6 Constant Parameter Models

In this section, we will adopt the foregoing model and the methodology to a specific case, in which all diffusion coefficients appeared in the dynamics of the assets are constants instead of deterministic functions. The following list if the summary of the underlying dynamics in this constant case, and note that all coefficients without argument notation $(\cdot)$ are all constants.

$$\frac{de(t)}{e(t)} = (\overline{\mu}_e + b_1(t)(r_d(t) - b_d) + b_2(t)(r_f(t) - b_f))dt + \sigma_e d\widehat{Z_e}(t),$$

$$dr_d(t) = a_d(b_d - r_d(t))dt - \sigma_{r_d} d\widehat{Z_{r_d}}(t), \tag{16}$$

$$M_d(t) = \exp\left(\int_0^t r_d(s)ds\right),$$

$$\begin{aligned}
\frac{dB_d(t,T_d)}{B_d(t,T_d)} &= r_d(t)dt + \sigma_{B_d}(T_d - t)(d\widehat{Z_{r_d}}(t) + \lambda_{r_d}dt), \\
B(T_d, T_d) &= 1,
\end{aligned}$$

where

$$\sigma_{B_d}(\tau) = \frac{1 - e^{-a_d \tau}}{a_d}\sigma_{r_d},$$

$$\frac{dS_d(t)}{S_d(t)} = (\mu_{S_d} + r_d(t))dt + \sigma_{S_d} d\widehat{Z_{S_d}}(t),$$

$$dr_f(t) = a_f(b_f - r_f(t))dt - \sigma_{r_f} d\widehat{Z_{r_f}}(t), \tag{17}$$

$$M_f(t) = \exp\left(\int_0^t r_f(s)ds\right),$$

$$\begin{aligned}
\frac{dB_f(t,T_f)}{B_f(t,T_f)} &= r_f(t)dt + \sigma_{B_f}(T_f - t)(d\widehat{Z_{r_f}}(t) + \lambda_{r_f}dt), \\
B(T_f, T_f) &= 1,
\end{aligned}$$

where

$$\sigma_{B_f}(\tau) = \frac{1 - e^{-a_f \tau}}{a_f}\sigma_{r_f},$$

$$\frac{dS_f(t)}{S_f(t)} = (\mu_{S_f} + r_f(t))dt + \sigma_{S_f} dZ_{S_f}(t),$$

$$\frac{d\widehat{M_f}(t)}{\widehat{M_f}(t)} = (\mu_e + r_f(t))dt + \sigma_e d\widehat{Z_e}(t),$$

$$\frac{d\widehat{S_f}(t)}{\widehat{S_f}(t)} = (\xi_f + r_f(t))dt + \sigma_{S_f} dZ_{S_f}(t) + \sigma_e d\widehat{Z_e}(t),$$

where

$$\xi_f = \overline{\mu}_e + b_1(r_d(t) - b_d) + b_2(r_f(t) - b_f) + \mu_{S_f} + \sigma_{e,S_f},$$

$$\frac{d\widehat{B_f}(t, T_f)}{\widehat{B_f}(t, T_f)} = (\zeta_f(t, T_f) + r_f(t))dt + \sigma_{B_f}(T_f - t)d\widehat{Z_{r_f}}(t) + \sigma_e d\widehat{Z_e}(t),$$

where

$$\zeta_f(t, T_f) = \overline{\mu}_e + b_1(r_d(t) - b_d) + b_2(r_f(t) - b_f) + \lambda_{r_f}\sigma_{B_f}(T_f - t) + \sigma_{e,B_f}(T_f - t),$$

and

$$\Theta(t) = \begin{bmatrix} \sigma_e & 0 & 0 & 0 & 0 \\ 0 & \sigma_{B_d}(t, T_d) & 0 & 0 & 0 \\ \sigma_e & 0 & \sigma_{B_f}(t, T_f) & 0 & 0 \\ 0 & 0 & 0 & \sigma_{S_d} & 0 \\ \sigma_e & 0 & 0 & 0 & \sigma_{S_f} \end{bmatrix},$$

