1 Introduction

International finance theory emphasizes the effectiveness of global diversification in achieving a higher expected return at a lower risk (Levy and Sarnat, 1970). Solnik (1974) derives an international Capital Asset Pricing Model that predicts that the share of wealth in the domestic market should be a constant multiple of the share of wealth invested in the foreign market. In this study, we study the optimal portfolio choice when markets are incomplete due to the lack of information regarding to the exchange rate risks, i.e., the state variable which determines the investment opportunity that set can not be observed in advance properly by the investors. In order to simplify our model, an international economy based on two-country scenario is selected and the optimal investment strategy is constructed for the investors given that the interest rate movements within two countries influence the exchange rate movements. According to the diversification effects in the international capital asset pricing model for the investors, the co-movements of the interest rates influence the optimal positions of the domestic and foreign currency holding significantly. In the meantime, when the holding position of the currency changes, the updated exchange rates are required to rebalance the holding for the consecutive rebalance date. Based on the market information, we assume that the change of the interest rates within two countries has significant influence upon the exchange rate movements and can be properly regarded as a proxy in forecasting the exchange rates in the international portfolio selection. Hence the main contribution of this study is to enhance the understanding of the co-movement effects between the interest rates and exchange rates in monitoring the optimal investment behaviors.

To explore the dynamic relationship between the exchange rate process and the interest rate process, the filtering approach is employed in solving the portfolio choice problem analytically. When the information of the interest rate process is disclosed, the exchange rate movements can be updated recursively using the interest rate as the proxy. Since interest rate and exchange rate dynamics are modeled through the stochastic processes, in response to the information from the exchange rate movements, the filtering methodology in Liptser and Shiryayev (2001) are adopted in this study to construct the optimal investment strategy. Investors use interest rates as the noisy signals to infer the exchange rate process (i.e., the unobservable state variable). The estimation process allows the agent to project the dynamics of the unobserved variables into the space of the securities. From the investors point of view, the financial markets are complete since the inferred processes for the state variables are spanned by the market securities.

Recent years have witnessed the introduction of foreign investment products aimed at attracting investors (i.e., life insurance underwriter, pension fund manager and wealthy individuals) who look for development of foreign financial market. The main characteristics of these products offer investment vehicles in hedging the domestic country risk with exposure to upside movements of the foreign markets within various time horizons. This paper discusses the issue of investors who want to find optimal international portfolio investment strategy which consider two countries' interest rate influence to the exchange rate mainly. As to cross-country investor, the hypothesis that change of the interest rate will influence the position that two countries' cash is held, at the same time when the position of the cash changes, the exchange rate will change. Under this point of view, we suppose that the change of two countries' interest rate have influence of a certain degree on the exchange rate change, and this one that influenced to investor's investment decision which is a topic of main research of this paper. The impact on exchange rate of the interest rate, we utilize the idea filtered, when the new information of the interest rate change which cause influence on the exchange rate change. Interest rate and exchange rate are all stochastic process, to this change course of exchange rate, the message of the interest rate needs receiving in, considering the influencing each other of two stochastic processes, we use the method of Liptser and Shiryayev (2001), at this assumption, we want to find optimal investment strategy. Here, parameter estimation can increase the new uncertain factor, and it turns into incomplete market. We suppose that the results of learning will tend towards a stable state, namely variation that parameter estimate is it can disappear get a regular value to count, can decrease the uncertain factor. We can utilize the assumption of the complete market to find out the final result.

Within this international economy, the changes of real exchange rates, real interest rates and stock prices follow the diffusion processes whose drifts and volatility parameters are driven by certain state variables. A country-specific representative individual trades on available assets to maximize the expected utility of her final wealth. The traditional solution to this problem is derived by using the stochastic dynamic programming technique pioneered in finance by Merton (1969, 1971). The investor's optimal portfolio strategy is known to contain a speculative element and as many hedge components as the number of state variables. However, while the speculative element is well identified and easy to interpret and work out, the implementation of the hedge components is problematic as the investor must first identify all the relevant state variables and then try to find their closed form expression by constructing the nonlinear Hamilton-Jacobi-Bellman (H-J-B) equations.

Instead of using stochastic control methods, the so-called martingale approach has been alternatively used by Pliska (1986), Karatzas et al. (1987) and Cox and Huang (1989, 1991) to study intertemporal consumption and portfolio policies when markets are complete, which was also the case in the earlier dynamic programming literature. The martingale technology describes the feasible investment strategy set by a single intertemporal budget equation and then solves the static investment problem in an infinite dimensional Arrow Debreu economy.

