第五章　租稅外部性、支出外部性及最適租稅

過去，有學者（如 Dahlby & Wilson）將支出外部性界定為地方公共支出對其他地區或不同層級政府稅基所產生的影響。在此，吾人則假設地方政府所提供之公共財具有外溢性（Spillover），會直接干擾其他地區居民的效用水準，故我們將原本之效用函數納入其他地方公共財，用以探討水平與支出外部性同時存在是否會對先前的結論有所改變。

第一節　稅基完全重疊下之總合分析

誠如上述，我們開始在模型運作裡加入支出外部性的考量，並假設該外溢效果為正，且繼續允許跨區購買的行為。如此，我們便可完整地分析租稅外部性與支出外部性的總合效果。

<u>地方政府的租稅政策</u>：

$$\underset{t_{1i},t_{2i}}{Max}\quad V\left(\underset{-}{q_i}, g_i, g_{i^*}, g_f\right)$$

s.t.
$$t_{1i}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{2i}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)=g_i$$

$$\sum_{i=1}^{2}\left[t_{1f}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{2f}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)\right]=2g_f$$

f.o.c.

$$\frac{\partial L_i}{\partial t_{ji}}=\frac{\partial V}{\partial q_{ji}}+MU_s\left[x_j\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)-\frac{1}{d}t_{ji}x_j+\sum_{k=1}^{2}t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}\right]$$

$$+MU_{s^*}\left(\frac{1}{d}t_{ji^*}x_{ji^*}\right)+\frac{1}{2}MU_f\left[\sum_{k=1}^{2}t_{kf}\frac{\partial x_{ki}}{\partial q_{ji}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)-\frac{1}{d}t_{jf}x_{ji}\right]=0\qquad j=1,2$$

(37)

利用 Roy's Identity 與 Slutsky Matrix

$$\Rightarrow -ax_{ji}+MU_s\left[x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)-\frac{1}{d}t_{ji}x_{ji}+\sum_{k=1}^{2}t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\left(\frac{\partial h_{ji}}{\partial q_{ki}}-x_{ji}\frac{\partial x_{ki}}{\partial y}\right)\right]$$

$$+MU_{s^*}\left(\frac{1}{d}t_{ji^*}x_{ji^*}\right)+\frac{1}{2}MU_f\left[\sum_{k=1}^{2}t_{kf}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\left(\frac{\partial h_{ji}}{\partial q_{ki}}-x_{ji}\frac{\partial x_{ki}}{\partial y}\right)-\frac{1}{d}t_{jf}x_{ji}\right]=0$$

$$\Rightarrow MRS_{sf}\sum_{k=1}^{2}t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial h_{ji}}{\partial q_{ki}}+\frac{1}{2}\sum_{k=1}^{2}t_{kf}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial h_{ji}}{\partial q_{ki}}$$

$$=x_{ji}\left\{\frac{a}{MU_f}-MRS_{sf}\left[\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)-\frac{1}{d}t_{ji}\right]+\sum_{k=1}^{2}\left(MRS_{sf}t_{ki}+\frac{1}{2}t_{kf}\right)\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial y}\right.$$

$$\left.+\frac{1}{d}t_{jf}-MRS_{s^*f}\frac{1}{d}t_{ji^*}\frac{x_{ji^*}}{x_{ji}}\right\}$$

$\because$ **Symmetric Equilibrium** $\therefore t_{ji}=t_{ji^*}$ **,** $x_{ji}=x_{ji^*}$

$$\Rightarrow MRS_{sf}\left[\left(\frac{1}{2}\right)\tilde{t}_{1s}h_{j1}+\left(\frac{1}{2}\right)\tilde{t}_{2s}h_{j2}\right]+\frac{1}{2}\left[\left(\frac{1}{2}\right)\tilde{t}_{1f}h_{j1}+\left(\frac{1}{2}\right)\tilde{t}_{2f}h_{j2}\right]=q_s'' \qquad (38)$$

$$q_s''=\left[\frac{a}{MU_f}-MRS_{sf}\left(\frac{1}{2}-\frac{1}{d}t_{js}\right)+\sum_{k=1}^{2}\left(MRS_{sf}t_{ks}+\frac{1}{2}t_{kf}\right)\left(\frac{1}{2}\right)\frac{\partial x_k}{\partial y}+\frac{1}{d}t_{jf}-MRS_{s^*f}\frac{1}{d}t_{js}\right]$$

