1 Introduction

A *mixed hypergraph* is a triple $\mathcal{H} = (X, \mathcal{C}, \mathcal{D})$, where X is the vertex set and each of $\mathcal{C}, \mathcal{D}$ is a list of subsets of X,the $\mathcal{C}$-*edges* and $\mathcal{D}$-*edges*, respectively, and we assume that all $\mathcal{C}$-edges and $\mathcal{D}$-edges have at least two elements. A *proper k-coloring* of a mixed hypergraph is a function, c, from the vertex set to a set of k colors so that each $\mathcal{C}$-edge has two vertices with a common color and each $\mathcal{D}$-edge has two vertices with distinct colors. A mixed hypergraph is *k-colorable* if it has a proper coloring with at most k colors. A *strict k-coloring* is a proper k-coloring using all k colors, it means that the function, c, is onto. In a colorable mixed hypergraph $\mathcal{H}$, the maximum(minimum) number of colors over all strict k-colorings is called the *upper(lower) chromatic number* of $\mathcal{H}$ and is denoted by $\bar{\chi}(\mathcal{H})(\chi(\mathcal{H}))$.

We use n to denote $|X|$ for the mixed hypergraph. Every proper k-coloring induces a partition of vertex set into color classes. Such partition, $\{X_1, X_2, \ldots, X_k\}$, is called a *feasible partition* with respect to the coloring. The number of feasible partitions into k colors is denoted by r_k. The vector $R(\mathcal{H}) = (r_1, \ldots, r_n)$ is called the *chromatic spectrum* of $\mathcal{H}$. The set of values k such that $\mathcal{H}$ has a strict k-coloring is the *feasible set* of $\mathcal{H}$, written $S(\mathcal{H})$; this is the set of indices i such that $r_i > 0$. A mixed hypergraph has a *gap at* k if its feasible set contains some elements larger and smaller than k but omits k, and we say that k is a gap of such mixed hypergraph. A subset of X is said to be *monochromatic* if they are assigned to the same color, and *polychromatic* if they are assigned to distinct colors.

Coloring mixed hypergraphs has been studied in several directions. If there exists a linear ordering of the vertex set X such that each edge represents an interval, then the hypergraph is said to be a *mixed interval hypergraph.* In [1], Bulgaru and Voloshin showed that in each colorable mixed interval hypergraph, the lower chromatic number is less than 3, and they introduced the *sieve-number* of mixed interval hypergraphs. A mixed *hypercacti* is a mixed hypergraph spanned by a cacti. Král', Kratochvíl, and Voss discussed the gap of a mixed hypercacti, and proved that there exist gaps of a non-planar graphs besides K_5 in [3].

The upper chromatic number is decided by $\mathcal{C}$-edges, so Voloshin introduced $\mathcal{C}$-*hypergraphs* which have no $\mathcal{D}$-edge, and he has an algorithm to find the upper chromatic number of a $\mathcal{C}$-hypergraph in [5]. Gionfriddo, Milazzo, and Voloshin defined a new edge coloring of multigraph, it can be considered as a coloring of a $\mathcal{C}$-hypergraph. They showed that how the maximum degree of the graph affects the upper chromatic number in [4].

Two strict colorings are different if there are two vertices which get the same color in one coloring but get different colors in the other coloring. The number of different strict k-colorings is denoted by r_k. The integer vector $R(\mathcal{H}) = (r_1, r_2, \ldots, r_n)$ is called the *chromatic spectrum* of $\mathcal{H}$. Jiang, Mubayi, Tuza, Voloshin, and West introduced some properties of spectrum. They can construct a mixed hypergraph with any given subset of positive integers excluding 1 as the feasible set by *doubling* and *shifting* in [2].

In section 2, we show that the minimum gap of a mixed hypergraph with more than 5 vertices is 3, and the maximum gap is $n-3$. Furthermore, for $3 \leq k \leq n-3$, we can find a mixed hypergraph which has a gap at k, in fact, there are only $k-1, k+1$ in the feasible set. In section 3, we consider a special case of mixed hypergraph. Suppose that the mixed hypergraph is spanned by a complete bipartite graph, $K_{s,t}$. If $2 = s \leq t$, then there is no gap. If $3 \leq s \leq t$, then the maximum gap is s, and we construct a mixed hypergraph, $\mathcal{H}$, which $S(\mathcal{H}) = \{2, s+1\}$.

Let $\mathcal{H} = (X, \mathcal{C}, \mathcal{D})$. Consider the size of each $\mathcal{C}$-edge and $\mathcal{D}$-edge. If each $C \in \mathcal{C}$ is formed by l vertices and each $D \in \mathcal{D}$ is formed by m vertices, then we say that this mixed hypergraph is (l, m)-uniform. In section 4, we find out when $2 \leq \lceil \frac{m}{2} \rceil < l$ then the minimum gap of $\mathcal{H}$ is l and the maximum gap has several situations. Besides, according to the minimum gap, we can prove that the minimum number of vertices of a (l, m)-uniform mixed hypergraph which has gaps is $s(l-1)+m$, where $s = \lceil \frac{m}{2} \rceil$. If $l = m = r$, then this mixed hypergraph is r-uniform. Therefore, a r-uniform mixed hypergraph has the same properties as (l, m)-uniform.