and

$$\Phi(t) = \Theta(t)^{-1} \begin{bmatrix} \overline{\mu}_e + b_1(t)(r_d(t) - b_d) + b_2(t)(r_f(t) - b_f) \\ \lambda_{r_d}\sigma_{B_d}(T_d - t) \\ \zeta_f(t, T_f) \\ \mu_{S_d}(t) \\ \xi_f(t) \end{bmatrix} + \Theta(t)^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} (r_f(t) - r_d(t)) .$$

$$= \Phi_1(t) + \Phi_2(t)(r_f(t) - r_d(t)). \quad (18)$$

According to Hull and White (1990), we have

$$\begin{aligned} B_d(t, T_d) &= A(t, T_d) \exp\left(-B(t, T_d) r\right) \quad (19) \\ B(t, T_d) &= (1 - \exp(-a_d(T - t)))/a_d, \\ A(t, T_d) &= \exp\left[\frac{(B(t, T_d) - T_d + t)(a_d\phi_d - \sigma_{r_d}^2/2)}{a_d^2} - \frac{\sigma_d^2 B\,(t, T_d)^2}{4a_d}\right] \\ \phi_d &= a_d b_d + \lambda_{r_d}\sigma_{r_d}, \end{aligned}$$

$$dr_d(t) = a_d(b_d - r_d(t))dt - \sigma_{r_d} d\widehat{Z_{r_d}}(t).$$

It can be written as

$$dr_d(t) = r_d(t)dH_d(t) + dN_d(t),$$

where $H_d(t) = -a_d t$ and $dN_d(t) = a_d b_d dt - \sigma_{r_d} d\widehat{Z_{r_d}}(t)$. Then

$$
\begin{aligned}
r_d(t) &= e^{H(t)}\left(r_d(0) + \int_0^t \exp(-H_d(s)dN_d(s)\right) \\
&= (r_d(0) - b_d)\exp(-a_d t) + b_d - \sigma_r \int_0^t \exp(a_d(s-t))d\widehat{Z_{r_d}}(s),
\end{aligned}
$$

and

$$
\int_t^T r_d(s)ds = D_d - \int_t^T g_d(s)d\widehat{Z_{r_d}}(s), \tag{20}
$$

where

$$
\begin{aligned}
D_d &= (r_d(t) - b_d)\frac{1-\exp(-a_d T)}{a_d} + b_d T = (r_d(t) - b_d)\frac{\sigma_{B_d}(T)}{\sigma_{r_d}} + b_d T, \\
g_d(s) &= \sigma_{r_d}\frac{1-\exp(-a_d(T-s))}{a} = \sigma_{B_d}(T-t).
\end{aligned}
$$

Thus, by substituting (19) and (20) into (12), we have

$$
\begin{aligned}
\theta(t,T) &= \exp\left\{-\int_t^T \Phi(\tau)^\intercal dW(\tau) - \int_t^T \left(r_d(\tau) + \frac{1}{2}\Phi(\tau)^\intercal\Phi(\tau)\right)d\tau\right\} B_d(t,T)^{-1} \\
&= \exp\left\{-\int_t^T \Phi(\tau)^\intercal dW(\tau) - \int_t^T \frac{1}{2}\Phi(\tau)^\intercal\Phi(\tau)d\tau\right\} \\
&\quad \times \exp\left\{D_d - \int_t^T g_d(s)d\widehat{Z_{r_d}}(s)\right\} \times A(t,T_d)\exp\left(-B(t,T_d)r\right).
\end{aligned}
$$

Upon inspection, only the first term in the last equality would generate stochastic components after taking conditional expectations. We proceed to carry out the calculation.

Applying the decomposition of $\Phi(t)$ in (10) to the integral

$$\begin{aligned}
& \exp\left\{\int_t^T \Phi(\tau)^\intercal dZ(\tau) + \int_t^T \frac{1}{2}\Phi(\tau)^\intercal \Phi(\tau) d\tau\right\} \qquad (21)\\
= \;& \exp\left\{\int_t^T \Phi_1(\tau)^\intercal dZ(\tau) + \int_t^T (r_f(\tau) - r_d(\tau))\Phi_2(\tau)^\intercal dZ(\tau)\right\} \\
& \times \exp\left\{\frac{1}{2}\int_t^T \Phi_1(\tau)^\intercal \Phi_1(\tau) d\tau + \int_t^T \Phi_1(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau)) d\tau\right\} \\
& \times \exp\left\{\frac{1}{2}\int_t^T \Phi_2(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau))^2 d\tau\right\}.
\end{aligned}$$