<u>中央政府的租稅政策</u>：

$$\underset{t_{1f},t_{2f}}{Max}\quad \sum_{i=1}^{2}V(\underset{-}{q_i},g_i,g_{i^*},g_f)$$

s.t. $$t_{1i}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{2i}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)=g_i$$

$$\sum_{i=1}^{2}\left[t_{1f}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{2f}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)\right]=2g_f$$

f.o.c.

$$\frac{\partial L_f}{\partial q_{ji}} = \frac{\partial V}{\partial q_{ji}} + MU_s\left[\sum_{k=1}^{2} t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}\right] + MU_{s^*}\left[\sum_{k=1}^{2} t_{ki^*}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\frac{\partial x_{ki^*}}{\partial q_{ji^*}}\right]$$

$$+MU_f\left[x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)+\sum_{k=1}^{2} t_{kf}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}\right]=0 \quad j=1,2 \qquad (39)$$

利用 Roy's Identity、Slutsky Matrix：

$$\Rightarrow -ax_{ji} + MU_s\left[\sum_{k=1}^{2} t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\left(\frac{\partial h_{ji}}{\partial q_{ki}} - x_{ji}\frac{\partial x_{ki}}{\partial y}\right)\right] + MU_{s^*}\left[\sum_{k=1}^{2} t_{ki^*}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\left(\frac{\partial h_{ji^*}}{\partial q_{ki^*}}\right.\right.$$

$$\left.\left.-x_{ji^*}\frac{\partial x_{ki^*}}{\partial y}\right)\right] + MU_f\left[\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)x_{ji} + \sum_{k=1}^{2} t_{kf}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\left(\frac{\partial h_{ji}}{\partial q_{ki}} - x_{ji}\frac{\partial x_{ki}}{\partial y}\right)\right]=0$$

在 Symmetric Equilibrium 下，$t_{ji}=t_{ji^*}=t_{js}$，$x_{ji}=x_{ji^*}$，$\frac{\partial x_{ki}}{\partial q_{ji}}=\frac{\partial x_{ki^*}}{\partial q_{ji^*}}$

$$\Rightarrow \left(MRS_{sf}+MRS_{s^*f}\right)\left[\left(\frac{1}{2}\right)\tilde{t}_{1s}h_{j1}+\left(\frac{1}{2}\right)\tilde{t}_{2s}h_{j2}\right]+\left[\left(\frac{1}{2}\right)\tilde{t}_{1f}h_{j1}+\left(\frac{1}{2}\right)\tilde{t}_{2f}h_{j2}\right]=q_f'' \quad (40)$$

$$q_f''=\left[\frac{a}{MU_f}+\sum_{k=1}^{2}\frac{1}{2}\left[\left(MRS_{sf}+MRS_{s^*f}\right)t_{ks}+t_{kf}\right]\frac{\partial x_k}{\partial y}-\frac{1}{2}\right]$$

$$\frac{1}{2}\tilde{t}_{1f}h_{j1}+\frac{1}{2}\tilde{t}_{2f}h_{j2}=\frac{2}{MRS_{sf}-MRS_{s^*f}}\left[MRS_{sf}q_f''-\left(MRS_{sf}+MRS_{s^*f}\right)q_s''\right]=\bar{q}_f$$

$$\frac{1}{2}\tilde{t}_{1s}h_{j1}+\frac{1}{2}\tilde{t}_{2s}h_{j2}=\frac{1}{MRS_{sf}-MRS_{s^*f}}\left(2q_s''-q_f''\right)=\bar{q}_s$$

$$\Rightarrow \frac{1}{2}\left(\tilde{t}_{1f}+\tilde{t}_{1s}\right)h_{j1}+\frac{1}{2}\left(\tilde{t}_{2f}+\tilde{t}_{2s}\right)h_{j2}=(\bar{q}_f+\bar{q}_s) \qquad j=1,2$$

