-\frac{k(k-1)}{2}\cdot\frac{1}{24}(k-2)(k-1)(k-3)(3k-4)$$
$$+\frac{1}{12}[\frac{k(k-1)}{2}]^2(k-2)(k-3)$$
$$\Rightarrow b_{k,k-3}=\frac{k!}{48}[(k-3)(k-2)(3k-5)-(k-1)(k-3)(3k-4)+k(k-1)(k-3)]$$
$$\Rightarrow b_{k,k-3}=\frac{k!}{48}(k-3)(k-2)(k-3)$$

比較等是兩邊 n^{k-4} 的係數

$$\frac{b_{k,k-4}}{(k-4)!}=\frac{1}{48}k(k-1)(k-2)(k-3)(k-2)(k-3)\cdot\frac{(k-3)(k-4)}{2}-\frac{1}{4!}k(k-1)(k-2)(3k-5)\cdot$$
$$\frac{1}{24}(k-3)(k-2)(k-4)(3k-7)+\frac{k-1}{2}\cdot k\cdot\frac{1}{48}(k-2)^2(k-1)^2(k-3)(k-4)-\frac{1}{5760}$$
$$(k-4)(k-3)(k-2)(k-1)k[-8-10(k-1)+15(k-1)^2+15(k-1)^3]$$
$$=k!\cdot(k-4)\cdot\frac{1}{2!4!5!}(15k^3-150k^2+485k-502)$$