We neglect the integrals $\frac{1}{2}\int_t^T \Phi_1(\tau)^\intercal \Phi_1(\tau) d\tau$, $\int_t^T \Phi_1(\tau)^\intercal dZ(\tau)$ on the right-hand side of (21) because of the deterministic contributions after taking the conditional expectations. There are three stochastic integrals left, namely,

$$\begin{aligned}
& \int_t^T (r_f(\tau) - r_d(\tau))\Phi_2(\tau)^\intercal dZ(\tau), \\
& \int_t^T \Phi_1(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau)) d\tau \text{ and} \\
& \frac{1}{2}\int_t^T \Phi_2(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau))^2 d\tau.
\end{aligned}$$

Note that, from (16) and (17) the dynamics of $r_f(\tau) - r_d(\tau)$ is

$$\begin{aligned}
r_f(\tau) - r_d(\tau) &= (r_d(0) - b_d)\exp(-a_d t) - (r_f(0) - b_f)\exp(-a_f t) + b_d - b_f \\
&\quad -\sigma_{r_d}\int_0^t \exp(a_d(s-t)) d\widehat{Z_{r_d}}(s) + \sigma_{r_f}\int_0^t \exp(a_f(s-t)) d\widehat{Z_{r_f}}(s), \qquad (22)\\
d(r_f(\tau) - r_d(\tau)) &= \left(a_d(b_d - r_d(\tau)) - a_f(b_f - r_f(\tau))\right) d\tau - \left(\sigma_{r_d} d\widehat{Z_{r_d}}(\tau) - \sigma_{r_f} d\widehat{Z_{r_f}}(\tau)\right) \qquad (23)\\
(d(r_f(\tau) - r_d(\tau)))^2 &= \left(\sigma_{r_d}^2 + \sigma_{r_f}^2\right).
\end{aligned}$$

Define $\Psi(t)$ such that

$$\Psi(t) = \int_0^t \Phi_2(\tau)^\intercal \Phi_2(\tau) d\tau. \tag{24}$$

Integration by parts and the application of Itô's lemma with $(r_f(\tau) - r_d(\tau))^2$ render the integral

$$\frac{1}{2}\int_t^T \Phi_2(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau))^2 d\tau,$$

into

$$\begin{aligned}
& \frac{1}{2}\int_t^T \Phi_2(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau))^2 d\tau \\
= \; & \frac{1}{2}(r_f(T) - r_d(T))^2 \Psi(T) - \frac{1}{2}(r_f(t) - r_d(t))^2 \Psi(t) \\
& - \int_t^T \Psi(\tau)(r_f(\tau) - r_d(\tau)) d(r_f(\tau) - r_d(\tau)) \\
& - \frac{1}{2}\int_t^T \Psi(\tau) \left(d(r_f(\tau) - r_d(\tau)) \right)^2 .
\end{aligned} \tag{25}$$

After substituting (15) into (17), it's clear that the only term we need to specify is

$$\int_t^T \Psi(\tau)(r_f(\tau) - r_d(\tau)) d(r_f(\tau) - r_d(\tau)),$$

and

$$
\begin{aligned}
& \int_t^T \Psi(\tau)(r_f(\tau)-r_d(\tau))d(r_f(\tau)-r_d(\tau)) \\
= & \int_t^T \Psi(\tau)(r_f(\tau)-r_d(\tau))\left(a_d(b_d-r_d(\tau))-a_f(b_f-r_f(\tau))\right)d\tau \\
& -\int_t^T \Psi(\tau)(r_f(\tau)-r_d(\tau))\sigma_{r_d}d\widehat{Z_{r_d}}(\tau)+\int_t^T \Psi(\tau)(r_f(\tau)-r_d(\tau))\sigma_{r_f}d\widehat{Z_{r_f}}(\tau) \\
= & (r_f(T)-r_d(T))\Upsilon(T)-(r_f(t)-r_d(t))\Upsilon(t) \\
& -\int_t^T \Upsilon(T)\left(a_d(b_d-r_d(\tau))-a_f(b_f-r_f(\tau))\right)d\tau \\
& -\int_t^T \Upsilon(T)\sigma_{r_d}d\widehat{Z_{r_d}}(\tau)+\int_t^T \Upsilon(T)\sigma_{r_f}d\widehat{Z_{r_f}}(\tau) \\
& -\int_t^T \Psi(\tau)(r_f(\tau)-r_d(\tau))\sigma_{r_d}d\widehat{Z_{r_d}}(\tau)+\int_t^T \Psi(\tau)(r_f(\tau)-r_d(\tau))\sigma_{r_f}d\widehat{Z_{r_f}}(\tau),
\end{aligned}
$$