由上述地方與中央的最適決策之一階條件可得：

$\tilde{t}_{jf}=t_j^*(\bar{q}_f)$，$\tilde{t}_{ji}=t_j^*(\bar{q}_s)$，$\tilde{t}_{jf}+\tilde{t}_{ji}=t_j^*\left(\bar{q}_f+\bar{q}_s\right)$

加入支出外部性後對於命題一的影響：

(A) 中央與地方政府有相同的租稅工具時（即兩者稅基完全相同），中央與地方對此二貨物的相對稅率仍應相等：

$$\frac{t_{1i}}{t_{2i}}=\frac{t_{1f}}{t_{2f}}$$

（B）地方公共財有提供過多的情況不一定成立。

將地方與中央的一階條件相減

$$\frac{\partial L_i}{\partial t_{ji}}=\frac{\partial V}{\partial q_{ji}}+MU_s\left[x_j\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)-\frac{1}{d}t_{ji}x_j+\sum_{k=1}^{2}t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}\right]$$

$$+MU_{s^*}\left(\frac{1}{d}t_{ji^*}x_{ji^*}\right)+\frac{1}{2}MU_f\left[\sum_{k=1}^{2}t_{kf}\frac{\partial x_{ki}}{\partial q_{ji}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)-\frac{1}{d}t_{jf}x_{ji}\right]$$

$$=\frac{\partial V}{\partial q_{ji}}+MU_s\left[\sum_{k=1}^{2}t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}\right]+MU_{s^*}\left[\sum_{k=1}^{2}t_{ki^*}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\frac{\partial x_{ki^*}}{\partial q_{ji^*}}\right]$$

$$+MU_f\left[x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)+\sum_{k=1}^{2}t_{kf}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}\right]=\frac{\partial L_f}{\partial t_{jf}}$$

∵Symmetric Equilibrium，上式可化簡如下：

$$\frac{1}{2}(MRS_{sf}-1)x_j=(MRS_{sf}-MRS_{s^*f})\frac{1}{d}t_{js}x_j+\frac{1}{2}t_{jf}x_j+\frac{1}{2}\sum_{k=1}^{2}\left(\frac{1}{2}\right)t_{kf}\frac{\partial x_k}{\partial q_j}+MRS_{s^*f}\sum_{k=1}^{2}\left(\frac{1}{2}\right)t_{ks}\frac{\partial x_k}{\partial q_j}$$

（41）

與（24）式相較，我們發現（41）式右方多了$\left[(MRS_{sf}-MRS_{s^*f})\frac{1}{d}t_{js}x_j\right]$**與**$\frac{1}{2}MRS_{s^*f}\sum_{k=1}^{2}t_{ks}\frac{\partial x_k}{\partial q_j}$**。接續本文先前的假設（即**$x_j,d,t_{js},t_{jf},MU_f,MU_s$**均大於零），此時只要本地公共財對消費者所產生的邊際效用（**MU_s**）大於外地公共財所引發的外溢效果（**MU_{s^*}**），就可能會強化水平外部性的力量，抵銷垂直外部性的扭曲效果，可是我們依舊不能由上述推導結果確定中央地方公共財的相對提供水準。**

從數學的推導結果來看，當兩級政府的稅基完全重疊時，加入支出外

部性的做法並不會對中央、地方的租稅政策造成任何扭曲效果，政策制定者仍然依照最適課稅法則來訂定本身的稅率。至於在公共支出水準方面，因為我們假設各地的地方公共財皆具正向外溢效果，所以地方政府往往忽略其所產生的外部利益，以致地方公共財呈現供給不足的狀態。由上可知，當地方公共財具正項外部性，且本地公共財產生的邊際效用大於外地公共財之外溢效果時（$MU_s > MU_{s^*}$），則支出外部性會與租稅競爭帶來同樣的效果，至於中央地方公共財的相對提供水準，仍需視這三種財政外部性的總合效果而定。

