Chapter 1.
INTRODUCTION

1.1 Concepts and Data types of Reliability Analysis

Reliability is the probability of a product or a system performing its function well for a specified period of time under regular operating conditions. Nowadays, all the customers expect the product they purchase is with "high-quality". What does "high-quality" mean? Basically, a high-quality product must be with high reliability. Therefore, reliability analysis becomes more and more important in manufacture and industry. It is also used in other areas, such as survival analysis and event-time analysis are synonyms of reliability analysis in biological and social sciences, respectively.

To assess the reliability of a product, reliability data needs to be collected first. There are three main types of reliability data, which are lifetime data (or failure time data), dichotomous data and degradation measurement data. The lifetime data is the observation of the time to failure, which is the most direct data type. However, it can not be obtained sometimes, for example, you will never know whether a stored match fails unless you fire it.

Firing a match, this kind of life test produces the dichotomous data, because we can only tell if the lifetime of a tested product exceeds the testing time or not. The dichotomous data can be regarded as a kind of censored data. There are two shortcomings for using dichotomous data. First, the maximum likelihood estimator (MLE) of the unknown parameters may not exist; second, the obtained estimator is less accurate. They are illustrated with the following example. Let the lifetime of a product follows an exponential distribution with mean $1/\alpha$. Assume that $\alpha = 0.1$. Consider these two experiments:

(1) we can observe the lifetime directly.

(2) the lifetime can not be observed directly but we can observe if the lifetime is longer than 10 or not.

In experiment (1), α is estimated by the reciprocal of sample mean (the MLE). In experiment (2), the MLE of α is $\frac{1}{10}\log\frac{n}{y}$, where n and y are the sample size and the number of products whose lifetime exceeds 10, respectively. It is easy to verify that when $y = n$ or 0 the MLE does not exist in case (2). It means when dichotomous data is used, the identical outcomes cause estimation problem. In addition, if the sample size is 10 in both cases, the mean square error (MSE) of using lifetime data is 1.66×10^{-3} and 2.24×10^{-3} for using dichotomous data (the MSE of using dichotomous data is calculated by conditioning on $y \neq n$ and 0, which is with probability 0.99).

Another commonly used data type in reliability analysis is degradation measurement. It is a measurement which is highly related to the lifetime, like the lumen to a fluorescent light bulb (Tseng et al., 1995), the fatigue crack-size of the alloy (Hudak et al., 1978). Usually, the failure of a product can be traced to the degradation process. When the amount of the measurement decreases (or increases) to certain value (the threshold), the physical or chemical property changes so that the item fails. If the degradation measurement is available, it provides more information than lifetime data. It can be used not only to tell if a product fails at a certain time but also to predict the residual lifetime, which is the time from the inspection to failure, by estimating when the degradation process reaches the threshold. Hence, degradation measurement data contains the most information among these three types of data.

1.2 Electro-Explosive Device and Thermal Transient Testing

An electro-explosive device (EED) transfers electricity into heat. When a current pulse is applied to an EED, the temperature of the bridgewire inside increases. Once the temperature of the bridgewire is high enough, the heat fires off the pyrotechnic powder around the bridgewire and explosion occurs. Nowadays, it is used not only to military affairs but in our daily life, like igniters of airbags.

It can not be judged if an EED is active by its appearance. From the traditional point of view, destructive explosive methods are needed to explore the reliability of an EED. That implies the EED is an one-shot device, which means after the test, a tested unit can not be utilized again. In the destructive experiment, a certain amount of experimental units are put into tests at prespecific times and the numbers of active and failed products are recorded. At a certain testing time, the number of active products follows a binomial distribution with sample size equaling the amount of the tested units, and success probability equaling the survival function of the product at the testing time. The likelihood of the dichotomous data then can be constructed by multiplying up those binomial densities, under the independence assumption. As mentioned, one of the drawbacks of using dichotomous data is that it provides the least information about its reliability (comparing to lifetime data and degradation measurements). The required sample size is large to reach a required accuracy.