through repeated integration by parts, where

$$
\Upsilon(T)=\int_0^t \Psi(\tau)\left(a_d(b_d-r_d(\tau))-a_f(b_f-r_f(\tau))\right)d\tau. \tag{26}
$$

Thus, we may summarize our results in the following lemmas.

Lemma 1 *With the assumptions of our financial model, there exist two deterministic functions* $\Psi(T)$ *in equation* (23) *and* $\Upsilon(T)$ *in equation* (26) *such that*

$$
\begin{aligned}
& \frac{1}{2}\int_t^T \Phi_2(\tau)^\intercal\Phi_2(\tau)(r_f(\tau)-r_d(\tau))^2 d\tau \qquad (27)\\
= & \frac{1}{2}(r_f(T)-r_d(T))^2\Psi(T)-\frac{1}{2}(r_f(t)-r_d(t))^2\Psi(t) \\
& -(r_f(T)-r_d(T))\Upsilon(T)-(r_f(t)-r_d(t))\Upsilon(t)+\sum_i\int_t^T(\cdot)\,dW_i(\tau)+(\cdot)\,,
\end{aligned}
$$

where $i = (r_d, r_f)$.

Lemma 2 *With the assumption of our financial model, there exist two deterministic functions* $\widetilde{\Psi}(T)$ *such that the integral*

$$\int_t^T \Phi_1(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau))d\tau,$$

may be treated in a similar fashion. The final result is

$$\begin{aligned} & \int_t^T \Phi_1(\tau)^\intercal \Phi_2(\tau)(r_f(\tau) - r_d(\tau))d\tau & (28) \\ = \;& (r_f(T) - r_d(T))\widetilde{\Psi}(T) - (r_f(t) - r_d(t))\widetilde{\Psi}(t) + \sum_i \sum_i \int_t^T (\cdot)\, dW_i(\tau) + (\cdot)\,, \end{aligned}$$

where $i = (r_d, r_f)$.

$$\widetilde{\Psi}(T) = \int_t^T \Phi_1(\tau)^\intercal \Phi_2(\tau) d\tau. \tag{29}$$

Lemma 3 *With the assumptions of our financial model, substituting the expression* (22) *into the stochastic integral*

$$\int_t^T (r_f(\tau) - r_d(\tau))\Phi_2(\tau)^\intercal dZ(\tau),$$

we have

$$
\begin{aligned}
& \int_t^T (r_f(\tau) - r_d(\tau))\Phi_2(\tau)^\intercal dZ(\tau) \qquad (30) \\
= & \int_t^T \begin{pmatrix} (r_d(0) - b_d)\exp(-a_d t) - (r_f(0) - b_f)\exp(-a_f t) + b_d - b_f \\ -\sigma_{r_d} \int_0^t \exp(a_d(s-t)) d\widehat{Z_{r_d}}(s) + \sigma_{r_f} \int_0^t \exp(a_f(s-t)) d\widehat{Z_{r_f}}(s) \end{pmatrix} \Phi_2(\tau)^\intercal dZ(\tau).
\end{aligned}
$$