第二節　支出外部性與地方稅基受限下之租稅政策

了解稅基完全重疊下支出外部性與租稅外部性的互動過程後，我們接著審視，當地方稅基受限時，兩級政府的財政政策是否會因支出外部性的加入而有所變化。

<u>地方政府的租稅政策</u>：

$$\underset{t_{1i}}{Max} \quad V\left(\underset{-}{q_i}, g_i, g_{i^*}, g_f\right)$$

s.t. $$t_{1i} x_{1i}(\underset{-}{q_i})\left(\frac{1}{2} + \frac{t_{1i^*} - t_{1i}}{d}\right) = g_i$$

$$\sum_{i=1}^{2}\left[t_{1f} x_{1i}(\underset{-}{q_i})\left(\frac{1}{2} + \frac{t_{1i^*} - t_{1i}}{d}\right) + t_{2f} x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}\right)\right] = 2g_f$$

f.o.c.

$$\frac{\partial L_i}{\partial t_{1i}} = \frac{\partial V}{\partial q_{1i}} + MU_s\left[x_{1i}\left(\frac{1}{2} + \frac{t_{1i^*} - t_{1i}}{d}\right) + t_{1i}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2} + \frac{t_{1i^*} - t_{1i}}{d}\right) - \frac{1}{d}t_{1i}x_{1i}\right] + MU_{s^*}\left[\frac{1}{d}t_{ji^*}x_{ji^*}\right]$$

$$+\frac{1}{2}MU_f\left[t_{1f}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)-\frac{1}{d}t_{1f}x_{1i}+\frac{1}{2}t_{2f}\frac{\partial x_{2i}}{\partial q_{1i}}\right]=0 \qquad (42)$$

<u>中央政府的租稅政策</u>：

$$\underset{t_{1f},t_{2f}}{Max} \quad \sum_{i=1}^{2}V(\underset{-}{q_i},g_i,g_{i^*},g_f)$$

s.t. $\quad t_{1i}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)=g_i$

$$\sum_{i=1}^{2}\left[t_{1f}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{2f}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}\right)\right]=2g_f$$

f.o.c.

$$\frac{\partial L_f}{\partial t_{1f}}=\frac{\partial V}{\partial q_{1i}}+MU_s\left[t_{1i}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)\right]+MU_{s^*}\left[t_{1i^*}\frac{\partial x_{1i^*}}{\partial q_{1i^*}}\left(\frac{1}{2}+\frac{t_{1i}-t_{1i^*}}{d}\right)\right]$$

$$+MU_f\left[x_{1i}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{1f}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+\frac{1}{2}t_{2f}\frac{\partial x_{2i}}{\partial q_{1i}}\right]=0 \qquad (43)$$

利用 Roy's Identity、Slutsky Matrix 及 Symmetric Equilibrium 的概念：

$$\Rightarrow\frac{1}{2}\left[\left(MRS_{sf}+MRS_{s^*f}\right)\tilde{t}_{1s}+\tilde{t}_{1f}\right]h_{11}+\frac{1}{2}\tilde{t}_{2f}h_{12}=q_{1f}$$

$$q_{1f}^{'}=\left\{\frac{a}{MU_f}+\left[\left(MRS_{sf}+MRS_{s^*f}\right)t_{1s}+t_{1f}\right]\frac{1}{2}\frac{\partial x_1}{\partial y}+\frac{1}{2}t_{2f}\frac{\partial x_2}{\partial y}-\frac{1}{2}\right\}$$

$$\frac{\partial L_f}{\partial t_{2f}}=\frac{\partial V}{\partial q_{2i}}+MU_s\left[t_{1i}\frac{\partial x_{1i}}{\partial q_{2i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)\right]+MU_{s^*}\left[t_{1i^*}\frac{\partial x_{1i^*}}{\partial q_{2i^*}}\left(\frac{1}{2}+\frac{t_{1i}-t_{1i^*}}{d}\right)\right]$$

$$+MU_f\left[t_{1f}\frac{\partial x_{1i}}{\partial q_{2i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+\frac{1}{2}x_{2i}+\frac{1}{2}t_{2f}\frac{\partial x_{2i}}{\partial q_{2i}}\right]=0 \qquad (44)$$