As mentioned in Vila and Zariphopoulou (1997), the martingale approach is appealing for two reasons. First, it can be used to solve for the asset demand under very general investment decisions regarding to the stochastic opportunity set. Second, and consequently, it can be applied in a general setting to solve for the equilibrium investment opportunity set (see Duffie and Huang (1985)).

Following the early research in Bawa et al. (1979), consider the parameter uncertainty influence that chooses to single period portfolio, and show that the prediction distribution of the returns can obtain different conditional distribution while the parameters uncertainty rather than certainty. The additional risk that increased by parameter uncertainty is called the estimation risk. Williams (1977) initiates an early work of estimation risk in a continuous time setting. Stulz (1986, 1987) studies the effect of learning about the monetary policy on interest and exchange rates. Detemple (1986) discusses estimation risk in a production economy, and Dothan and Feldman (1986) and Feldman (1992) discuss the term structure and interest rate dynamics with learning. Gennotte (1986) is a good example of a study of signal filtration and learning in a dynamic portfolio choice setting.

In recent years, Kandel and Stambaugh (1996) explore the economic importance of stock return predictability and the effect of estimate risk as assets remuneration can partially predict and the coefficient of the predictive relation are estimated but not know. Given that the exact predictive relation is unknown and that the investor is assumed to trade only at discrete one-month intervals, uncertainty about the parameters of the conditional return distribution—estimation risk—affects the investor's optimal portfolio decision. Although this simple framework point out the economic importance of stock return predictability given estimation risk, the one period investment horizon assumption prevents considerations of the dynamic effects induced both by full information hedging and by learning through time, which may be important for investors with longer horizons.

Gennotte (1986) and Feldman (1992) find that consider security prices in continuous time following diffusion processes that gets different result when the parameter uncertainty in discrete-time single-period model. In particular, the Bawa et al. (1979) estimation risk effect disappears, so that an investor with logarithmic utility ignores stochastic variation in the future investment opportunity set. An investor with utility that is not the logarithm must consider the uncertain parameter, not because it influences the best base efficient file of change immediately, but because when time passes, he will understand the parameter in detail. The value of his estimates of unknown parameters is "state variable" in his dynamic optimization problem and it is need to take precautions against unanticipated changes in these state variables that influence the best portfolio choice. Poncet (2003) have found the optimal portfolio about international asset allocation but the predictability is not considered.

The effect of stock return predictability for an investor with a long investment horizon in the absence of parameter uncertainty is studied in several recent papers, including Brennan, Schwartz and Lagnado (1997), Campbell and Viceira (1999), Brandt (1999), Lynch and Balduzzi (1998), and Liu (1998). However, none of these papers take into account the fact that the underlying predictive relation is uncertain. Barberis (2000) studies a problem that is closely related to the one analyzed in this paper. He derives a dynamic strategy in a discrete time setting with estimation risk; however, in doing so, he simplifies the problem by ignoring the possibility that the investor will learn more about the predictive relation as time passes. Xia (2001) examines the optimal dynamic portfolio strategy for a long horizon investor who takes into account the uncertain evidence of stock return predictability. Following Xia (2001), the Kalman filter approach is employed under the Bayesian methodology to consider two sequences which one of the two process can be used to predict the another. Given the prior information, we can find the posterior mean and variance of the entire unknown state process.

The paper adds to the above literature by providing further understanding of the importance of prediction effects with respect to the international portfolio selection problem. The features of our models are summarized in the following

1. There are five potential sources of uncertainty in the model economy that we have described: interest rate risks represented by the innovations for the domestic and foreign markets, the unanticipated stock returns for the domestic and foreign markets, and the exchange rate risk.

2. We study the effect of parameter uncertainty and its associated hedge component in a dynamic continuous time context with a time-variant investment opportunity set, which is induced by possible exchange rate predictability.

3. The hedging demands for currency risk and interest rate risk are fully investigated through dynamic fund separation theorems proposed in Merton (1971). Certain mutual funds are constructed to demonstrate the proposed demands.

The rest of this paper is organized as follows. Section 2 describes the financial market and the proposed model, starting from the basic framework and followed by the dynamics of invested opportunity set and the martingale constraints. Section 3 use learning effect to find the process about predictive variable. Section 4 explores its explicit characteristics regarding the fund wealth on the optimal investment decision incorporating the exchange rate and interest rate risks. Section 5 presents the closed-form solution for the model with constant parameters. A special case with simplified assumptions is fully explored in Section 6. Conclusions are presented in Section 7.