Collecting all the results of (27), (28) and (30) obtained above, we compute $J(\gamma; t, T)$ as

$$
\begin{aligned}
J(\gamma; t, T) &= E_t\left[\theta(t,T)^{\frac{\gamma}{\gamma-1}}\right] \\
&= K(\gamma; t, T)) \exp\left\{\frac{\gamma}{2(\gamma-1)}(r_f(t) - r_d(t))^2 (\Psi(T) - \Psi(t))\right\} \\
&\quad \times \exp\left\{\frac{\gamma}{\gamma-1}(r_f(t) - r_d(t))\left(\widetilde{\Psi}(T) - \widetilde{\Psi}(t) - \Upsilon(T) + \Upsilon(t)\right)\right\},
\end{aligned}
$$

where $K(\gamma; t, T))$ is a deterministic function. Here we utilize the independence property of $(r_f(T) - r_d(T)) - (r_f(t) - r_d(t))$ with respect to the conditional expectation operator $E_t[\cdot]$

because of the expression (22) . Applying Itô's lemma, we have

$$\begin{aligned}
& \frac{dJ(\gamma;t,T)}{J(\gamma;t,T)} \\
= \; & \frac{\gamma}{\gamma-1}\left\{(\Psi(T)-\Psi(t))\left(r_f(T)-r_d(T)\right)+\left(\widetilde{\Psi}(T)-\widetilde{\Psi}(t)-\Upsilon(T)+\Upsilon(t)\right)\right\} \\
& \times d\left(r_f(t)-r_d(t)\right)+(\cdot)dt \\
= \; & \frac{\gamma}{\gamma-1}\left\{(\Psi(T)-\Psi(t))\left(r_f(T)-r_d(T)\right)+\left(\widetilde{\Psi}(T)-\widetilde{\Psi}(t)-\Upsilon(T)+\Upsilon(t)\right)\right\} \\
& \times\left[-\left(\sigma_{r_d}d\widehat{Z_{r_d}}(\tau)-\sigma_{r_f}d\widehat{Z_{r_f}}(\tau)\right)\right]+(\cdot)dt.
\end{aligned}$$

We immediately obtain the following proposition.

Proposition 4 *The instantaneous conditional* $\left(\frac{\gamma}{\gamma-1}\right)$ *moment of the Arrow-Debreu prices of the reference country bond of maturity* T *is given by*

$$\begin{aligned}
J(\gamma;t,T) &= E_t\left[\theta(t,T)^{\frac{\gamma}{\gamma-1}}\right] \\
&= K(\gamma;t,T))\exp\left\{\frac{\gamma}{2(\gamma-1)}\left(r_f(t)-r_d(t)\right)^2\left(\Psi(T)-\Psi(t)\right)\right\} \\
&\quad\times\exp\left\{\frac{\gamma}{\gamma-1}\left(r_f(t)-r_d(t)\right)\left(\widetilde{\Psi}(T)-\widetilde{\Psi}(t)-\Upsilon(T)+\Upsilon(t)\right)\right\},
\end{aligned}$$

where $K(\gamma;t,T))$ *is a deterministic function. The diffusion vector* $\sigma_J(\gamma;t,T)^\intercal$ *of the process* *of $\frac{dJ(\gamma;t,T)}{J(\gamma;t,T)}$ is given by*

$$
\begin{aligned}
& \sigma_J(\gamma;t,T)^\intercal \qquad (31)\\
= \; & \frac{\gamma}{\gamma-1}\left\{(\Psi(T)-\Psi(t))\left(r_f(T)-r_d(T)\right)+\left(\widetilde{\Psi}(T)-\widetilde{\Psi}(t)-\Upsilon(T)+\Upsilon(t)\right)\right\} \\
& \times \begin{bmatrix} 0 & \sigma_{r_d} & \sigma_{r_f} & 0 & 0 \end{bmatrix}.
\end{aligned}
$$

Substituting the expressions of $\Psi(t)$, $\Upsilon(t)$ and $\widetilde{\Psi}(t)$ in (24), (26) and (29), respectively and (14), (18) and (31) into (15), we obtain the expression of optimal allocation strategy $\pi(t)$ of risky assets.

In this case, we have the following proposition.