同理

$$\Rightarrow \frac{1}{2}\left[\left(MRS_{sf}+MRS_{s^*f}\right)\tilde{t}_{1s}+\tilde{t}_{1f}\right]h_{21}+\frac{1}{2}\tilde{t}_{2f}h_{22}=q_{2f}$$

$$q_{2f}^{'}=\left\{\frac{a}{MU_f}+\left[\left(MRS_{sf}+MRS_{s^*f}\right)t_{1s}+t_{1f}\right]\frac{1}{2}\frac{\partial x_1}{\partial y}+\frac{1}{2}t_{2f}\frac{\partial x_2}{\partial y}-\frac{1}{2}\right\}=q_{1f}^{'}=\frac{1}{2}q_f^{'}$$

由中央政府的一階條件，我們可求得此二商品的稅率水準：

$\tilde{t}_{1s}+\tilde{t}_{1f}=t_1^*(q_f^{'})+\left[1-\left(MRS_{sf}+MRS_{s^*f}\right)\right]\tilde{t}_{1s}$**，及**$\tilde{t}_{2f}=t_2^*(q_f^{'})$

加入支出外部性後對於原模型命題二的影響：

同樣地，將中央與地方對於第一種商品的一階條件相減

$$\frac{\partial L_i}{\partial t_{1i}}=\frac{\partial V}{\partial q_{1i}}+MU_s\left[x_{1i}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{1i}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)-\frac{1}{d}t_{1i}x_{1i}\right]+MU_{s^*}\left[\frac{1}{d}t_{ji^*}x_{ji^*}\right]$$

$$+\frac{1}{2}MU_f\left[t_{1f}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)-\frac{1}{d}t_{1f}x_{1i}+\frac{1}{2}t_{2f}\frac{\partial x_{2i}}{\partial q_{1i}}\right]$$

$$=\frac{\partial V}{\partial q_{1i}}+MU_s\left[t_{1i}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)\right]+MU_{s^*}\left[t_{1i^*}\frac{\partial x_{1i^*}}{\partial q_{1i^*}}\left(\frac{1}{2}+\frac{t_{1i}-t_{1i^*}}{d}\right)\right]$$

$$+MU_f\left[x_{1i}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+t_{1f}\frac{\partial x_{1i}}{\partial q_{1i}}\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+\frac{1}{2}t_{2f}\frac{\partial x_{2i}}{\partial q_{1i}}\right]=\frac{\partial L_f}{\partial t_{1f}}$$

∵ Symmetric Equilibrium，所以上式可改寫如下：

$$\frac{1}{2}\left(MU_f-MU_s\right)x_1+MU_{s^*}\left(\frac{1}{2}t_{1s}\frac{\partial x_1}{\partial q_1}-\frac{1}{d}t_{1s}x_1\right)+\left(1-\frac{1}{2}\right)MU_f\left[\frac{1}{2}t_{1f}\frac{\partial x_1}{\partial q_1}-\frac{1}{d}t_{1f}x_1\right.$$

$$\left.+\frac{1}{2}t_{2f}\frac{\partial x_2}{\partial q_1}\right]+\frac{1}{d}MU_s t_{1s}x_1+\frac{1}{d}t_{1f}x_1=0 \qquad (45)$$

(A) 與之前加入租稅競爭假設的結論相同，即使$MRS_{sf} > 1$且$\partial x_{2i} / \partial q_{1i} \leq 0$，中央政府也不一定要對財貨 1 採用補貼政策才能消除地方課稅所引發的財政外部性。