In order to test a stored EED, a nondestructive test, thermal transient testing (TTT) was developed. The test is based on the response curve of the voltage over time. Theoretically, the curve of an active product increases until reaches its maximum and then maintains the maximum to the end. Hence, an experienced engineer can tell the goodness of a tested unit according to its response curve. However, the judgment by response curve sometimes does not reflect the real state of the EED. For example, an EED with an erratic curve may still function well. Besides, this method is very subjective and depends highly on the experience. So far this graphic method is only used to detect whether a product has failed but has not yet been used to predict the residual lifetime of a stored EED.

In addition to this graphic outcome, several continuous measurements are provided as well, such as the resistance (R), the maximum voltage change at bridgewire terminals (Vm), the temperature rise ($Theta$) and the thermal conductance ($Gamma$), which are parameters reflecting the current condition of an EED at the testing time. Hence they should contain some information of the reliability. For more details about TTT, see Murphy and Menichelli (1979).

1.3 A Motivating Example

We are interested in determining the reliability of a MK71 electric detonator which is a type of EED's. The reliability of one-shot devices has been studied and the destructive experiments are usually used to collect dichotomous data. As mentioned in the previous section that if a degradation measurement or lifetime data is available, it provides more information than dichotomous data.

We knew that the direct lifetime data for an EED was unobservable. Hence we were interested in using degradation measurement to assess the reliability. Having discussed with engineers, we realized that no measurement has been used as a degradation measurement to assess the reliability of MK71 electric detonators. We were also told that the TTT is a nondestructive test and has been used to test if a MK71 electric detonator is still active. As mentioned, the TTT also provides some continuous measurements over time. Thus, we were wondering if one of these measurements can be used as a degradation measurement. That is the value of the measurement can properly represent the condition of the tested MK71 electric detonator.

Therefore, our goal is to design an experiment which can help us to assess the reliability of an EED and to look for a suitable degradation measurement among the continuous measurements provided by TTT at the same time. Once a degradation is available, it can be used not only to test the goodness of an EED but to predict their residual lifetime. By obtaining the information of residual lifetime, the replacement will be more efficient.

Beside an EED's regular characters, MK71 electric detonators are highly reliable due to military standards. The lifetime of a regular MK71 electric detonator is usually years or decades. It is almost impossible to observe failure occurs under a regular condition, so an accelerated experiment is needed. In the experiment, different experimental stresses are applied to experimental units. For each experimental unit, four measurements, R, Vm, $Theta$ and $Gamma$ were recorded over time. At the end of the experiment, a destructive test was applied to examine if the unit is under a good condition.

Though we do not know if any one of the continuous measurements can be a good degradation measurement, we can regard these continuous measurements as auxiliary measurements. The observations of one of the continuous measurement in association with the dichotomous result will be contained in a new reliabiity model, DwACM (see Chapter 3). From the model, the relationship of the measurement and lifetime distribution can be established. We also provide a criterion to select the measurement with the strongest relationship as a degradation measurement in Chapter 4. If the relationship is strong, the estimation accuracy of lifetime will be increased as well.

1.4 Overviews

This article is organized as follows: Chapter 2 gives a brief introduction to the statistical methods which are used in this dissertation. In Chapter 3, a new reliability model, DwACM, is applied to modeling a data set which contains dichotomous results and observations of an auxiliary continuous measurement. A criterion, CCP, is proposed in Chapter 4 to select the best measurement as a degradation measurement among all possible candidates. In Chapter 5, we provide the estimation methods to the DwACM model under both frequentist and Bayesian frameworks. In Chapter 6, we construct an experiment to collect the data we need, and use a simulation study to illustrate the estimation method. A procedure is also proposed in Chapter 6. The conclusion is drawn in Chapter 7, which also contains some directions for future researches.