Proposition 5 ***(Optimal Asset Allocation Strategy in the Constant Case)*** *The domestic isoelastic investor's optimal allocation strategy $\pi(t)$ of risky assets is given by*

$$
\begin{aligned}
\pi(t) &= \frac{1}{1-\gamma}\Theta(t)^{-1}\Phi(t)-\frac{\gamma}{1-\gamma}\Theta(t)^{-1}\begin{bmatrix} 0 \\ \sigma_{B_d}(t,T_d) \\ 0 \\ 0 \\ 0 \end{bmatrix}+\frac{\gamma}{\gamma-1}\Lambda(t,T)\Theta(t)^{-1}\begin{bmatrix} 0 \\ \sigma_{r_d} \\ \sigma_{r_f} \\ 0 \\ 0 \end{bmatrix} \qquad (32)\\
&= \pi_1+\pi_2+\pi_3,
\end{aligned}
$$

where

$$\Lambda(t,T) = \left\{ (\Psi(T) - \Psi(t))\,(r_f(T) - r_d(T)) + \left(\widetilde{\Psi}(T) - \widetilde{\Psi}(t) - \Upsilon(T) + \Upsilon(t) \right) \right\},$$

and

$$\begin{aligned}
\Psi(t) &= \int_0^t \Phi_2(\tau)^\intercal \Phi_2(\tau) d\tau, \\
\widetilde{\Psi}(T) &= \int_t^T \Phi_1(\tau)^\intercal \Phi_2(\tau) d\tau, \\
\Upsilon(T) &= \int_0^t \Psi(\tau)\,(a_d(b_d - r_d(\tau)) - a_f(b_f - r_f(\tau)))\,d\tau.
\end{aligned}$$

$$
\begin{aligned}
\pi_1 &= \frac{1}{1-\gamma}\Theta(t)^{-1}\Phi(t) = \frac{a}{1-\gamma}\cdot \mathbf{w}_M, \\
\mathbf{w}_M &= \frac{\Theta(t)^{-1}\Phi(t)}{1_5^\top \Theta(t)^{-1}\Phi(t)}, \\
a &= 1_5^\top \Theta(t)^{-1}\Phi(t), \\
\pi_2 &= -\frac{\gamma}{1-\gamma}\Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{B_d}(t,T_d) & 0 & 0 & 0 \end{bmatrix}^\top = \frac{-b\gamma}{1-\gamma}\cdot \mathbf{w}_Y, \\
\mathbf{w}_Y &= \frac{\Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{B_d}(t,T_d) & 0 & 0 & 0 \end{bmatrix}^\top}{1_5^\top \Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{B_d}(t,T_d) & 0 & 0 & 0 \end{bmatrix}^\top}, \\
b &= 1_5^\top \Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{B_d}(t,T_d) & 0 & 0 & 0 \end{bmatrix}^\top, \\
\pi_3 &= \frac{\gamma}{1-\gamma}\Lambda(t,T)\Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{r_d} & \sigma_{r_f} & 0 & 0 \end{bmatrix}^\top = \frac{c\gamma}{1-\gamma}\cdot \mathbf{w}_E, \\
\mathbf{w}_E &= \frac{\Lambda(t,T)\Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{r_d} & \sigma_{r_f} & 0 & 0 \end{bmatrix}^\top}{1_5^\top \Lambda(t,T)\Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{r_d} & \sigma_{r_f} & 0 & 0 \end{bmatrix}^\top}, \\
c &= 1_5^\top \Lambda(t,T)\Theta(t)^{-1}\begin{bmatrix} 0 & \sigma_{r_d} & \sigma_{r_f} & 0 & 0 \end{bmatrix}^\top.
\end{aligned}
$$

given a, b, and c are real constants.

This is related to the four-fund theorem addressed in Rudolf and Ziemba (2004). In four-fund theorem, the international portfolio invests in the following four funds to maximize the expected utility:

1. The international myopic portfolio $\mathbf{w}_M$ with level $\frac{a}{1-\gamma}$.

2. The domestic interest rate hedge portfolio $\mathbf{w}_Y$ with level $\frac{-b\gamma}{1-\gamma}$.

3. The cross country interest rate differential hedge portfolio $\mathbf{w}_E$ with level $\frac{c\gamma}{1-\gamma}$.

4. Finally, the domestic riskless asset with level $1 - \frac{a}{1-\gamma} + \frac{b\gamma}{1-\gamma} - \frac{c\gamma}{1-\gamma}$.