（B）從（45）式，我們無法由中央地方公共財的相對提供情況來判斷地方課稅對於中央稅收所產生的垂直外部性。

根據先前的數學推導與假設，我們知道當地方公共財具有正的外部利益時，支出外部性會強化租稅競爭的效果，使得地方公共財提供不足的情況更為嚴重。因此，接續著之前的結論，當租稅外部性與支出外部性並存時，中央地方資源分配即使未達最適水準，兩級政府的租稅政策還是有可能達成最適課稅的條件。所以，在地方稅基受限的情形下，Hoyt 認為稅率與支出水準同時遭受扭曲的說法不一定會成立。

第三節　支出外部性與移轉制度之設計

區域間租稅競爭並未對中央政府的補助制度產生明顯之變革，但當地方公共財具有外溢效果時，有可能影響消費者的效用水準，進而衝擊到補助制度的設計方式。故本文接下來即以支出外部性對補助制度所造成的影響作為討論之重點。

<u>中央政府的租稅政策</u>：

$$\underset{t_{1f}, t_{2f}, t_{3f}, m_1, m_2}{Max} \quad \sum_{i=1}^{2} V(\underline{q}_i, g_i, g_{i^*}, g_f)$$

s.t.
$$\sum_{i=1}^{2} \left[(t_{1f} - t_{1i} m_1) x_{1i}(\underline{q}_i) \left(\frac{1}{2} + \frac{t_{1i^*} - t_{1i}}{d} \right) + (t_{2f} - t_{2i} m_2) x_{2i}(\underline{q}_i) \left(\frac{1}{2} + \frac{t_{2i^*} - t_{2i}}{d} \right) \right.$$

$$+ t_{3f} x_{3i}(\underset{-}{q_i})(\frac{1}{2}) - S_i - g_f \Bigg] = 0$$

$$(1+m_1)t_{1i}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+(1+m_2)t_{2i}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)+S_i = g_i$$

f.o.c.

$$\frac{\partial L_f}{\partial t_{jf}} = \frac{\partial V}{\partial q_{ji}} + MU_s\left[\sum_{k=1}^{2}(1+m_k)t_{ki}\frac{\partial x_{ki}}{\partial q_{ji}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\right] + MU_{s^*}\left[\sum_{k=1}^{2}(1+m_k)t_{ki^*}\frac{\partial x_{ki^*}}{\partial q_{ji^*}}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\right]$$

$$+ MU_f\left[\sum_{k=1}^{2}(t_{kf}-m_k t_{ki})\frac{\partial x_{ki}}{\partial q_{ji}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)+x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)+\frac{1}{2}t_{3f}\frac{\partial x_{3i}}{\partial q_{ji}}\right] = 0$$

$$j = 1,2 \qquad (46)$$

$$\frac{\partial L_f}{\partial t_{3f}} = \frac{\partial V}{\partial q_{3i}} + MU_s\left[\sum_{k=1}^{2}(1+m_k)t_{ki}\frac{\partial x_{ki}}{\partial q_{3i}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\right] + MU_{s^*}\left[\sum_{k=1}^{2}(1+m_k)t_{ki^*}\frac{\partial x_{ki^*}}{\partial q_{3i^*}}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\right]$$

$$+ MU_f\left[\sum_{k=1}^{2}(t_{kf}-m_k t_{ki})\frac{\partial x_{ki}}{\partial q_{3i}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)+\frac{1}{2}x_{3i}+\frac{1}{2}t_{3f}\frac{\partial x_{3i}}{\partial q_{3i}}\right] = 0 \qquad (47)$$

$$\frac{\partial L_f}{\partial S} = MU_s + MU_{s*} - MU_f = 0 \qquad (48)$$

$$\frac{\partial L_f}{\partial m_j} = MU_s\left[t_{ji}x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)\right] + MU_{s*}\left[t_{ji^*}x_{ji^*}\left(\frac{1}{2}+\frac{t_{ji}-t_{ji^*}}{d}\right)\right] - MU_f\left[t_{ji}x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)\right] = 0$$

