II. The residual income valuation model

We brief the derivation of Ohlson's residual income valuation model in the following for the convenience of readers.

A. Assumptions

To formulate the valuation model, three analytical assumptions are used. First, stock dividends satisfy the clean surplus relation: they reduce the book value without affecting current earnings. Second, the risk free rate is chosen as the discount rate to meet the risk neutrality requirement. Third, the time-series behavior of abnormal earnings can be captured by a linear model. These three assumptions lead to a linear close-form valuation solution: the intrinsic value equals the book value plus a linear function of the present value of future abnormal earnings. The intrinsic value can also be regarded as a weighted average of book value and current earnings minus current dividends.

B. The valuation model

The clean surplus relation can be described as follows:

$$y_t = y_{t-1} + x_t - d_t, \tag{3}$$

where x_t is the earning in period $[t-1, t]$, y_t is the book value at time t, and d_t is the dividend payout at time t. Equation (3) satisfies the following properties:

$$\begin{aligned} \partial y_t / \partial d_t &= -1 \\ \partial x_t / \partial d_t &= 0 \end{aligned} . \tag{4}$$

These properties imply that dividends reduce book value without affecting current earnings. We define abnormal earnings, denoted by $x_t^{\,a}$, as the earnings minus the cost of capital:

$$x_t^{\,a} = x_t - (R_f - 1) y_{t-1} , \tag{5}$$

where R_f is the risk free rate plus one. If $x_t^{\,a}$ is positive, then the company is making a profit at time t; otherwise, it has a loss. Applying equation (5) to equation (3), we get

$$d_t = x_t^{\,a} - y_t + R_f y_{t-1} . \tag{6}$$

For simplicity, we assume that the term structure of the interest rate is flat and non-stochastic. The intrinsic value V_t^* at time t can then be expressed as

$$V_t^{\,*} = y_t + \sum_{t=1}^{\infty} R_f^{-t} E_t[\tilde{x}_{t+t}^{a}] , \tag{7}$$

in which

$$E_t[\tilde{x}_{t+x}^{a}] / R_f^{\,t} \rightarrow 0, as\, t \rightarrow \infty . \tag{8}$$

C. Forecasting abnormal earnings

Since y_t and the discount rate is observable, we only need to forecast abnormal earnings. As stated earlier, the time-series behavior of abnormal earnings is framed as a linear model as follows:

$$\tilde{x}_{t+1}{}^{a} = w x_t{}^{a} + v_t + \tilde{e}_{1t+1}\,, \tag{9.1}$$

and

$$\tilde{v}_{t+1} = g v_t + \tilde{e}_{2t+1}\,, \tag{9.2}$$

where v_t results from other information besides abnormal earning and (e_{1t}, e_{2t}) are random errors with mean zero. Since v_t originates from something other than abnormal earnings, it is independent of $x_t{}^{a}$. Parameters w and g can be estimated from historical data and restricted between zero and one. We further assume that g equals to one. Equation (9.2) implies that $v_1 = v_2 = \cdots = 0$ when $v_0 = e_{2t} = 0$ and that v_t is independent to each other. With these assumptions, the sequence {$x_t{}^{a}$} follows an AR (1) process. If sequence {$x_t{}^{a}$} and {d_t} for $t = 0,1,\cdots$ are known, then the book value y_t under initial condition $y_0 = -d_0$ **can be derived** by equation (10)

$$y_t = x_t{}^{a} + R_f\, y_{t-1} - d_t\,. \tag{10}$$

Combining equation (9.1) with equation (10), we get

$$E_t[\tilde{x}_{t+1}] = (R_f - 1)y_t + wx_t^a + v_t \text{ ,} \tag{11}$$

where $E_t[.]$ is the expected operator based on information at time t.