利用 Symmetric Equilibrium

$$\Rightarrow MU_s + MU_{s*} = MU_f \qquad (49)$$

同理，利用中央政府租稅政策的一階條件，並套用 Symmetric Equilibrium 的概念，中央的一階條件可改寫如下：

$$\Rightarrow \frac{1}{2}\left[\tilde{t}_{1f} + \tilde{t}_{1s} + (MRS_{sf} + MRS_{s*f} - 1)(1+m_1)\tilde{t}_{1s}\right]h_{j1}$$

$$+ \frac{1}{2}\left[\tilde{t}_{2f} + \tilde{t}_{2s} + (MRS_{sf} + MRS_{s*f} - 1)(1+m_2)\tilde{t}_{2s}\right]h_{j2} + \frac{1}{2}\tilde{t}_{3f}\, h_{j3} = q_{jf}$$

$$q_{jf}=\left\{\frac{a}{MU_f}+\sum_{k=1}^{2}\left[(MRS_{sf}+MRS_{s^*f})(1+m_k)t_{ks}+(t_{kf}-m_k t_{ks})\right]\left(\frac{1}{2}\right)\frac{\partial x_k}{\partial y}-\frac{1}{2}+\frac{1}{2}t_{jf}\frac{\partial x_j}{\partial y}\right\}$$

$$j=1,2,3$$

<u>地方政府的租稅政策</u>：

$$\underset{t_{1i},t_{2i}}{Max}\quad V\left(\underset{-}{q_i},g_i,g_{i^*},g_f\right)$$

s.t. $$\frac{1}{2}\sum_{i=1}^{2}\left[(t_{1f}-t_{1i}m_1)x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+(t_{2f}-t_{2i}m_2)x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)\right.$$

$$\left.+t_{3f}x_{3i}(\underset{-}{q_i})(\frac{1}{2})-S_i\right]=g_f$$

$$(1+m_1)t_{1i}x_{1i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{1i^*}-t_{1i}}{d}\right)+(1+m_2)t_{2i}x_{2i}(\underset{-}{q_i})\left(\frac{1}{2}+\frac{t_{2i^*}-t_{2i}}{d}\right)+S_i=g_i$$

f.o.c.

$$\frac{\partial L_s}{\partial t_{ji}}=\frac{\partial V}{\partial q_{ji}}+MU_s\left[\sum_{k=1}^{2}(1+m_k)t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}+(1+m_j)\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)x_{ji}-\frac{1}{d}(1+m_j)t_{ji}x_{ji}\right]$$

$$+MU_{s^*}\left[(1+m_j)t_{ji^*}x_{1i^*}\frac{1}{d}\right]+\frac{1}{2}MU_f\left[\sum_{k=1}^{2}(t_{kf}-m_k t_{ki})\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}-m_j x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)\right.$$

$$\left.-(t_{jf}-t_{ji}m_j)\frac{1}{d}+\frac{1}{2}t_{3f}\frac{\partial x_{3i}}{\partial q_{ji}}\right]=0\qquad j=1,2\qquad\textbf{(50)}$$

對原模型命題三的影響：

因為假設地方公共財具有正的外溢效果（即$MU_{s^*}>0$），而根據（48）與（49）兩式$MU_s+MU_{s^*}=MU_f$，則$MU_s\neq MU_f$（若兩者相等，$MRS_{sf}=1$，

那麼$MU_{s*}=0$**與假設不合），再將中央與地方的一階條件相減可得**

$$\frac{\partial L_f}{\partial t_{jf}}=\frac{\partial V}{\partial q_{ji}}+MU_s\left[\sum_{k=1}^{2}(1+m_k)t_{ki}\frac{\partial x_{ki}}{\partial q_{ji}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\right]+MU_{s*}\left[\sum_{k=1}^{2}(1+m_k)t_{ki^*}\frac{\partial x_{ki^*}}{\partial q_{ji^*}}\left(\frac{1}{2}+\frac{t_{ki}-t_{ki^*}}{d}\right)\right]$$

$$+MU_f\left[\sum_{k=1}^{2}(t_{kf}-m_kt_{ki})\frac{\partial x_{ki}}{\partial q_{ji}}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)+x_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)+\frac{1}{2}t_{3f}\frac{\partial x_{3i}}{\partial q_{ji}}\right]$$

$$=\frac{\partial V}{\partial q_{ji}}+MU_s\left[\sum_{k=1}^{2}(1+m_k)t_{ki}\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}+(1+m_j)\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)x_{ji}-\frac{1}{d}(1+m_j)t_{ji}x_{ji}\right]$$

$$+MU_{s*}\left[(1+m_j)t_{ji^*}x_{1i^*}\frac{1}{d}\right]+\frac{1}{2}MU_f\left[\sum_{k=1}^{2}(t_{kf}-m_kt_{ki})\left(\frac{1}{2}+\frac{t_{ki^*}-t_{ki}}{d}\right)\frac{\partial x_{ki}}{\partial q_{ji}}-m_jx_{ji}\left(\frac{1}{2}+\frac{t_{ji^*}-t_{ji}}{d}\right)\right.$$

$$\left.-\left(t_{jf}-t_{ji}m_j\right)\frac{1}{d}+\frac{1}{2}t_{3f}\frac{\partial x_{3i}}{\partial q_{ji}}\right]=\frac{\partial L_s}{\partial t_{ji}}\qquad j=1,2$$

$\because$ Symmetric Equilibrium**，故上式可化簡為**

$$\left(1-\frac{1}{2}\right)MU_f\left\{\left[\sum_{k=1}^{2}\frac{1}{2}(t_{kf}-m_kt_{ks})\frac{\partial x_k}{\partial q_j}-\frac{1}{2}m_jx_j-\frac{1}{d}(t_{jf}-m_jt_{js})x_j+\frac{1}{2}t_{3f}\frac{\partial x_3}{\partial q_j}\right]\right.$$

$$\left.+x_j+m_jx_j+\frac{2}{d}(t_{jf}-m_jt_{js})x_j\right\}-MU_s\left[(1+m_j)\frac{1}{2}x_j-\frac{1}{d}(1+m_j)t_{js}x_j\right]+MU_{s^*}\left[\sum_{k=1}^{2}\frac{1}{2}(1+m_k)t_{ks}\frac{\partial x_k}{\partial q_j}\right.$$

$$\left.-\frac{1}{d}(1+m_j)t_{js}x_j\right]=0\qquad j=1,2\qquad\textbf{(51)}$$

(A) 補助制度的條件轉變為$MU_s+MU_{s*}=MU_f$**，沒有任何一個地方政府會產生財政外部性的說法亦非最適補助制度之必要條件。**

(B) 在$MU_s+MU_{s*}=MU_f$**的條件下，中央與地方的混何稅率與單一政府的稅率水準相同，並符合最適課稅的條件。**

與前面兩個模型相比較，納入支出外部性後使得補助制度的條件產生著重大轉變，由原本的$MU_s = MU_f$變成$MU_s + MU_{s*} = MU_f$。這主要的原因就是消費者的效用函數受到了影響，本地與外地的公共財皆為其帶來了正的邊際效用，且兩者的性質有所相同。此時消費者的效用係同時來自兩個地方所提供之公共支出，故當中央考量兩級政府的資源配置時，會將其他地方政府的外溢效果一併考慮進來，而最適補助制度的條件也就隨之轉變成$MU_s + MU_{s*} = MU_f$之情形。

至於 Hoyt 提及地方政府的租稅政策不會對中央產生任何財政外部性亦為最適補助制度的必要條件，在加入租稅競爭與支出外部性的因素可能不再成立，因為地方課稅所引起的垂直外部性最終為租稅競爭與支出外部性兩者的總效果所消除。